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Comment on "Log or Linear? Distinct Intuitions of the Number Scale in Western and Amazonian Indigene Cultures"

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Dehaene *et al*. (Reports, 30 May 2008, p. 1217) argued that native speakers of Mundurucu, a language without a linguistic numerical system, inherently represent numerical values as a logarithmically spaced spatial continuum. However, their data do not rule out the alternative conclusion that Mundurucu speakers encode numbers linearly with scalar variability and psychologically construct space-number mappings by analogy.

The Mundurucu language bears an unusual linguistic phenomenon: The language includes number words only for the numbers one to five, whereas numerical values greater than five are labeled with approximate quantifiers like "some" or "many" (*1*). Consequently, native speakers of Mundurucu present a rare opportunity to study the nature of human numerical concepts in the absence of a robust verbal numerical system. Dehaene *et al*. (*2*) tested Mundurucu- and English-speaking participants on the number-line estimation task developed by Siegler and colleagues (*3*, *4*). In (*2*), participants positioned numerical values, presented either as nonsymbolic values (dot arrays or tones) or as symbolic, spoken number words, on a line anchored at both ends with fixed numerical values. Both groups positioned the numerical values ordinally, from small to large, along the length of the line. English speakers positioned the symbolic number words at linear intervals, but they positioned most of the nonsymbolic numerical values at logarithmically spaced intervals. In contrast, Mundurucu speakers positioned both the nonsymbolic and symbolic numerical values logarithmically. The authors thus concluded that a linear numerical code is unique to cultures that engage in formal education and that space-number mappings like those reported for Western societies (*5*–*7*) are culturally universal. Here, we offer an alternative account for each of these conclusions.

Dehaene *et al*. (*2*) tested Mundurucu and English speakers' numerical performance against the predictions of a precisely linear numerical code (Fig. 1A) and a logarithmic numerical code (Fig. 1B). Based on goodness of fit, the authors concluded that, unlike English speakers, Mundurucu speakers psychologically encode both symbolic and nonsymbolic numbers logarithmically. However, a third possibility was not tested: the possibility that Mundurucu speakers psychologically encode numbers linearly with scalar variability (Fig. 1C) (*8*).

The logarithmic code (Fig. 1B) and the linear-scalar code (Fig. 1C) predict similar outcomes in numerical performance. Both codes predict that smaller numbers are easier to distinguish than larger numbers. Both codes also predict that the midpoint between two numerical anchors is at the geometric mean rather than the arithmetic mean. Thus, Dehaene *et al*.'s finding that the Mundurucu indicate that the "middle of the interval 1 through 10 is 3 or 4, not 5 or 6" (*2*) is consistent with either code. Finally, both codes predict responses on the number-line task that conform to a logarithmic function over and above a precisely linear function. A logarithmic behavioral response function would emerge from a logarithmic code because of the compressed scaling of numbers in psychological space. Under a linear-scalar code, a logarithmic response function would emerge from noise that increases proportionally with number, combined with a ratio comparison process between the anchor and intermediate probe values.

Unfortunately, the behavioral predictions from the logarithmic and linear-scalar codes are virtually impossible to distinguish from the subjective scaling data obtained by Dehaene *et al*. (*2*) and previous studies (*3*, *4*) that made similar claims (*9*, *10*). In fact, some have argued that the only class of experimental data that can disambiguate the underlying nature of approximate numerical representations is one derived from arithmetic operations (*8*, *9*, *11*).

As in previous studies that employed the same subjective scaling paradigm (*3*, *4*), Dehaene *et al*.'s report actually contrasts an approximate numerical code (either a logarithmic code or a linear code with scalar variability) with an exact numerical code (a precisely linear code). Their findings are consistent with previous research on Mundurucu speakers (*1*) that showed that both small symbolic numbers and nonsymbolic numerosities are represented approximately. The data are not informative as to which approximate numerical code

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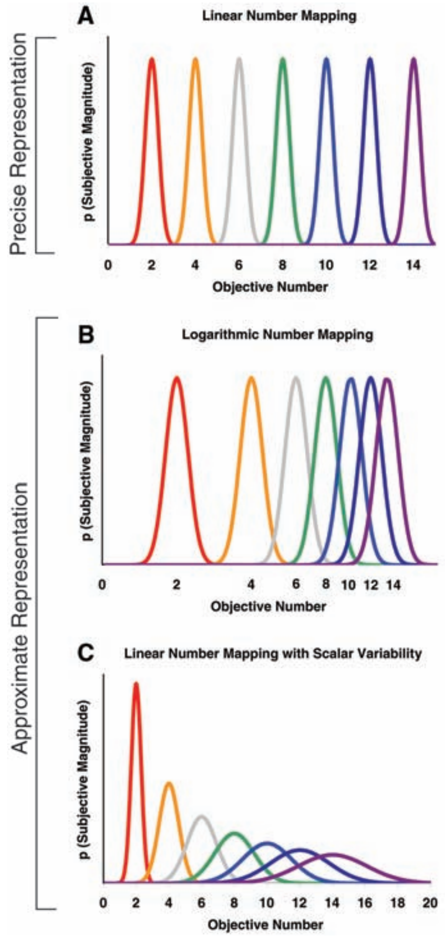

Fig. 1. The three hypothetical psychological codes underlying numerical representation. A precise linear code (**A**) posits equal spacing between values on the subjective number line and little variability. This code yields an exact mapping between objective and subjective number and thus allows one to appreciate that 99 and 100 differ by the same amount as 9 and 10. Under the logarithmic code (**B**), numerical values are psychologically compressed logarithmically with a constant amount of noise. Under this system, numerical representations become increasingly less distinct as objective number increases because they become closer together in psychological space. The linear numerical code with scalar variability (**C**) represents numerical values with equal psychological distances between adjacent values, and the amount of noise in the numerical representation increases proportionally with its value. Like the logarithmic code, the linear-scalar code predicts that confusion between neighboring values increases with magnitude, not because of the subjective spacing of the values but because of the increased variability with which each value is represented. Whereas the precisely linear code (A) represents objective numbers precisely, the logarithmic (B) and linear-with-scalar variability (C) codes represent numbers approximately.

underlies nonsymbolic numerical judgments: logarithmic or linear with scalar variability. In this sense, the question of "log or linear?" remains unanswered.

Dehaene *et al*. (*2*) also reported that nonliterate Munduruc*u* speakers represent numbers on a spatial continuum. Such a finding would indeed be remarkable given that a psychological mapping between numbers and space has never been demonstrated in a nonliterate human or nonverbal animal. Previous work has shown that educated adults map numbers onto a one-dimensional spatial continuum that is strongly influenced by the culturally defined direction of reading (*5*–*7*). For example, when required to make a parity judgment, adults who read from left to right are faster at responding with their left hand to small values but, with their right hand to large values suggesting that the mapping of numbers onto space is defined by reading direction (*5*).

A critical feature of previously demonstrated spatial-numeric mapping effects is that the spatial property of participants' numerical responses emerged implicitly—that is, under circumstances in which a space-to-number mapping was not overtly required. In contrast, Dehaene *et al*. (*2*) presented subjects with the overtly spatial task of positioning numbers on a line between two numerical anchors. Under these conditions, the spatial property of participants' numerical responses may be only superficially similar to the number-space mapping evidenced in literate adults.

Spatial responses in the number-line estimation task need not reflect an inherent space-number mapping. Humans are masters of analogy, even early in development. For example, by 3 years of age, children can map the concepts "daddy," "mommy," and "baby" onto a large, a medium, and a small flower pot (*12*). Yet, we would not conclude from this finding that children's underlying psychological representation of a family is fundamentally mapped to three different-sized flower pots. Similarly, adult humans are skilled at mapping between unidimensional properties. For example, adults can adjust the loudness of a sound, the length of a line, or the size of a numerical value to match the brightness of a light (*13*). However, the ability to map between these dimensions does not imply that the psychological foundation of brightness perception is loudness, length, or number. Instead, mappings between dimensions that all have something in common can be accomplished analogically, in this case using the property of unidimensionality. Thus, the results of Dehaene *et al*. provide evidence that Munduruc*u* speakers can map between the unidimensional properties of length and number, but this is not evidence that Munduruc*u* number representations are deeply, fundamentally, or intuitively spatial.

In short, Dehaene *et al*.'s (*1*, *2*) extraordinary studies of the Munduruc*u* mind offer a provocative set of hypotheses regarding the universal underlying nature of human numerical representation. However, there is still room for debate as to whether a linear numerical continuum is strictly a cultural invention and whether the mind inherently maps numbers onto space.

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