MATH4426
Robert Gross
Homework 9
Due November 20, 2020

Your answers must be in the form of a typed PDF file, and must be e-mailed to me by 5PM EST on November 20. Please name your file hw09-lastname-firstname.pdf. My solution file is hw09-gross-robert.pdf.

I will try to acknowledge receipt of each e-mail.

1. Suppose that

$$f(x,y) = \begin{cases} c\left(x^2 + \frac{xy}{3}\right) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

is a joint probability density function for random variables X and Y.
(a) Compute the value of c.
(b) What is the marginal density $f_X(x)$?
(c) What is the marginal density $f_Y(y)$?
(d) Are X and Y independent?
(e) What is $E[X]$?
(f) What is $E[Y]$?
(g) What is $P\{X > 0.5 | Y < 1\}$?

2. Suppose that X and Y are independent exponential random variables with parameters λ and μ respectively.
(a) Let $Z = X/Y$. What is the density function $f_Z(z)$?
(b) What is $P\{X < Y\}$?
In both cases, your answers should be given in terms of λ and μ.

3. If θ is any real number, the *Cauchy distribution* with parameter θ has density function

$$f(x) = \frac{1}{\pi(1 + (x - \theta)^2)}.$$

Suppose that X and Y are independent standard normal random variables. Let $U = X$ and $V = X/Y$.
(a) Compute the joint density function $f_{U,V}(u, v)$.
(b) Show that the marginal density function $f_V(v)$ has a Cauchy distribution.

4. Suppose that X and Y are independent normal random variables. Suppose that X has mean 10 and variance 1 and Y has mean 10 and variance 4.
(a) What is $P\{X > 11\}$
(b) What is $P\{Y > 11\}$
(c) What is $P\{X + Y > 22\}$?

5. Suppose that X and Y are discrete random variables, each taking the values 1, 2, and 3, with this mass function:

	$p(x,y)$	$y = 1$	$y = 2$	$y = 3$
x	1	0.08	0.06	0.10
	2	0.11	0.13	0.06
	3	0.16	0.20	0.10

(a) What is $E[X]$?
(b) What is $E[Y]$?
(c) What is $\text{Var}(X)$?
(d) What is $\text{Var}(Y)$?
(e) What is $\text{Cov}(X, Y)$?
(f) What is $\rho(X, Y)$, the correlation between X and Y?

6. Suppose that X and Y are integer-valued random variables. We define

$$p(i,j) = P\{X = i, Y = j\}$$
$$p(i|j) = P\{X = i|Y = j\}$$
$$q(j|i) = P\{Y = j|X = i\}$$

Show that

$$p(i,j) = \frac{p(i|j)}{\sum_i \frac{p(i|j)}{q(j|i)}}.$$