MATH4426
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Homework 7
Due October 30, 2020

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EDT on October 30. Please name your file hw07-lastname-firstname.pdf. My solution file is hw07-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. A newsboy purchases newspapers for \$1.50 and sells them for \$2.00. He may not return unsold newspapers. Suppose that the daily demand is a binomial random variable, $(10, 0.4)$. How many newspapers should he buy daily to maximize his expected profit? *Note*: You might find it helpful to use a spreadsheet to simplify some of your calculations.

2. Suppose that the probability density function for a random variable X is given by

$$f(x) = \begin{cases} \dfrac{C}{x^2} & x > 20 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is C?
(b) Let $F(x)$ be the cumulative distribution function for X. What is $F(x)$? Be sure to make sure to define the function for all real numbers x.
(c) What is $P\{X > 35\}$?

3. Suppose that the number of arrivals in an emergency room every hour is a Poisson random variable with $\lambda = 2.8$.
(a) Find the probability that at least 3 people appear in the next hour.
(b) Suppose that at least 1 person showed up last hour. Find the probability that at least 3 people showed up last hour.

4. Suppose that X is a continuous random variable with probability density function

$$f(x) = \begin{cases} c(x^3 - 1) & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is c?
(b) What is the cumulative distribution function for X?

5. Let Y be a random variable. Show that

$$E[Y] = \int_0^\infty P(Y > y)\,dy - \int_0^\infty P(Y < -y)\,dy$$

by showing that

$$\int_0^\infty P(Y > y)\,dy = \int_0^\infty x f_Y(x)\,dx$$
$$\int_0^\infty P(Y < -y)\,dy = -\int_{-\infty}^0 x f_Y(x)\,dx.$$

In these formulæ, $f_Y(x)$ is the probability density function for Y.

6. If X is a random variable with density function $f(x)$, show that

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx.$$

Hint: The previous exercise shows that

$$E[g(X)] = \int_0^{\infty} P(g(X) > y)\,dy - \int_0^{\infty} P(g(X) < -y)\,dy.$$

Now proceed as we did in class.

7. This problem gives an alternate proof of the formula $E[X+Y] = E[X] + E[Y]$ when X and Y are integer-valued random variables. You therefore may not use that formula when doing this problem. All sums below are over all integers—positive, negative, and 0.

(a) Let k be an integer. Show that

$$P\{X+Y=k\} = \sum_j P\{X=j, Y=k-j\}.$$

(b) Show that

$$E[X+Y] = \sum_k \sum_j kP\{X=j, Y=k-j\}.$$

(c) Show that

$$E[X+Y] = \sum_n \sum_m (m+n)P\{X=m, Y=n\}.$$

Hint: Make a change of variables in the formula in part (b).

(d) Show that

$$P\{X=m\} = \sum_n P\{X=m, Y=n\}$$

$$P\{Y=n\} = \sum_m P\{X=m, Y=n\}$$

(e) Put all of this together to conclude that $E[X+Y] = E[X] + E[Y]$.