Flexible Top Trading Cycles and Chains Mechanism: Maintaining Diversity in Erasmus Student Exchange

Umut Dur   Onur Kesten   Utku Ünver

Econometric Society World Congress’15
New Market-Old Mechanism

Analyze the student exchange program in Europe.

Propose a variant of the Top Trading Cycles and Chains mechanism to address the observed problems.
New Market-Old Mechanism

Analyze the student exchange program in Europe
New Market-Old Mechanism

Analyze the student exchange program in Europe

Propose a variant of the Top Trading Cycles and Chains mechanism to fix the observed problems
Student Exchange in Europe

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  - More than 4,000 member colleges from 33 countries in Europe.
  - Each year, around 300,000 students benefit from it.
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  - Each year, around 300,000 students benefit from it.

The purpose of Erasmus is to improve the quality of higher education and strengthen its European dimension. It does this by encouraging transnational cooperation between universities.

European Comission’s Website
How Successful is the Program?

- Less than 1.5% of the college students benefit from the program
How Successful is the Program?

- Lack of diversity in exchanges:
  - Western European countries exchanging students between each other
  - Eastern European countries exchanging students between each other
How Successful is the Program?

## Difference between the number of incoming & outgoing exchange students

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- Since exchange students pay tuition to their **home colleges**, people in countries with more incoming students are not happy with the program.
How Successful is the Program?

Although participation in Erasmus and other European programmes would continue, the imbalances in student flows would remain and probably increase, perhaps intensifying the simmering resentment that UK universities are losing more than they are gaining from these programmes.

timeshighereducation.co.uk

Danish taxpayers should not have to pay for educating exchange students in Denmark.

universitypost.dk
How Successful is the Program?

Under the current practice

- **Low participation**: Less than 1.5% of the students benefiting
- **Lack of diversity in exchanges**: Western Europe vs. Eastern Europe
- **Huge imbalance** between the number of incoming and outgoing students
In This Paper

- In this paper our objectives are:
  - Increase the number of exchanges
  - Increase the level of diversity
  - Maintain a reasonable level of difference between the # of incoming and outgoing students

We introduce TTCC mechanism under various constraints such as College specific exchange quota, Student type specific minimum quota.

To our knowledge, first application of TTCC in many to one matching markets.

We show that TTCC performs great in terms of fairness, efficiency, strategy-proofness, achieving distributional goals.
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- We show that TTCC performs great in terms of
  - fairness
  - efficiency
  - strategy-proofness
  - achieving distributional goals
Outline

- Current Procedure
- Model
- Generalized Top Trading Cycles and Chains
- Minimum Quota
How Does It Work?

- Currently, in Erasmus programme the exchanges are done through 3 step procedure
  - Bilateral Agreement
  - Application
  - Assignment
How Does It Work?

- **Bilateral Agreement**
  - Each member college signs bilateral agreement with other members
  - Colleges agree on the maximum number of students to be exchanged
  - Bilateral agreements give reserved seats to the colleges at the other ones
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  - Each student applies to his home college
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- **Assignment**
  - Each college ranks its own applicants according to GPA, written exam, etc...
  - Each college assigns its own students to the reserved seats via Serial Dictatorship mechanism
How Does It Work?

- Bilateral agreement stage is crucial
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Once a college signs bilateral contracts it loses its control over the number of incoming students
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- Once a college signs bilateral contracts it loses its control over the number of incoming students
- Moreover, a college cannot force its own students to participate in the program
- A college may have more incoming students than outgoing and this causes
  - Increase in class size
  - Need more instructors
  - Financial burden: Less tuition for more students
What the colleges do to prevent imbalances?

- They limit the number of students to be exchanged
- They do not sign bilateral agreement with some colleges
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Limiting the number of students to be exchanged: Low participation rate
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- They limit the number of students to be exchanged
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Limiting the number of students to be exchanged: **Low participation rate**

Not signing bilateral agreement with some colleges: **Lack of diversity**
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A student exchange problem consists of

- a set of colleges $C = \{c_1, \ldots, c_m\}$,
- a set of students $S = \bigcup_{c \in C} S_c$,
- an aggregate quota vector $Q = (Q_c)_{c \in C}$,
- a list of student preferences $P = (P_s)_{s \in S}$,
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  - \( q_{c,d} \) : max number of college \( d \) students that college \( c \) would like to import
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- a list of college **internal priority order** over own students
  \( \succ = (\succ_c)_{c \in C} \).
A (feasible) matching is a function $\mu : S \rightarrow C \cup \emptyset$ such that for any $c, d \in C$

- $|\mu^{-1}(c) \cap S_d| \leq q_{c,d}$.
- $|\mu^{-1}(c)| \leq Q_c$ and
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A matching \( \mu \) is **fair** if there does not exist a student pair \((s, s')\) such that (1) \( \{s, s'\} \subseteq S_c \) (2) \( s \succ_c s' \) and (3) \( \mu(s')P_s \mu(s) \).
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A matching is Pareto efficient if there does not exist another matching such that all students students are (weakly) better-off.

A mechanism is strategy-proof if students cannot benefit from misreporting their preferences.
Outline

- Current Procedure
- Model
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- Minimum Quota
Top Trading Cycles and Chains introduced by Roth, Sonmez and Unver (2004) for kidney exchange

- A **cycle** is a list of colleges and students \((c_1, s_1, c_2, s_2, ..., c_k, s_k)\) each agent points to the adjacent agent and the last student points to the first college.

- A **chain** is a list of colleges and students \((c_1, s_1, c_2, s_2, ..., s_{k-1}, c_k)\) each agent points to the adjacent agent.
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- Next, we define the generalized version of TTCC for student exchange programs
Step 0: Assign a tie-breaking number to each college
**gTTCC**

- **Step 0:** Assign a tie-breaking number to each college
- **Step 1:** Each college points to its own student with the highest internal priority
Step 0: Assign a tie-breaking number to each college

Step 1: Each college points to its own student with the highest internal priority

Each pointed student points to her favorite college which has
- available positions for students from her home college,
- available positions for all students, and
- imbalance below its tolerance level or remaining students.

If there is a cycle: Execute each cycle. Remove assigned students. Update the quotas. Return to Step 1. Otherwise proceed to Step 2.

Step 2: If no student left, done. Otherwise, select the chain starting with the college with the best tie-breaking number and execute it. Update the quotas. Remove assigned students. Return to Step 1.
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- $\emptyset$ points to the students pointing itself.

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Update the quotas. Remove assigned students. Return to Step 1.
Properties of gTTCC

Theorem

gTTCC is fair, Pareto efficient and strategy-proof.
Properties of gTTCC

Theorem

gTTCC is fair, Pareto efficient and strategy-proof.

- Independent of the quotas set, each college has
  - a balance no more than its tolerable level
  - incoming students no more than its aggregate quota

- No need to set lower quotas $\implies$ More exchanges compared to the current procedure
Simulations

By using simulations, we compare current procedure and gTTCC under various scenarios.

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<th>Exchange under gTTCC</th>
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$\alpha$ is the degree of correlation in preferences.
Simulations

By using simulations, we compare current procedure and gTTCC under various scenarios

- Fraction of exchanged students is more under gTTCC
- More students prefer their assignment under gTTCC

\[ \alpha \] is the degree of correlation in preferences
Outline

- Current Procedure
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- Generalized Top Trading Cycles and Chains
- Minimum Quota
Minimum Quota

- We observe lack of diversity in exchanges
We observe lack of diversity in exchanges

In order to increase the diversity (having incoming students from more countries) in the exchange we add minimum quota requirement to our model

- $m_{c,d}$: is the minimum number of college $d$ students to be assigned to college $c$. 
Minimum Quota

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  - $m_{c,d}$: is the minimum number of college $d$ students to be assigned to college $c$.

**Assumption:** If $m_{c,d} > 0$, then each student from college $d$ prefers college $c$ to $\emptyset$
Adding one more step to gTTCC

- We modify gTTCC by adding an initial step in which before students point, we check whether
  - there is enough students left from each college,
  - there is enough available seats left in each college

to satisfy all minimum quota requirement.
Extensions

*gTTCC with Minimum Quota*

**Theorem**

*gTTCC with minimum quota* is fair, strategy-proof and its outcome cannot be Pareto dominated by another matching satisfying minimum quota requirement.
Tie breaking: C-D-B-A

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Tie breaking: C-D-B-A
Other Extensions

- gTTCC mechanism can deal with many other constraints without losing its desirable properties
  - Limit on the number of imports from a specific country/region
  - Limit on the number of imports from a specific field (engineering, management)
  - Limit on the number of exports
  - Tolerable Balance between incoming and outgoing students within each field
Consider a matching market which has not been studied to our knowledge

Use TTCC mechanism in a many to one matching problem
  - Fair (across to same types)
  - Strategy-proof
  - Efficient
  - Welfare gains compared to the Current System without any cost
Back Up Slides

- Simulation Setup
- Example for Extensions
- gTTCC with diversity
Simulations

- 10 Colleges and 100 Students. Each college has 10 students.
- Bilateral quotas: drawn from the discrete uniform distribution on the interval [0, 4]
- Tolerance: drawn from the discrete uniform distribution on the interval [2, 10]
- Aggregate quota: drawn from the discrete uniform distribution on the interval [Tolerance, 10]
- Student preferences: $U_{i,c} = \alpha \times Z_c + (1 - \alpha) \times Z_{i,c}$
  - $Z_c$ is an i.i.d standard uniformly distributed random variable and represents the common tastes of students on college $c$.
  - $Z_{i,c}$ is also an i.i.d standard uniformly distributed random variable and represents the tastes of student $i$ on college $c$.
  - The correlation in the students preferences is captured by $\alpha \in [0, 1]$. 
Simulations

- \( g^{TTCC} \) is calculated based on the true quotas and preferences.
- In order to calculate the current system outcome we randomly create a bilateral quota vector, \((\tilde{q}_{c,c'})_{c' \in C}\). For each college \(c\) such that \[\sum \tilde{q}_{c,c'} = b_c\]
Other Extensions
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Other Extensions

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
 & $a_1$ & $a_2$ & $a_3$ & $b_1$ & $b^m$ & $c_1$ & $c^m$ \\
\hline
A & 0 & 1 & 1 & 1 & 2 & 0 \\
B & 0 & 1 & 0 & 1 & 0 & 2 \\
C & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
\end{tabular}
\end{table}
Other Extensions

\[\begin{array}{cccc|cccc}
a_1^e & a_2^e & a_3^e & b_1^m & c_1^m & q_{..A} & q_{..B} & q_{..C} \\
B & C & C & A & A & - & 0 & 1 \\
C & B & B & \emptyset & \emptyset & 0 & - & 1 \\
C & 1 & 1 & - & 1 & 0 & 1 & 1
\end{array}\]
Other Extensions

\[ \begin{array}{c|c|c|c|c|c|c|c} 
 a^e_1 & a^e_2 & a^m_3 & b^m_1 & c^m_1 & q_{A,1} & q_{B,1} & q_{C,1} \\
 B & C & C & A & A & - & 0 & 1 \\
 C & B & B & & & 0 & 1 & 0 \\
 & & & 1 & 1 & & & 1 \\
 \end{array} \]
Adding one more step to gTTCC

- We modify gTTCC by adding an initial step in which before students point, we check whether
  - there is **enough students** left from each college,
  - there is **enough available** seats left in each college

To satisfy all **minimum quota** requirement.
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  - If # of remaining $c$ students is equal to # of unfilled reserved seats for $c$, then restrict all $c$ students to point to the colleges with unfilled reserved seats for $c$
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To satisfy all **minimum quota** requirement.

In particular,

- If # of remaining $c$ students is equal to # of unfilled reserved seats for $c$, then restrict all $c$ students to point to the colleges with unfilled reserved seats for $c$

- If # of seats left in $c$ is equal to # of unfilled reserved seats at $c$ then $c$ can be pointed by the students from colleges which have not satisfied the minimum quota requirement