Flexible Top Trading Cycles and Chains Mechanism: Maintaining Diversity in Erasmus Student Exchange

Umut Dur   Onur Kesten   Utku Ünver

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New Market-Old Mechanism

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Analyze the student exchange program in Europe

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Propose a variant of the Top Trading Cycles and Chains mechanism to fix the observed problems

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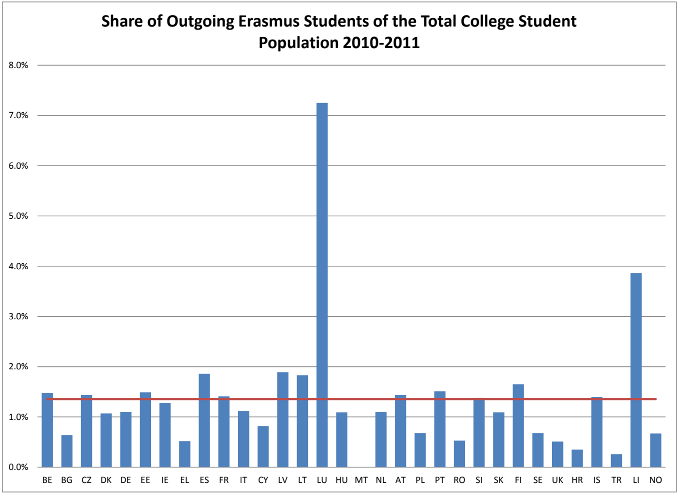
> *The purpose of Erasmus is to improve the quality of higher education and strengthen its European dimension. It does this by encouraging transnational cooperation between universities.*
>
> *European Comission's Website*

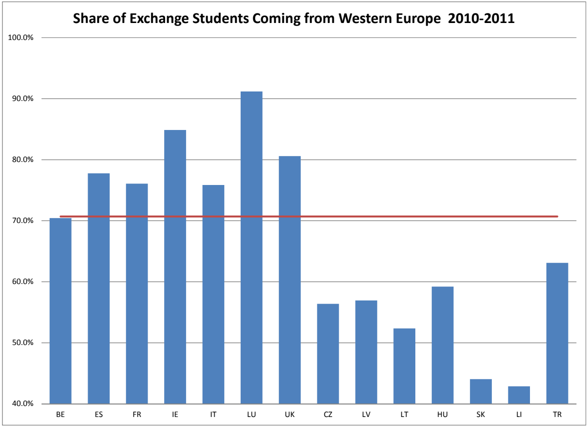
How Successful is the Program?

- Less than 1.5% of the college students benefit from the program



How Successful is the Program?

- Lack of diversity in exchanges:
  - Western European countries exchanging students between each other
  - Eastern European countries exchanging students between each other



How Successful is the Program?

Difference between the number of incoming & outgoing exchange students

	2004/05	2005/06	2006/07	2007/08	2008/09	2009/10	2010/11	Total
UK	9052	9,264	9,273	8,452	8,636	8,770	8,927	62,374
Sweden	3,928	4,532	4,827	5,403	5,793	6,060	6,348	36,891
Denmark	2,087	2,684	2,958	3,292	3,625	3,934	4,262	22,842
Poland	-6,058	-6,911	-7,489	-7,744	-7,256	-6,079	-4,640	-46,177
Germany	-5,154	-5,959	-6,006	-5,752	-5,685	-6,102	-6,059	-40,717
Turkey	-843	-2,024	-3,117	-4,475	-4,560	-5,117	-5,209	-25,345

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- Since exchange students pay tuition to their **home colleges**, people in countries with more incoming students are not happy with the program

How Successful is the Program?

Although participation in Erasmus and other European programmes would continue, the imbalances in student flows would remain and probably increase, perhaps intensifying the simmering resentment that UK universities are losing more than they are gaining from these programmes.

timeshighereducation.co.uk

Danish taxpayers should not have to pay for educating exchange students in Denmark.

universitypost.dk

How Successful is the Program?

Under the current practice

- **Low participation:** Less than 1.5% of the students benefiting
- **Lack of diversity in exchanges:** Western Europe vs. Eastern Europe
- Huge **imbalance** between the number of incoming and outgoing students

In This Paper

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  - Increase the number of exchanges
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  - College specific exchange quota
  - Student type specific minimum quota
- To our knowledge, first application of TTCC in many to one matching markets
- We show that TTCC performs great in terms of
  - fairness
  - efficiency
  - strategy-proofness
  - achieving distributional goals

Outline

- Current Procedure
- Model
- Generalized Top Trading Cycles and Chains
- Minimum Quota

How Does It Work?

- Currently, in Erasmus programme the exchanges are done through 3 step procedure
  - Bilateral Agreement
  - Application
  - Assignment

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  - Each member college signs bilateral agreement with other members
  - Colleges agree on the maximum number of students to be exchanged
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- Assignment
  - Each college ranks its own applicants according to GPA, written exam, etc...
  - Each college assigns its own students to the reserved seats via Serial Dictatorship mechanism

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- Once a college signs bilateral contracts it loses its control over the number of incoming students
- Moreover, a college cannot force its own students to participate in the program
- A college may have more incoming students than outgoing and this causes
  - Increase in class size
  - Need more instructors
  - Financial burden: Less tuition for more students

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    - They limit the number of students to be exchanged
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- Limiting the number of students to be exchanged: **Low participation rate**
- Not signing bilateral agreement with some colleges: **Lack of diversity**

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A **student exchange problem** consists of

- a set of **colleges** $C = \{c_1, ..., c_m\}$,
- a set of **students** $S = \bigcup_{c \in C} S_c$,
- an **aggregate quota** vector $Q = (Q_c)_{c \in C}$,
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- a list of college **internal priority order over own students** $\succ = (\succ_c)_{c \in C}$.

Definitions

- A **(feasible) matching** is a function $\mu : S \to C \cup \emptyset$ such that for any $c, d \in C$
  - $|\mu^{-1}(c) \cap S_d| \leq q_{c,d}$,
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- A matching is **Pareto efficient** if there does not exist another matching such that all students students are (weakly) better-off
- A mechanism is **strategy-proof** if students cannot benefit from misreporting their preferences.

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gTTCC

Top Trading Cycles and Chains introduced by Roth, Sonmez and Unver (2004) for kidney exchange

- A cycle is a list of colleges and students $(c_1, s_1, c_2, s_2, ..., c_k, s_k)$ each agent points to the adjacent agent and the last student points to the first college
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- Next, we define the generalized version of TTCC for student exchange programs

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Properties of gTTCC

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- Independent of the quotas set, each college has
  - a balance no more than its tolerable level
  - incoming students no more than its aggregate quota
- No need to set lower quotas $\Longrightarrow$ More exchanges compared to the current procedure

Simulations

By using simulations, we compare current procedure and gTTCC under various scenarios

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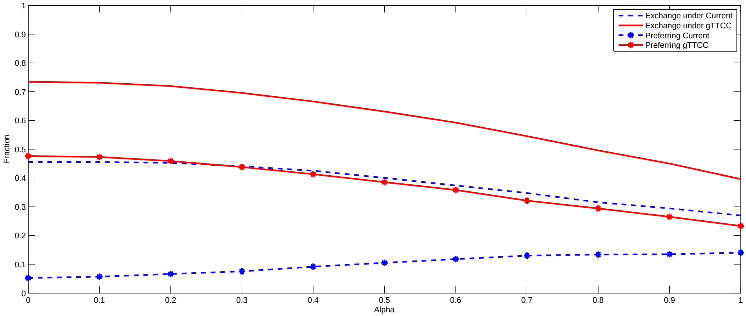
- Fraction of exchanged students is more under gTTCC
- More students prefer their assignment under gTTCC



α is the degree of correlation in preferences

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Assumption: If $m_{c,d} > 0$, then each student from college d prefers college c to $\emptyset$

Adding one more step to gTTCC

- We modify gTTCC by adding an initial step in which before students point, we check whether
    - there is **enough students** left from each college,
    - there is **enough available** seats left in each college

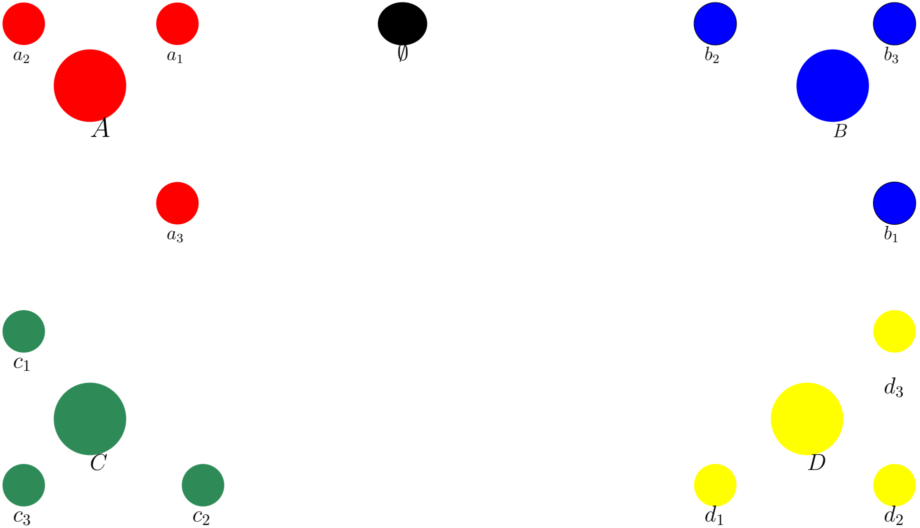
to satisfy all minimum quota requirement.

gTTCC with Minimum Quota

Theorem

gTTCC with minimum quota *is fair, strategy-proof and its outcome cannot be Pareto dominated by another matching satisfying minimum quota requirement.*

Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	2	4	1	–	1	0	0
B	1	–	1	1	3	1	0	–	0	1
C	2	1	–	2	2	0	0	0	–	0
D	1	1	1	–	2	1	0	0	0	–

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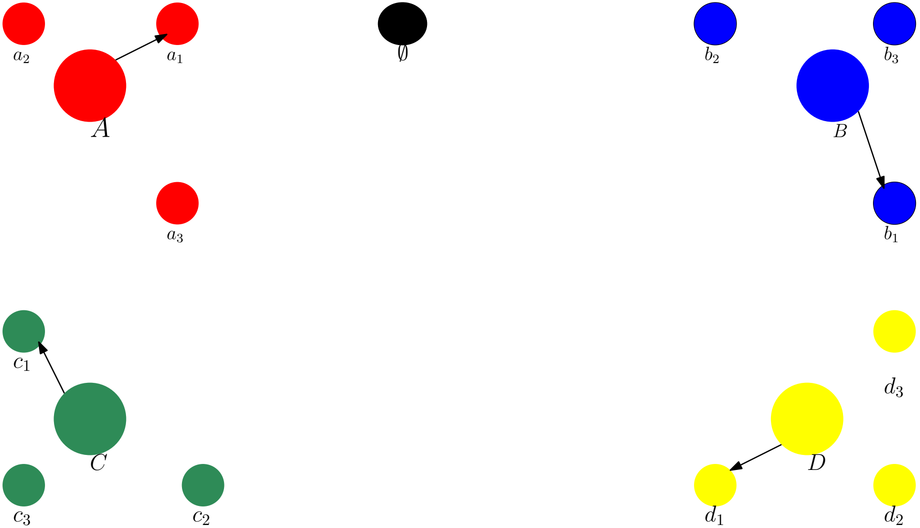

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B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

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B	1	–	1	1	3	1	0	–	0	1
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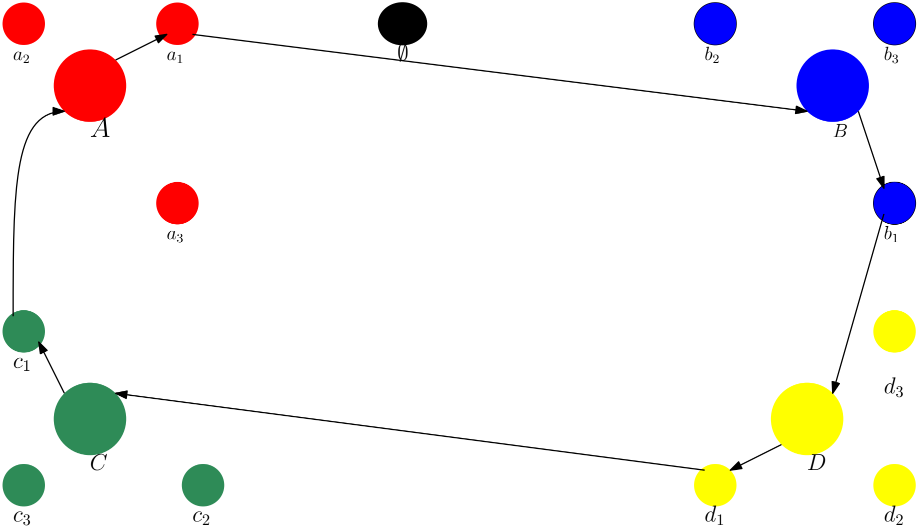

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B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
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$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

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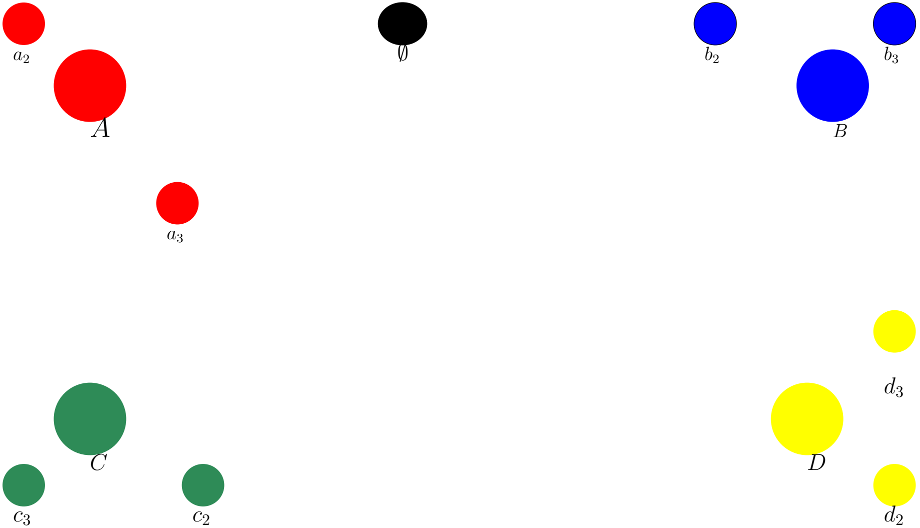

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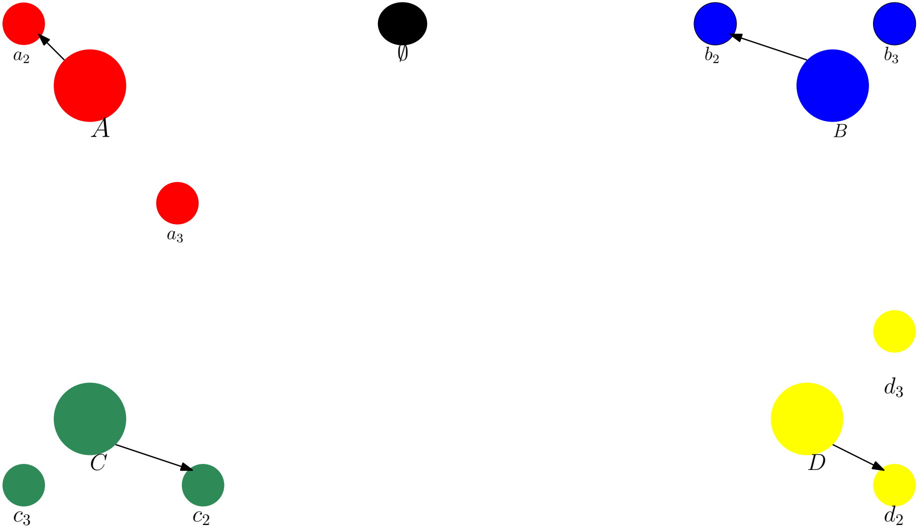

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D	1	0	1	–	1	1	0	0	0	–

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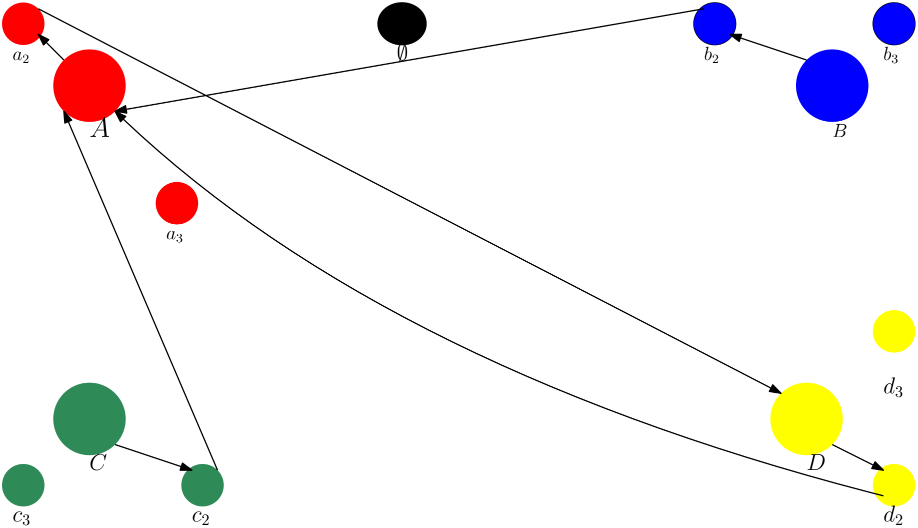

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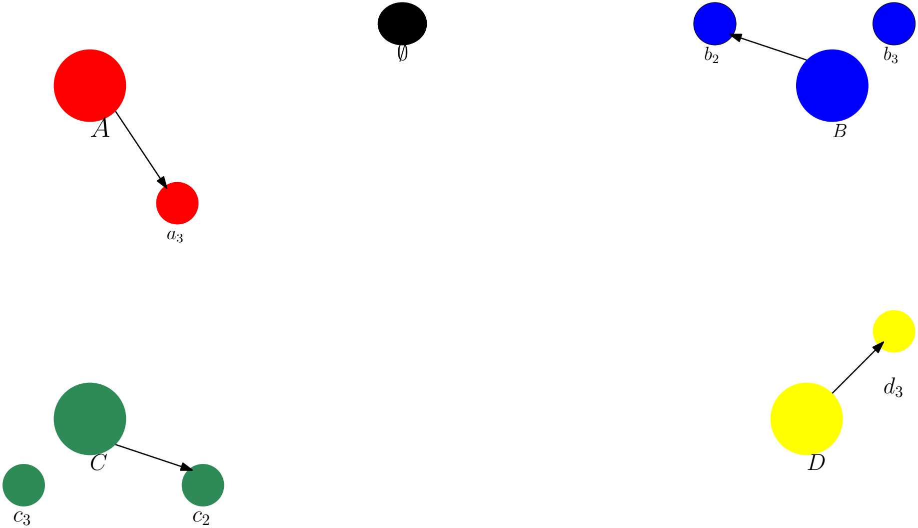
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	1	0	1	–	1	1	0	0	0	–

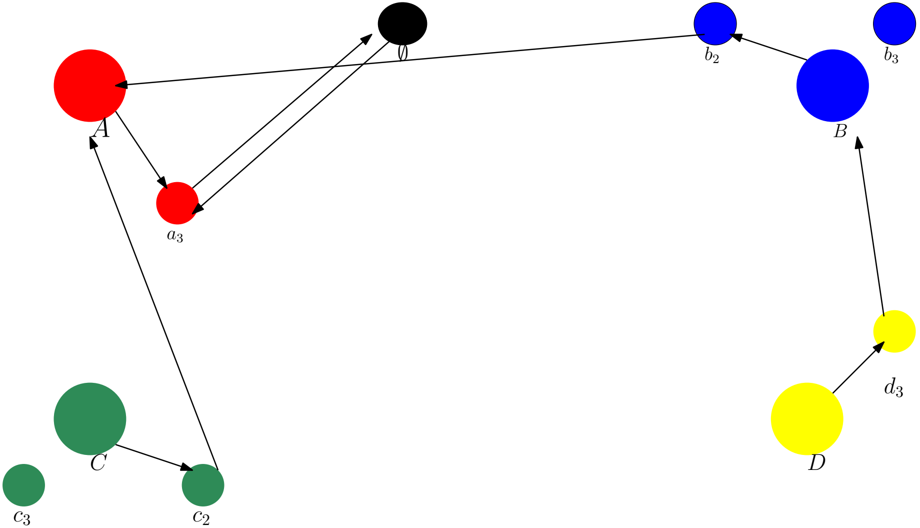
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	1	2	1	–	1	0	0
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

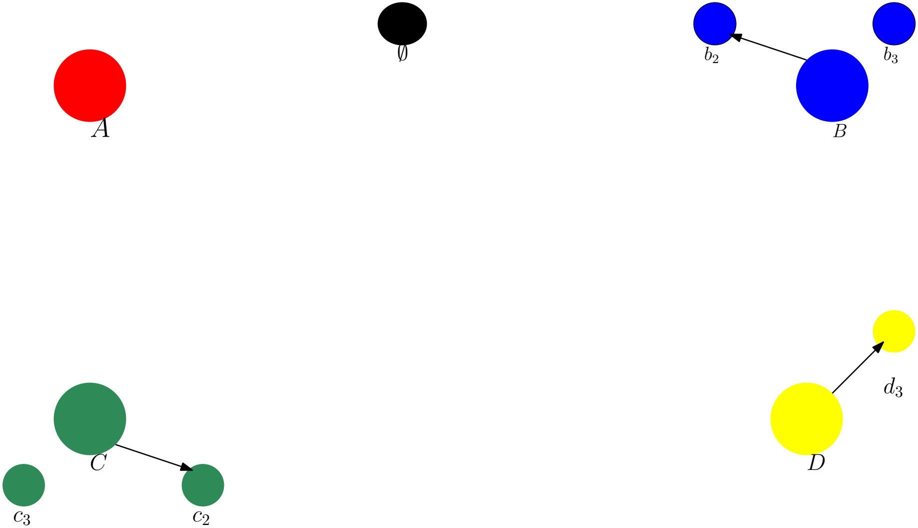
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	1	2	1	–	1	0	0
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

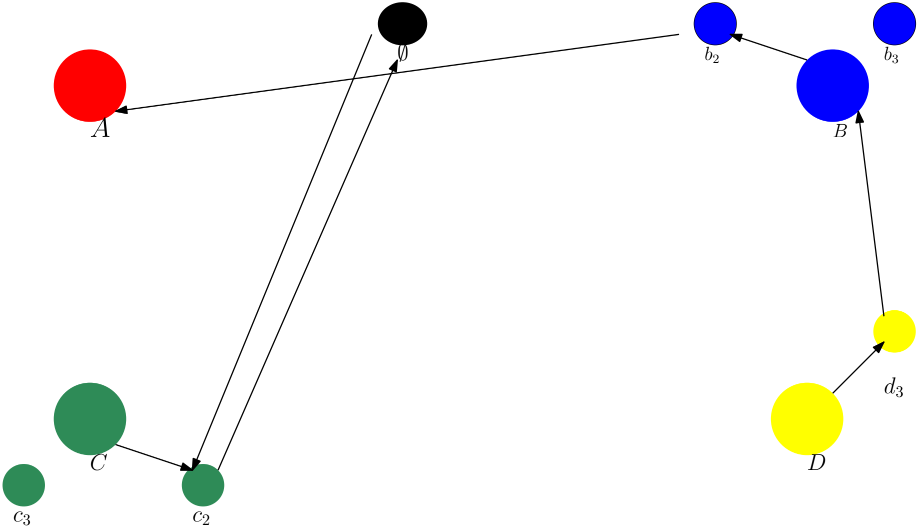
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	1	2	1	–	1	0	0
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

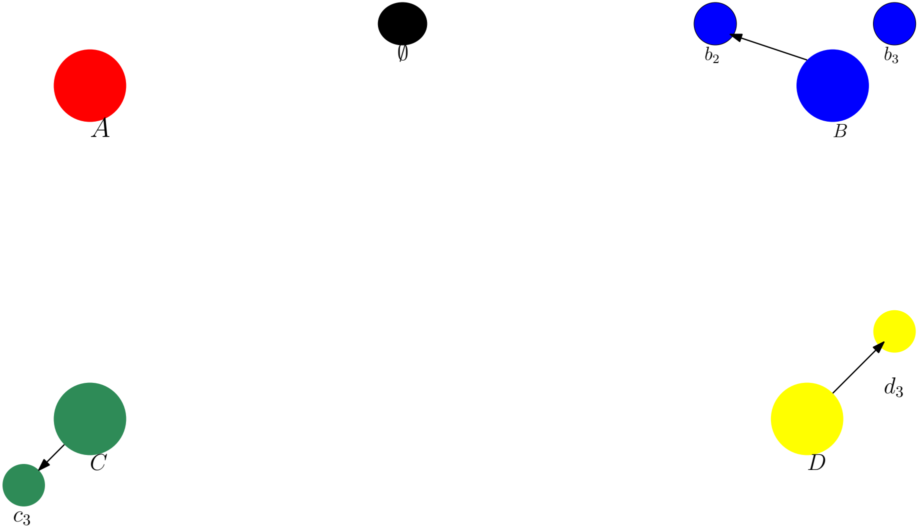
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	1	2	1	–	1	0	0
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

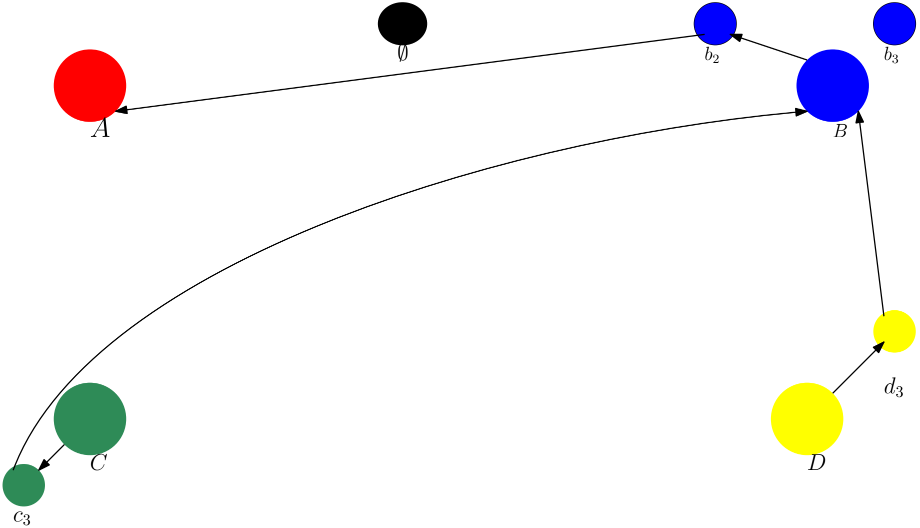
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	1	2	1	–	1	0	0
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

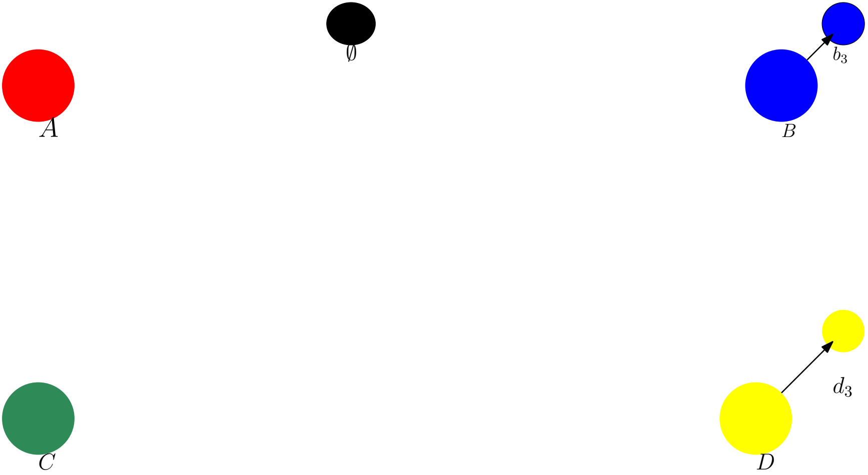
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	1	1	1	2	1	–	1	0	0
B	0	–	1	1	2	1	0	–	0	1
C	2	1	–	1	1	0	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

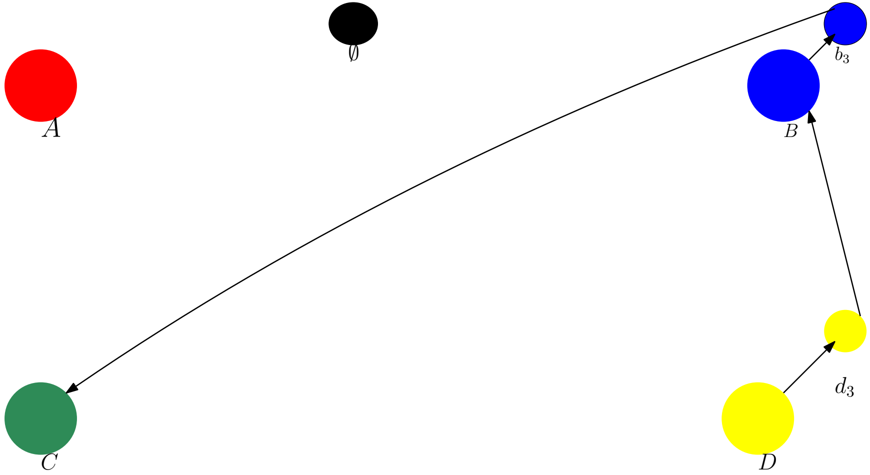
Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	$q_{\cdot,D}$	Q	b	$m_{\cdot\cdot A}$	$m_{\cdot\cdot B}$	$m_{\cdot\cdot C}$	$m_{\cdot\cdot D}$
A	–	0	1	1	1	0	–	0	0	0
B	0	–	0	1	1	1	0	–	0	1
C	2	1	–	1	1	1	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

Tie breaking:C-D-B-A



a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3
B	B	B	D	A	A	A	A	D	C	A	C
C	D	D	A	B	C	B	$\emptyset$	B	B	B	B
D	$\emptyset$	$\emptyset$	C	$\emptyset$	$\emptyset$	D		$\emptyset$	A	C	$\emptyset$
$\emptyset$			$\emptyset$			$\emptyset$			$\emptyset$	$\emptyset$	

	$q_{.,A}$	$q_{.,B}$	$q_{.,C}$	$q_{.,D}$	Q	b	$m_{..A}$	$m_{..B}$	$m_{..C}$	$m_{..D}$
A	–	0	1	1	1	0	–	0	0	0
B	0	–	0	1	1	1	0	–	0	1
C	2	1	–	1	1	1	0	0	–	0
D	0	0	1	–	0	1	0	0	0	–

Other Extensions

- gTTCC mechanism can deal with many other constraints without losing its desirable properties
  - Limit on the number of imports from a specific country/region
  - Limit on the number of imports from a specific field (engineering, management)
  - Limit on the number of exports
  - Tolerable Balance between incoming and outgoing students within each field

Conclusion

- Consider a matching market which has not been studied to our knowledge
- Use TTCC mechanism in a many to one matching problem
  - Fair (across to same types)
  - Strategy-proof
  - Efficient
  - Welfare gains compared to the Current System without any cost

Back Up Slides

- Simulation Setup
- Example for Extensions
- gTTCC with diversity

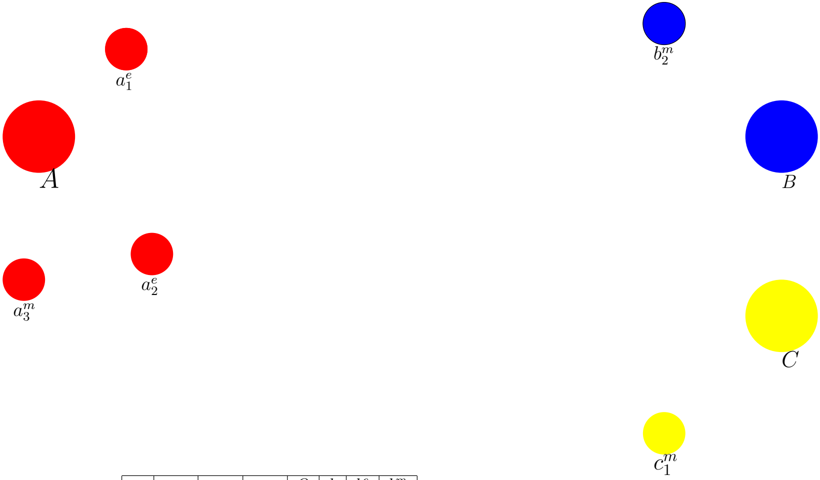
Simulations

- 10 Colleges and 100 Students. Each college has 10 students.
- Bilateral quotas: drawn from the discrete uniform distribution on the interval [0,4]
- Tolerance: drawn from the discrete uniform distribution on the interval [2,10]
- Aggregate quota: drawn from the discrete uniform distribution on the interval [Tolerance,10]
- Student preferences: $U_{i,c} = \alpha \times Z_c + (1 - \alpha) \times Z_{i,c}$
    - Z_c is an i.i.d standard uniformly distributed random variable and represents the common tastes of students on college c.
    - $Z_{i,c}$ is also an i.i.d standard uniformly distributed random variable and represents the tastes of student i on college c.
    - The correlation in the students preferences is captured by $\alpha \in [0, 1]$.

Simulations

- gTTCC is calculated based on the true quotas and preferences
- In order to calculate the current system outcome we randomly create a bilateral quota vector, $(\tilde{q}_{c,c'})_{c' \in C}$. for each college c such that $\sum \tilde{q}_{c,c'} = b_c$

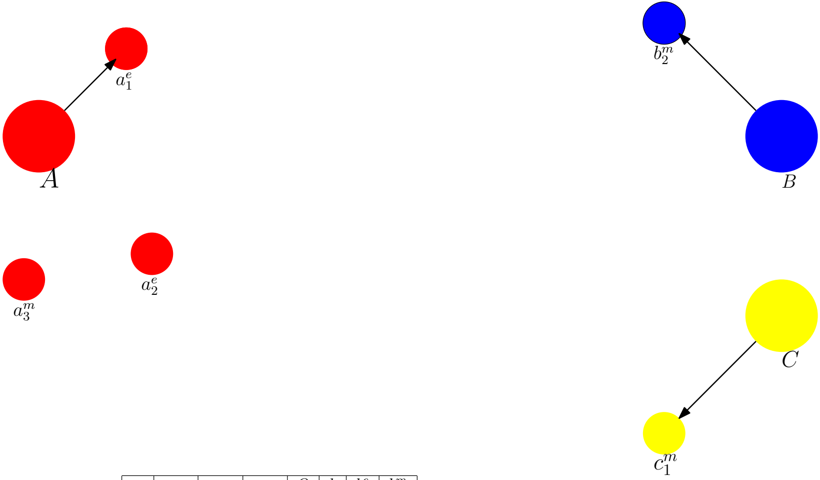
Other Extensions



a_1^e	a_2^e	a_3^m	b_1^m	c_1^m
B	C	C	A	A
C	B	B	$\emptyset$	$\emptyset$

	$q_{.,A}$	$q_{.,B}$	$q_{.,C}$	Q	b	b^e	b^m
A	–	1	1	2	1	1	1
B	1	–	1	1	1	1	1
C	1	1	–	1	0	1	1

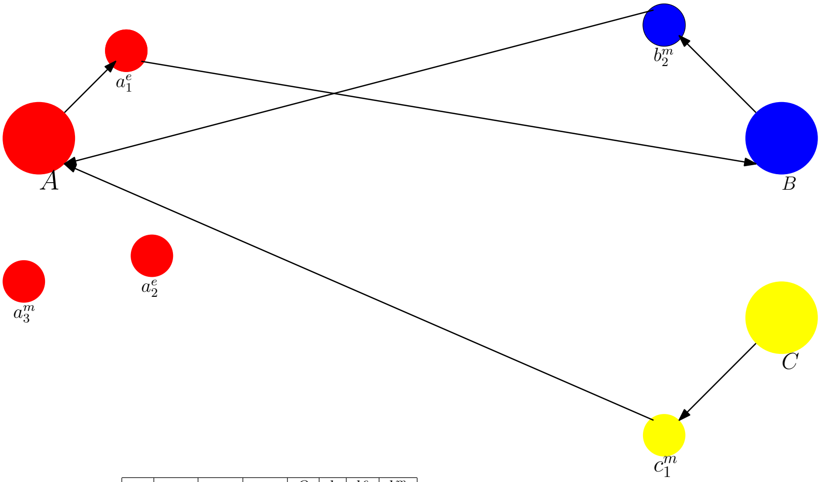
Other Extensions



a_1^e	a_2^e	a_3^m	b_1^m	c_1^m
B	C	C	A	A
C	B	B	$\emptyset$	$\emptyset$

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	Q	b	b^e	b^m
A	–	1	1	2	1	1	1
B	1	–	1	1	1	1	1
C	1	1	–	1	0	1	1

Other Extensions



a_1^e	a_2^e	a_3^m	b_1^m	c_1^m
B	C	C	A	A
C	B	B	$\emptyset$	$\emptyset$

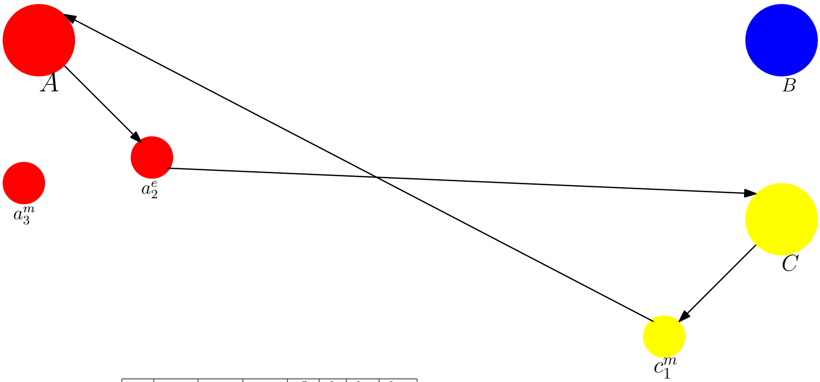
	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	Q	b	b^e	b^m
A	–	1	1	2	1	1	1
B	1	–	1	1	1	1	1
C	1	1	–	1	0	1	1

Other Extensions

a_1^e	a_2^e	a_3^m	b_1^m	c_1^m
B	C	C	A	A
C	B	B	$\emptyset$	$\emptyset$

	$q_{.,A}$	$q_{.,B}$	$q_{.,C}$	Q	b	b^e	b^m
A	–	0	1	1	1	2	0
B	0	–	1	0	1	0	2
C	1	1	–	1	0	1	1

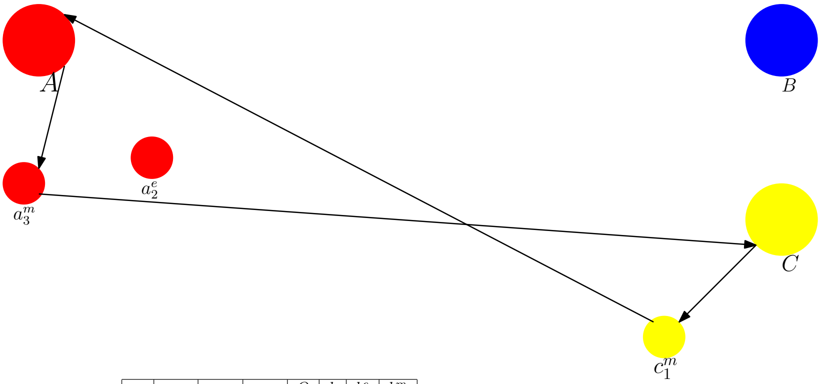
Other Extensions



a_1^e	a_2^e	a_3^m	b_1^m	c_1^m
B	C	C	A	A
C	B	B	$\emptyset$	$\emptyset$

	$q_{\cdot,A}$	$q_{\cdot,B}$	$q_{\cdot,C}$	Q	b	b^e	b^m
A	–	0	1	1	1	2	0
B	0	–	1	0	1	0	2
C	1	1	–	1	0	1	1

Other Extensions



a_1^e	a_2^e	a_3^m	b_1^m	c_1^m
B	C	C	A	A
C	B	B	$\emptyset$	$\emptyset$

	$q_{.,A}$	$q_{.,B}$	$q_{.,C}$	Q	b	b^e	b^m
A	–	0	1	1	1	2	0
B	0	–	1	0	1	0	2
C	1	1	–	1	0	1	1

Adding one more step to gTTCC

- We modify gTTCC by adding an initial step in which before students point, we check whether
  - there is **enough students** left from each college,
  - there is **enough available** seats left in each college

to satisfy all minimum quota requirement.

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  - there is **enough students** left from each college,
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to satisfy all minimum quota requirement.

In particular,

- If # of remaining c students is equal to # of unfilled reserved seats for c, then restrict all c students to point to the colleges with unfilled reserved seats for c

Adding one more step to gTTCC

- We modify gTTCC by adding an initial step in which before students point, we check whether
  - there is **enough students** left from each college,
  - there is **enough available** seats left in each college

to satisfy all minimum quota requirement.
In particular,

- If # of remaining c students is equal to # of unfilled reserved seats for c, then restrict all c students to point to the colleges with unfilled reserved seats for c
- If # of seats left in c is equal to # of unfilled reserved seats at c then c can be pointed by the students from colleges which have not satisfied the minimum quota requirement