MATH4426
Robert Gross
Homework 5
Due October 16, 2020

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EDT on October 16. Please name your file hw05-lastname-firstname.pdf. My solution file is hw05-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. Suppose that Team A and Team B play games against each other repeatedly until one team has won k games. Suppose that team A wins each game with probability p, and team B wins each game with probability $1-p$, and that each game is an independent event. Let X be the total number of games played.
(a) Compute $E[X]$ when $k=2$. Your answer will depend on p.
(b) Compute $E[X]$ when $k=3$. Your answer will depend on p.
(c) Show that your answers to (a) and (b) are maximized when $p=0.5$.

2. Define a random variable X with the formula

$$P\{X=k\}=\log_{10}\left(\frac{k+1}{k}\right), \qquad k=1,2,3,\ldots,9$$

(a) Show that this is indeed a probability mass function by showing that $\sum_k P\{X=k\}=1$.
(b) Compute $E[X]$.

3. Suppose that X has the following probability mass function:

$$p(0)=\tfrac{1}{3} \qquad p(\pm 1)=\tfrac{13}{55} \qquad p(\pm 2)=\tfrac{1}{11} \qquad p(\pm 3)=\tfrac{1}{165}$$

Compute $P\{X=k|X>0\}$ for $k=1,2,3$.

4. Suppose that E, F, and G are events. Show that

$$\frac{P(F|E)}{P(G|E)}=\frac{P(F)P(E|F)}{P(G)P(E|G)}.$$

You may assume that all of the probabilities in this expression are non-zero.

5. An urn contains 12 balls: 8 black, 4 white. Two players, A and B, take turns drawing one ball from the urn and replacing it until one of them gets a white ball. That player is the winner. What is the probability that B is the winner?

6. An urn contains 20 black and 10 white balls. Balls are removed one at a time randomly without replacement. What is the probability that all of the black balls are removed before all of the white balls?

7. Suppose that E and F are events. Show that the probability that precisely one of the two events occurs is $P(E)+P(F)-2P(EF)$.