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Children's understanding of fraction and decimal symbols and the notation-specific relation to pre-algebra ability

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ABSTRACT

Fraction and decimal concepts are notoriously difficult for children to learn yet are a major component of elementary and middle school math curriculum and an important prerequisite for higher order mathematics (i.e., algebra). Thus, recently there has been a push to understand how children think about rational number magnitudes in order to understand how to promote rational number understanding. However, prior work investigating these questions has focused almost exclusively on fraction notation, overlooking the open questions of how children integrate rational number magnitudes presented in distinct notations (i.e., fractions, decimals, and whole numbers) and whether understanding of these distinct notations may independently contribute to pre-algebra ability. In the current study, we investigated rational number magnitude and arithmetic performance in both fraction and decimal notation in fourth- to seventh-grade children. We then explored how these measures of rational number ability predicted pre-algebra ability. Results reveal that children do represent the magnitudes of fractions and decimals as falling within a single numerical continuum and that, despite greater experience with fraction notation, children are more accurate when processing decimal notation than when processing fraction notation. Regression analyses revealed that both magnitude and arithmetic performance predicted pre-algebra ability, but magnitude understanding may be particularly unique and depend on notation. The

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educational implications of differences between children in the current study and previous work with adults are discussed.

Introduction

Rational number instruction is a major component of most elementary and middle school curricula (National Governors Association Center for Best Practices, 2010; National Mathematics Advisory Panel., 2008). However, despite the focus on teaching these concepts, children and adults still encounter significant difficulty in understanding rational numbers. Recently, educational and psychological research has paid particular attention to fraction understanding to identify exactly how it is that adults and children think about rational numbers in terms of both magnitude and arithmetic (e.g., DeWolf, Grounds, Bassok, & Holyoak, 2014; Hurst & Cordes, 2016; Iuculano and Butterworth, 2011; Lortie-Forgues, Tian, & Siegler, 2015; Meert, Gregoire, & Noel, 2010; Schneider & Siegler, 2010; Siegler, Thompson, & Schneider, 2011; Wang & Siegler, 2013) and has identified a link between rational number understanding and more advanced math learning, including algebra (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; DeWolf, Bassok, & Holyoak, 2015b; Hurst & Cordes, 2017a, 2017b; Siegler et al., 2012). Surprisingly, however, much of this research has focused on how understanding of fraction notation specifically may contribute to algebra learning and ability, overlooking the fact that the same magnitudes can be represented using distinct notations (e.g., the magnitude of half can be written as 0.5 or 1/2) and that these notations are typically taught in different ways and at different grade levels. Thus, there remain open questions as to whether the demonstrated relation between rational number understanding and algebra may be notation dependent and whether this may change across educational stages.

In the current study, we aimed to characterize fourth- to seventh-grade children's processing of symbolic rational numbers across distinct notations and to clarify its relationship to other math domains, specifically pre-algebra ability. Critically, given that fraction instruction and decimal instruction typically begin in different grades, span several years, and are often taught separately (e.g., National Governors Association Center for Best Practices, 2010), it is possible that children learning these concepts may differentially rely on fraction or decimal notation across distinct contexts (e.g., DeWolf, Bassok, & Holyoak, 2015a; Iuculano and Butterworth, 2011; Rapp, Bassok, DeWolf, & Holyoak, 2015). As such, children might not immediately understand how to integrate decimal and fraction information (i.e., treat decimals and fractions as falling along distinct numerical continua), making some aspects of notation-dependent rational number learning more critical for advanced mathematical thinking than others. Thus, this age group was chosen to provide a window into fraction and decimal understanding in children who have recently learned and are currently learning these concepts.

In this study, we had two main goals: First, we aimed to shed light on children's processing of symbolic magnitude by examining their ability to compare rational number magnitudes presented in distinct notations (fraction, decimal, and whole number). Second, we aimed to clarify the relation between rational number understanding and pre-algebra ability in school-aged children in two ways: (a) by directly comparing the relative strength of rational number magnitude understanding and rational number arithmetic proficiency in predicting pre-algebra performance and (b) by looking at the relation to pre-algebra performance for both fraction and decimal notations in particular.

Symbolic magnitudes

Substantial research has investigated how children and adults process symbolic whole numbers (i.e., 1, 2, 3, etc.) using a variety of tasks. One task used to investigate magnitude understanding is the number comparison task, in which the participant is asked to rapidly choose which of two symbols represents the larger magnitude. Data from this task robustly reveal systematic responding based on the ratio of the two values being compared (combining both size effects and distance effects; e.g., Moyer & Landauer, 1967). For example, comparing 12 and 13 would be more difficult (i.e., result in lower accuracy and/or slower reaction time) than comparing 7 and 8 (same distance between values, smaller numerical magnitudes) or than comparing 11 and 15 (larger distance, similar magnitude). Although there is substantial debate over the model that best characterizes the exact nature of magnitude representations (Balci & Gallistel, 2006; Cordes, Gallistel, Gelman, & Latham, 2007; Dehaene, 1992, 2001; Siegler & Opfer, 2003; Verguts, Fias, & Stevens, 2005), it is generally agreed that the presence of ratio effects suggests that these magnitudes are mentally represented within an ordered, approximate, and analog system (Dehaene, Bossini, & Giraux, 1993; Gallistel & Gelman, 1992; Meck & Church, 1983; Moyer & Landauer, 1967, 1973). Thus, the existence of ratio effects in numerical comparison tasks can inform our understanding of how numerical magnitudes are mentally represented.

Recently, researchers have used these same numerical comparison tasks to explore magnitude comparisons of rational number values. These studies have yielded similar behavioral patterns of ratio-dependent responding in children (Meert et al., 2010; Siegler et al., 2011; Wang & Siegler, 2013) and adults (DeWolf et al., 2014; Hurst & Cordes, 2016; Schneider & Siegler, 2010; Varma & Karl, 2013). However, unlike whole number ratio effects, the ratio effects observed for fractions and decimals may be less robust and more dependent on particular aspects of the task and/or stimuli. For example, individuals may rely on decimal length as a cue to relative magnitude or may compare fractions on a component-by-component basis without regard for the underlying magnitudes. As such, research suggests that decimal and fraction magnitude processing is not automatic (Kallai & Tzelgov, 2009, 2014) but instead requires deliberate effort in contexts where the use of alternative, non-magnitude-based strategies is prevented (Gabriel, Szucs, & Content, 2013; Ganor-Stern, 2013; Ganor-Stern, Karasik-Rivkin, & Tzelgov, 2011; Meert et al., 2010; Schneider & Siegler, 2010). Importantly, however, understanding how individuals compare each notation in isolation is not sufficient for understanding the rational number system as a whole. Rather, children must come to learn how these three notations are related to each other and have an integrated conceptualization of the rational number system. Although many people have noted the importance of understanding children's abilities to translate among different representations (e.g., Berch, 2016; Moss, 2005; Vamvakoussi & Vosniadou, 2010), there remains a relative lack of empirical work investigating how children think about these notations in distinct ways. As such, determining whether ratio effects are found when children compare values across notations (e.g., 0.38 vs. 1/3) as well as within each notation (e.g., 1/2 vs. 1/4 and 0.5 vs. 0.75) may provide insight into the degree to which children may integrate fraction and decimal magnitude concepts.

Despite the fact that in many cases decimals and fractions represent identical magnitudes (e.g., 1/4 = 0.25), the use of distinct notations may lead children to represent these magnitudes differently, making understanding the relations among fractions, decimals, and whole numbers particularly difficult. For example, when adolescents in one study were asked what kind of number could go between two other numbers, they insisted that only fractions could fall between two fractions and that only decimals could fall between two decimals (Vamvakoussi & Vosniadou, 2010). Relatedly, a greater reliance on one notation may impede learning of other rational number notations, making it difficult to understand how one may be related to the other. For example, evidence suggests that children's prior understanding of whole numbers interferes with their learning of fractions and decimals. Termed the "whole number bias" (Ni & Zhou, 2005), this inappropriate application of whole number knowledge and counting routines has been shown to negatively influence children's understanding of symbolic fractions and nonsymbolic proportions (e.g., Boyer, Levine, & Huttenlocher, 2008; Jeong, Levine, & Huttenlocher, 2007; Ni & Zhou, 2005). Thus, although fractions, decimals, and whole numbers all are used to represent rational number magnitudes, children's pattern of errors reveals difficulty in relating these notational systems.

On the other hand, recent evidence reveals that educated adults are capable of representing the magnitudes of these values in an integrated way across distinct notations, as demonstrated by ratio-dependent responding when comparing rational number magnitudes both within and between notations (Hurst & Cordes, 2016). That is, the speed with which they compare a fraction with a decimal, a fraction with a whole number, or a whole number with a decimal increases as the ratio between these two values increase (i.e., performance improves for larger ratios), revealing that responding is magnitude dependent and consistent with the idea that adults may represent values presented across these distinct notations within the same ordered and approximate analog magnitude system. In light of this, it may be that children's responding will similarly reveal a pattern of ratio dependence consistent with a fully integrated representational system. Along these lines, Siegler and colleagues (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011) recently proposed an integrated theory of number development positing that across schooling children progressively learn to represent more classes of numbers (i.e., whole numbers, fractions, decimals, and negative numbers) as numerical magnitudes. In fact, other data suggest that children who are actively learning fraction and decimal notation may be better than adults at actively inhibiting competing whole number information while thinking about fractions (Meert et al., 2010; Opfer & DeVries, 2008), suggesting that adults' whole number heuristic-based responding may be more practiced and ingrained. Therefore, we might expect children to also show ratio-dependent responding across distinct notations, providing evidence for an integrated representation of the magnitudes represented by symbolic rational numbers even early on in symbolic fraction and decimal learning.

In the current study, we aimed to investigate fourth- to seventh-grade children's abilities to compare symbolic magnitudes across distinct notations in order to evaluate whether children are capable of rapidly accessing the magnitudes associated with fraction and decimal notation and whether these representations of magnitude are integrated with each other, as evidenced by ratio-dependent responding in cross-notation comparisons.

Rational numbers and algebra

The recent surge in research on fraction and decimal understanding is motivated in part by evidence suggesting that rational number understanding is related to more advanced math learning, including algebra (Booth & Newton, 2012; Booth et al., 2014; DeWolf et al., 2015b; Hurst & Cordes, 2017a, 2017b; Siegler et al., 2012). For example, children's abilities to map symbolic fractions onto number lines has been shown to be predictive of algebra readiness (Booth & Newton, 2012) and learning of algebra equation solving (Booth et al., 2014). But what exactly is it about children's rational number knowledge that is important for algebra performance?

On the one hand, fractions play a substantial and direct role in many algebra contexts. For example, understanding rates of change involves creating a ratio between two values (slope = change in y/change in x), which necessarily involves working with fractions. Thus, at least part of the relation may be very direct and due to the cumulative nature of math and recognizing fractions as relevant and useful in algebra contexts. On the other hand, there are also indirect conceptual links between algebra and rational numbers that may provide relations that are not attributable to the direct overlap in fraction knowledge. In particular, it may be that having a strong understanding of rational number magnitudes is critical for algebra. That is, understanding algebra often involves understanding relations among elements. Thus, children who have a better grasp of how the numerator and denominator in a fraction relate to one another in order to represent a single value may also be more likely to pick up on other relations, for example, between values in an algebraic equation. In addition, an ability to solve algebraic equations requires an understanding that variables represent unknown quantities (Linchevski, 1995), including values beyond the whole number count list (Christou and Vosniadou, 2012). Thus, at a fundamental level, a conceptual understanding of algebraic variables may be facilitated when children have a strong grasp of rational number magnitudes (Christou and Vosniadou, 2012).

There is evidence in support of this proposed relation between algebra and rational number magnitudes, suggesting that an understanding of rational number magnitudes is related to algebra ability (Booth & Newton, 2012; Booth et al., 2014; DeWolf et al., 2015b; Hurst & Cordes, 2017a, 2017b) but that this relation may be primarily driven by decimal magnitude understanding specifically (DeWolf et al., 2014; Hurst & Cordes, 2017a). For example, Hurst and Cordes (2017a) found that adults' performance on a decimal comparison task was uniquely predictive of algebra ability but that fraction magnitude performance was not. Relatedly, DeWolf et al. (2015b) found that the ability to map decimals onto number lines (another measure of magnitude understanding) was a better predictor of algebra performance than mapping fractions onto number lines. Importantly, however, those studies that have directly compared decimal and fraction understanding have done so with adults (Hurst & Cordes, 2017a) and older children (seventh graders; DeWolf et al., 2015b), who have had substantial practice with both fraction and decimal notation. Adults show better performance on magnitude tasks involving decimals (DeWolf et al., 2014; Hurst & Cordes, 2016), a notation that children typically learn after fraction notation and, thus, have less exposure to early in rational number instruction. Thus, it may be that these individuals have gained practice with decimals as a way to overcome their limitations with fractions. Furthermore, recent work has emphasized the importance of not just understanding rational numbers in distinct notations but also being able to translate between notations (e.g., Berch, 2016). Thus, in the current study, we aimed to investigate the relation between fourth- to seventh-grade children's pre-algebraic ability and their ability to compare rational number magnitudes *across* notation. By doing so, we extended previous work focusing on magnitude understanding within individual notations to investigate rational number magnitude understanding across fraction, decimal, and whole number notations in a group of children who were still in the process of acquiring rational number concepts.

In addition, an ability to work with rational numbers in arithmetic procedures may also be particularly critical for algebra. Working with rational numbers and algebraic problem solving both require an understanding of complex symbolic manipulation and an ability to follow abstract rules and procedures (e.g., Carraher, Schliemann, & Brizuela, 2000; MacGregor & Price, 1999). In this case, we might expect that children's ability to successfully manipulate complex fraction and decimal notation in an arithmetic operation (e.g., finding common denominators to perform fraction addition) would be related to their ability to solve complex algebraic equations. Prior work with adults supports this claim, revealing that both fraction arithmetic performance and decimal arithmetic performance uniquely predict algebra ability over and above magnitude understanding (Hurst & Cordes, 2017a). Research with seventh graders has also found that conceptual understanding of the components of fraction notation (e.g., understanding equivalent fractions, the inverse of fractions, and ratio) was more predictive than magnitude understanding of fraction notation (DeWolf et al., 2015b). Thus, it may be that the ability to engage in complex symbolic manipulation, as required by both fraction and decimal arithmetic, may be critically important for pre-algebra in ways that are not captured by rational number magnitude understanding. However, some evidence suggests that adults may be better at fraction arithmetic than decimal arithmetic despite being better at thinking about magnitude using decimal notation compared with fraction notation (Hurst & Cordes, 2016), potentially because they are more able to rely on rules of memorization and less able to rely on a deep conceptual understanding. Given that children may show a different pattern early on in their fraction learning, the relative contribution of fraction arithmetic and decimal arithmetic abilities in predicting pre-algebraic understanding in children may differ from that in adults.

The current study aimed to explore these questions in children who are actively learning these concepts (fraction and decimal magnitudes and arithmetic) to see how rational number magnitude understanding and arithmetic understanding relate to pre-algebra ability in children and how this relation may differ from that found in adults, who have already completed instruction on these topics. Given that children may still be actively engaging with the conceptual aspects of fraction and decimal notation and arithmetic, their performance may reveal different preferences and patterns across the two notations and types of knowledge being measured, resulting in a relation between rational number and algebra performance distinct from that reported in previous work with adults and older children.

The current study

Here, we measured children's abilities to use decimal and fraction notation both as a representation of magnitude (in a number comparison task) and in simple addition. Child participants completed a number comparison task (comparing across fractions, decimals, and whole numbers), two rational number addition assessments (fractions and decimals), and a pre-algebra assessment in order to address two major research aims. First, we aimed to better characterize children's processing of rational number symbolic magnitudes by assessing whether fourth to seventh graders exhibit ratio-dependent responding when comparing rational numbers both within notation (e.g., fraction vs. fraction, decimal vs. decimal, whole number vs. whole number) and between notations (e.g., fraction vs. decimal, fraction vs.

whole number, decimal vs. whole number). That is, do children, who are still in the process of learning rational number concepts, conceptualize symbolic rational number magnitudes as falling along an integrated continuum? Second, we aimed to clarify the nature of the reported relation between rational number and algebra abilities in children by investigating how distinct knowledge types across distinct notations may be differentially implicated in the relation. That is, when measuring rational number understanding in terms of both symbolic magnitude understanding across distinct notations (decimal vs. fraction, whole number vs. fraction, and whole number vs. decimal) and arithmetic ability (fraction addition and decimal addition), what are the relative contributions to pre-algebra ability in a group of children who are in the process of actively learning these concepts?

Method

Participants

A total of 41 children in Grades 4–7 participated. Children were divided into two grade groups: lower grades (fourth and fifth graders; $n = 22$; M_{age} = 10.6 years, range = 9.75–11.83; 11 boys) and upper grades (sixth and seventh graders; $n = 19$; M_{age} = 12.9 years, range = 11.58–14.40; 15 boys). Data from 1 additional child were excluded because of random responding resulting from a failure to follow directions (e.g., selecting responses without looking at the questions).

Children were recruited from the greater Boston, MA area in the northeastern United States and tested at approved testing locations, including local after-school programs, the participants' homes, and our laboratory. Children received either a small prize or $10 for their participation, in accordance with the regulations set by specific testing locations. All procedures were approved by the Boston College institutional review board, all parents provided written consent, and children provided written assent. Funding for this work was provided by Boston College.

Stimuli

Number comparison task

On each trial in the number comparison task (adapted from Hurst and Cordes, 2016), children were tasked to indicate which of two rational numbers (decimal, fraction, and/or whole number) represented the greater numeric value. There were six distinct notation comparison types, each presented in a separate block, with the order of the blocks randomized across participants: fraction versus fraction (FvF; e.g., 1/2 vs. 3/4), decimal versus decimal (DvD; e.g., 0.50 vs. 0.75), whole number versus whole number (NvN; e.g., 5 vs. 2), whole number versus decimal (NvD; e.g., 5 vs. 2.10), whole number versus fraction (NvF; e.g., 4 vs. 9/2), and decimal versus fraction (DvF; e.g., 0.89 vs. 4/5). The ratio between the two numerical values (larger value/smaller value) presented on each trial was chosen from one of two approximate numerical ratio bins: 1.5 (exact ratios tested ranged from 1.35 to 1.67) and 2.5 (exact ratios tested ranged from 2.20 to 2.92). Each ratio bin for each comparison type included 4 unique numeric pairs shown twice (once with the largest on the left and once with the largest on the right). Thus, each block contained a total of 16 trials (4 pairs × 2 [shown twice] × 2 ratio bins), making the entire task 96 trials (16 trials × 6 blocks).

For fraction stimuli, the values of all numerators and denominators were between 1 and 10 and no number appeared twice in the same comparison (e.g., 3/4 vs. 4/7 would not occur). This was done to prevent the use of overt componential whole number strategies (as in Schneider & Siegler, 2010). For decimal stimuli, all numbers were presented to the hundredths digit (i.e., two digits after the decimal point), with a whole number (sometimes 0) always before each decimal point (e.g., 0.20, 1.25). Decimal values ranged from 0.20 to 4.50, fraction values ranged from 1/5 to 9/2, and whole numbers ranged from 1 to 30. Between trials, a 32-point font (0.5 cm^2) fixation point appeared in the center of the screen.

All stimuli were made using Adobe Illustrator in 72-point regular Arial font and presented on a 13-in. (33 cm) screen (1200 × 800 pixels). Whole number stimuli were approximately 2.5 cm high × 1.5 cm wide, decimal stimuli were approximately 6 cm wide × 2 cm high, and fraction stimuli were approximately 3 cm high × 6 cm wide.

Pre-algebra assessment

The pre-algebra assessment was composed of 7 questions (with subparts, resulting in a total of 13 questions) adapted from the Early Algebra Research Project (Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). The questions involved solving for unknowns within an expression and finding patterns in data tables (see Appendix for a complete set of questions). Importantly, no decimals or fractions were included in any questions on the pre-algebra assessment; thus, any relation found between performance on the pre-algebra assessment and our rational number measures was not due to a direct overlap in skills assessed.

Rational number arithmetic assessments

The fraction arithmetic assessment and decimal arithmetic assessment each consisted of five addition problems presented in two separate paper booklets (see Appendix for a full set of problems). All problems were presented horizontally, and fractions were presented in their formal vertical format (e.g., $\frac{2}{3} + \frac{5}{6}$). The booklets contained enough space for children to work out solutions and record their responses in the "answer box" provided on the page.

Procedure

Children completed the three tasks in the following order: (1) number comparison task, (2) pre-algebra assessment, and (3) fraction and decimal arithmetic assessments (with the order of the two arithmetic assessments counterbalanced). The experimenter remained with the children throughout the experiment and provided general encouragement ("All right!", "Good job!") but did not provide specific feedback.

Number comparison task

Children were shown two numbers on the screen and asked to press a key on the keyboard (right arrow or left arrow for right or left stimulus, respectively) to indicate the larger numerical value as quickly as possible while also trying their best to do so correctly. Children were instructed to keep their hand on the keyboard throughout the session. Numbers remained on the screen until the participants selected their response, and a fixation cross appeared between trials in the middle of the screen (1000 ms) to direct attention back to the middle before new numbers became visible.

The comparison task consisted of six blocks of trials. Within each block, the type of comparison presented was held constant; however, each block presented a different comparison type (DvD, NvN, FvF, NvF, NvD, or DvF). The blocks were presented in a random order. Prior to each block, children saw an instruction screen followed by two practice trials (same comparison type as the other trials in that block), where they were given corrective feedback ("Good job! That's correct" or "I'm sorry! That's not correct").

Pre-algebra assessment

The experimenter read each problem instruction with the children and then allowed them to complete the problem on their own. Children were allowed to work out the problem on paper. Once the children completed the problem, the experimenter moved on to read the instructions for the next problem.

Arithmetic assessments

Children were presented with the fraction addition assessment and the decimal addition assessment in separate booklets. The problems in each booklet were presented in a set order, with the order of the booklets counterbalanced across participants. Children were allowed to work out the problems on paper and recorded their answers in the "answer box" on the paper. Children had as much time as they needed to complete the assessments.

Data analyses

Accuracy was used as the primary dependent variable on all tasks. On the number comparison task, reaction time (RT) was also recorded and analyzed. For RT analyses, only RTs from correct responses and those within 3 standard deviations of that individual's mean RT for the Comparison Type × Ratio combination were included. Following these exclusion criteria, in order for data from a Comparison Type × Ratio combination to be included, the participant needed to have scored at least 5 of 8 correct (to ensure that the RTs were reflective of at least a basic understanding of the task) and have a minimum of 2 included data points (correct and within 3 standard deviations of the individual's mean). Given the relatively low accuracy on comparisons involving fractions, this resulted in only 20 participants having complete RT data across both ratios and all six blocks of notations.

Two children (both in fifth grade) did not complete the arithmetic assessments and so were not included in those analyses or the regression analyses, leaving 39 children (20 lower grade and 19 upper grade) in the regression analyses.

Results

Rational number magnitude performance

We investigated performance on the magnitude comparison task across notation (6: FvF, DvD, NvN, NvD, NvF, or DvF) and ratio (2: small or large) using a repeated-measures analysis of variance (ANOVA) on accuracy, with grade level (2: lower or upper) as a between-participant factor. There was a main effect of notation (reporting the Huynh–Feldt correction for a violation of sphericity), $F(2.98, 116.3) = 29.5$, $p < .001$, partial $\eta^2 = .43$; whole number comparisons resulted in the highest accuracy and fraction comparisons resulted in the lowest accuracy ($M_{FvF} = 76\%$, $M_{DvF} = 78\%$, $M_{NvF} = 81\%$, $M_{DvD} = 93\%$, $M_{NvD} = 93\%$, $M_{NvN} = 98\%$). There was also a main effect of ratio, $F(1, 39) = 30.9$, $p < .001$, partial $\eta^2 = .40$, with higher accuracy on the larger ratio ($M = 90\%$) compared with the smaller ratio ($M = 83\%$). However, there was also a Notation × Ratio interaction (reporting Huynh–Feldt correction for a violation of sphericity), $F(3.6, 141.5) = 11.2$, $p < .001$, partial $\eta^2 = .20$. Follow-up tests (p values reflecting paired t tests) revealed that there was a significant ratio effect on the FvF comparisons ($M_{Small} = 63\%$, $M_{Large} = 88\%$, $p < .001$, Cohen's $d = 0.95$) and on the DvF comparisons ($M_{Small} = 74\%$, $M_{Large} = 83\%$, $p < .01$, Cohen's $d = 0.40$). There was a marginal effect on the NvF comparisons ($M_{Small} = 78\%$, $M_{Large} = 83\%$, $p = .07$, Cohen's $d = 0.29$) and nonsignificant differences across ratio on DvD ($M_{Small} = 92\%$, $M_{Large} = 95\%$, $p = .16$, Cohen's $d = 0.22$), NvN ($M_{Small} = 97\%$, $M_{Large} = 99\%$, $p = .30$, Cohen's $d = 0.12$), and NvD ($M_{Small} = 93\%$, $M_{Large} = 92\%$, $p = .60$, Cohen's $d = 0.08$).

Lastly, there was not a main effect of grade, $p = .13$, partial $\eta^2 = .06$, Grade × Ratio interaction, $p = .24$, partial $\eta^2 = .04$, or Grade × Ratio × Notation interaction, $p = .56$, partial $\eta^2 = .02$. However, there was a marginal Grade × Notation interaction, $p = .07$, partial $\eta^2 = .06$. Follow-up independent t tests suggested that children in the upper grades were significantly more accurate than children in the lower grades on NvF comparisons ($p = .04$) and marginally more accurate on FvF ($p = .09$), but did not perform differently on any other comparison type (ps > .30) (see Fig. 1).

Notably, in the previous analyses based on accuracy, the NvN, DvD, and NvD comparisons (which did not show significant ratio effects) were those comparisons resulting in the highest accuracy, with average performance above 90% on all three blocks. Thus, a failure to obtain ratio effects may have been a function of near-ceiling performance on those comparisons. Thus, we repeated the same analysis using average RT on correct responses for these comparison blocks (1 participant had extremely low accuracy on the DvD block, resulting in too few meaningful RT data points to be included; $N = 40$).

Again, there was a main effect of notation, $F(2, 76) = 23.7$, $p < .001$, partial $\eta^2 = .39$, with performance on the NvN comparisons ($M = 940$ ms) taking significantly less time than performance on the DvD ($M = 1398$ ms) and NvD ($M = 1364$ ms) comparisons (paired t tests, ps < .001), but DvD and NvD were not significantly different from each other ($p = .80$). There was a main effect of ratio, $F(1, 38) = 18.9$, $p < .001$, partial $\eta^2 = .33$, with performance taking longer on the smaller ratio ($M = 1294$ ms) compared with the larger ratio ($M = 1175$ ms). Furthermore, there was a marginal interaction between ratio and notation, $F(1.78, 67.8) = 3.1$, $p = .06$, partial $\eta^2 = .08$. In particular, the ratio effect

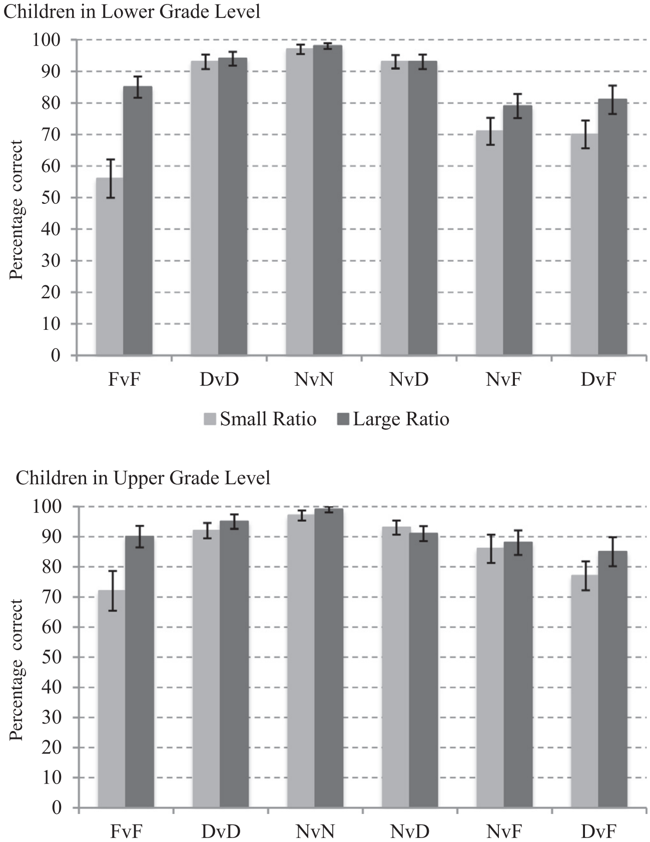

Fig. 1. Performance on the magnitude comparison task across all six notation types, both ratios, and children from both lower grade levels (fourth and fifth grades; upper panel) and upper grade levels (sixth and seventh grades; lower panel). FvF, fraction versus fraction; DvD, decimal versus decimal; NvN, whole number versus whole number; NvD, whole number versus decimal; NvF, whole number versus fraction; DvF, decimal versus fraction.

found in DvD trial data was significantly larger than the ratio effect in NvN ($p = .02$), although NvD was in between these and was not significantly different from DvD ($p = .10$) or NvN ($p = .57$). However, in all three trial types, we did see a similar pattern, with significant ratio effects in NvN ($M_{\text{Small–Large}} = 65$ ms, paired *t* test, $p = .007$) and DvD ($M_{\text{Small–Large}} = 208$ ms, paired *t* test, $p = .001$) comparisons and marginally significant ratio effects in NvD ($M_{\text{Small–Large}} = 84$ ms, paired *t* test, $p = .06$).

The ANOVA on RT also revealed a main effect of grade, $F(1, 38) = 7.2$, $p < .01$, partial $\eta^2 = .16$, with children in the lower grades taking longer to respond than children in the upper grades ($M_{\text{Lower}} = 1390$ ms, $M_{\text{Upper}} = 1078$ ms) and a significant Notation × Grade interaction, $F(2, 76) = 6.09$, $p = .004$, partial $\eta^2 = .14$. In particular, children in the upper and lower grade levels did not show significantly different RTs on NvN ($p = .20$) or DvD ($p = .10$) comparisons, but children in upper grades were significantly faster than children in lower grades on NvD comparisons ($M_{\text{Lower}} = 1664$ ms, $M_{\text{Upper}} = 1085$ ms, $p = .001$). Grade did not interact with ratio ($p = .60$, partial $\eta^2 < .01$) or with Ratio × Notation (three-way interaction, $p = .09$, partial $\eta^2 = .07$).

Arithmetic performance

We compared performance on the fraction and decimal arithmetic tests across both the lower and upper grade levels. Overall, children in upper grades performed significantly better than children in lower grades, $F(1, 37) = 4.8$, $p = .03$, partial $\eta^2 = .11$, and children performed significantly better on decimal addition compared with fraction addition, $F(1, 37) = 8.9$, $p = .005$, partial $\eta^2 = .19$. There was not a significant interaction between grade level and notation, $p = .36$, partial $\eta^2 = .02$ (lower grade level: $M_{\text{Fractions}} = 53\%$, $M_{\text{Decimals}} = 77\%$; upper grade level: $M_{\text{Fractions}} = 78\%$, $M_{\text{Decimals}} = 90\%$).

Individual differences in predicting math ability

First, we looked at the relative contribution of rational number magnitude understanding and rational number arithmetic, across multiple notations, for predicting arithmetic ability. We used regression analyses to assess whether accuracy in flexibly representing rational number magnitudes (composite score computed as average performance on DvF, NvF, and NvD; Model 1) and accuracy on rational number addition (composite score computed as average accuracy on fraction and decimal addition assessments; Model 2) predicted performance on the pre-algebra assessment. In addition to investigating these two predictors separately (controlling only for grade), in Model 3 we included both magnitude and addition measures in the same model to investigate the relative impact of both types of knowledge.

Rational number magnitude ability (RN magnitude) was calculated as a composite of the scores (i.e., average percentage correct across all relevant blocks) on the three comparisons that required children to compare between distinct notations (DvF, NvF, and NvD) because this has been argued to be a critical component of rational number sense (e.g., Berch, 2016). Rational number arithmetic (RN arithmetic) was calculated as a composite across both decimal and fraction addition assessments (i.e., average percentage correct across both assessments). See Table 1 for simple correlations and descriptive statistics, and see Table 2 for complete regression statistics for all models.

Grade level was a significant predictor of pre-algebra performance (Model 0, $p = .04$), and so it was included in all subsequent regression models in order to control for educational differences in pre-algebra ability. When looking at RN magnitude and RN arithmetic independently, controlling only for grade, both knowledge types explained significant additional variance over and above grade (ps $< .001$). Furthermore, when both measures were included in the same model (in addition to grade), RN magnitude continued to explain significant unique variance ($p = .004$) and the RN arithmetic score explained marginally significant unique variance ($p = .057$) over and above each other.

Next, we were specifically interested in whether the relations between rational number understanding and pre-algebra ability are dependent on fraction and decimal notation in particular. That is, beyond rational number understanding across notation (as in the previous analyses), we aimed to better isolate the separate contributions of understanding fraction and decimal notation for both magnitude and arithmetic separately. Thus, we performed regression analyses with fraction performance and decimal performance predicting pre-algebra ability, controlling for grade level. When looking at the magnitude measures, the overall model was significant, $R^2 = .42$, $F(3, 35) = 8.3$, $p < .001$, reinforcing the importance of magnitude understanding as shown in the previous analyses using different measures. However, only FvF magnitude performance predicted significant unique variance, $\beta = 0.58$, $t(38) = 4.2$, $p < .001$, whereas DvD magnitude, $\beta = -0.06$, $t(38) = -0.44$, $p = .66$, and grade level, $\beta = 0.17$, $t(38) = 1.3$, $p = .22$, did not. Similarly, results of the regression using the arithmetic measures to predict pre-algebra performance show that the overall model was significant, $R^2 = .38$, $F(3, 35) = 7.1$, $p < .001$, aligning with the previous analyses involving the composite of these measures. Furthermore,

Table 1
Descriptive statistics and bivariate correlations for all variables used in the regression analyses ($N = 39$).

	Mean (*SD*) (%)	Grade level	RN arithmetic	RN magnitude	Pre-algebra
Grade level	–	–			
RN arithmetic	74 (29)	0.34*	–		
RN magnitude	83 (12)	0.24+	0.56**	–	
Pre-algebra	84 (21)	0.34*	0.58**	0.56**	–

Note. RN, rational number.
+ $p < .10$ (two-tailed).
* $p < .05$ (two-tailed).
** $p < .001$ (two-tailed).

Table 2
Full regression statistics for all three models ($N = 39$).

		Beta	Statistic
Grade only (Model 0)			
$R^2 = .11$, $F(1, 37) = 4.7$, $p = .04$	Grade level	0.36	$t(38) = 4.7$, $p = .04$
Magnitude model (Model 1)			
$R^2 = .44$, $F(2, 36) = 14.2$, $p < .001$	Grade level	0.19	$t(38) = 1.5$, $p = .14$
	RN magnitude	0.59	$t(38) = 4.6$, $p < .001$
Arithmetic model (Model 2)			
$R^2 = .36$, $F(2, 36) = 10.2$, $p < .001$	Grade level	0.16	$t(38) = 1.1$, $p = .30$
	RN arithmetic	0.53	$t(38) = 3.7$, $p < .001$
Overall model (Model 3)			
$R^2 = .50$, $F(3, 35) = 11.5$, $p < .001$	Grade level	0.13	$t(38) = 1.0$, $p = .30$
	RN magnitude	0.44	$t(38) = 3.1$, $p = .004$
	RN arithmetic	0.29	$t(38) = 2.0$, $p = .057$

Note. The outcome variable in all models is percentage correct on the pre-algebra measure.

decimal arithmetic, $\beta = 0.35$, $t(38) = 2.2$, $p = .03$, was a significant unique predictor of pre-algebra performance, and fraction arithmetic, $\beta = 0.29$, $t(38) = 1.9$, $p = .06$, shows a similar, although only marginal, relation. Again, grade level was not significant, $\beta = 0.14$, $t(38) = 1.0$, $p = .30$.

Discussion

We used a rational number magnitude comparison task and assessments of fraction and decimal addition to (a) better characterize how children process fraction and decimal notation in terms of both magnitude information and arithmetic and (b) investigate how magnitude understanding and arithmetic ability across distinct notations may be differentially implicated in the relation between rational number ability and pre-algebra ability in children.

Rational number magnitude understanding

Children did show evidence of systematic ratio-dependent responding when comparing magnitudes within the same notation (whole numbers vs. whole numbers, decimals vs. decimals, and fractions vs. fractions) and across different notations (decimals vs. fractions, although only marginally for whole numbers vs. fractions and whole numbers vs. decimals). However, the behavioral level at which this ratio effect was observed and the size of these ratio effects differed across notation; fraction comparisons showed ratio dependence in accuracy measures, whereas comparisons not involving fractions showed ratio dependence in RT measures. The distinct levels of behavior that reveal ratio-dependent responding likely reflect differential fluency with these number systems at this point in the children's education. In particular, consistent with adult data, children were less proficient (i.e., lower accuracy) in judging numerical magnitudes presented in fraction notation than those presented in either decimal notation or whole numbers. As such, ratio effects in accuracy on fraction comparisons and ratio effects in RT on non-fraction comparisons may reflect the largest individual differences across distinct ratios. That is, when children are first learning (and show worse performance with) a notation, they may show ratio dependence in terms of accuracy, but as children's accuracy improves and approaches ceiling levels, ratio effects in terms of accuracy may be weaker and less robust. However, children did show ratio-dependent responding in all single notation comparisons and when comparing between fractions and decimals, suggesting that they are able to compare magnitudes of rational numbers in distinct notations and represent the magnitudes in an integrated analog magnitude system, similar to previous work with adults (Hurst & Cordes, 2016). When comparing between whole numbers and either fractions or decimals, however, children showed only small marginally significant ratio effects. The smaller marginal ratio effect in fraction versus whole number (NvF) and decimal versus whole number (NvD) comparisons may reflect a larger tendency to use heuristics in contexts where non-whole number rational numbers (which children may assume are typically less than 1) and whole numbers are being compared, perhaps reflecting that integrating these notations is more difficult than integrating the other notations (e.g., Ni & Zhou, 2005).

Of course, it is possible that participants may have invoked magnitude-based heuristics across all comparison types when performing our task. For example, it may be that when two values are on opposite sides of half or when the fraction is being compared with 1 (as opposed to another whole number such as 5), the numerical magnitudes chosen may affect performance less (resulting in a lower ratio effect) because people rely on a benchmark strategy (more vs. less than half) or learned heuristics ("fractions are less than 1"). In the current study, we did not systematically manipulate the stimuli or include enough trials to look at the influence of these potential heuristics on ratio-dependent responding, although this may be an interesting direction for future research. In addition, one potential issue with our current design is that we did not control for the length of decimal values. Thus, it is impossible to know for certain whether children treated the decimals like decimals or, instead, treated them like whole numbers. Although our data do suggest that in the current study decimals and whole numbers were not treated the same (DvD trials had larger ratio effects and slower performance than NvN trials, and children performed fairly well on NvD trials, which would not be the case if decimals were treated like whole numbers), future work could further extend these findings by investigating strategies and performance differences across specific magnitudes both within and between notations. Overall, however, data suggest that children can conceive of numerical magnitudes presented in distinct notations as falling within an integrated continuum (in line with Siegler et al., 2011) despite difficulties in explicitly understanding how these notations are related (e.g., Vamvakoussi & Vosniadou, 2010).

In contrast to adults who performed better on fraction arithmetic assessments (Hurst & Cordes, 2016), the current data revealed that children performed better on decimal addition relative to fraction addition. One explanation for this difference between the child findings (the current study) and the adult findings (Hurst & Cordes, 2016) may be that decimal arithmetic has the potential to be easier early on in the learning process, but remembering and/or executing procedures long after formal instruction is completed may be easier for fraction notation relative to decimal notation. Alternatively, there might not be a difference in the relative performance in decimal and fraction arithmetic across educational levels. Instead, it should be noted that the child findings (the current study) and the adult findings (Hurst & Cordes, 2016) relied on the different arithmetic operations used in the assessments; adults in Hurst and Cordes (2016) received addition, subtraction, multiplication, and division, whereas children in the current study received only addition. Some evidence suggests that a conceptual understanding of rational number multiplication and division is particularly difficult, and this understanding can be dissociated from an understanding of the individual magnitudes and procedural rules for these symbols (e.g., Lortie-Forgues & Siegler, 2017; Siegler & Lortie-Forgues, 2014). Thus, different arithmetic operations may highlight distinct patterns in the relation between rational number understanding and algebra than the one reported here with addition only. Future work should further investigate these differences by looking at distinct arithmetic operations as well as potential educational differences in how these arithmetic operations are treated.

The results of the current study, revealing that children show an early preference for decimal notation for both magnitude understanding and arithmetic, are consistent with arguments suggesting that children would benefit from instruction that involves aspects of decimal notation (e.g., percentages) before fraction instruction (Moss & Case, 1999). In particular, if it is the case that children are able to quickly grasp decimal notation for rational number magnitudes, then it may be more effective to incorporate understanding of the relational aspect of proportional values through fraction notation only after children have some competence with rational number values themselves (in decimal notation). Future research should continue to investigate whether this preference for decimal notation seen in children (for magnitude and addition; the current study) and in adults (for magnitude only; DeWolf et al., 2014; Hurst & Cordes, 2016) can be leveraged early in instruction to encourage better understanding of rational number values.

Relation to pre-algebraic ability

The current data suggest both notational and knowledge-level differences in how rational number ability predicts pre-algebra ability in fourth- to seventh-grade children. First, children's rational number sense, operationalized as their ability to compare magnitudes across notations, was a significant and unique predictor of pre-algebra ability over and above rational number addition performance and grade level. Thus, in line with other research, rational number magnitude may be particularly important for algebra (e.g., DeWolf et al., 2015b; Hurst & Cordes, 2017a). Although the unique contribution of rational number addition was only statistically marginal, rational number addition also significantly predicted algebra performance. Together, these results suggest some role for both types of knowledge, in line with previous work using a similar magnitude comparison task with adults (Hurst & Cordes, 2017a).

Analyses evaluating notation differences within magnitude and arithmetic understanding revealed some evidence of notation-dependent relations. Although arithmetic understanding in both fraction and decimal notation uniquely predicted pre-algebra ability (even though fraction notation was only marginal), this was not the case for magnitude. Specifically, when directly comparing the contributions of fraction magnitude understanding and decimal magnitude understanding, only children's understanding of fraction magnitudes, and not their understanding of decimal magnitudes, explained significant and unique variance in pre-algebra ability. These findings contrast with recent studies suggesting that decimal magnitude understanding (through either a number line task or a numerical comparison task) is a better predictor of algebra ability than fraction notation in older children and adults (DeWolf et al., 2015b; Hurst & Cordes, 2017a). On the one hand, it may be that the decimal comparisons used in the current study were too easy for children, resulting in whole-number-like performance that was almost at ceiling. In this case, it may be that the number line task or more difficult comparisons used by others (DeWolf et al., 2015b; Hurst & Cordes, 2017a) better captured decimal magnitude knowledge. On the other hand, it may be that the relation between rational number magnitude understanding and algebra may differ earlier in the educational cycle, when children are in the midst of learning these concepts. There are critical factors of early fraction and decimal education that may predict this difference across children and adults in different educational stages. In particular, it is likely that the adults (Hurst & Cordes, 2017a) and older children (seventh graders; DeWolf et al., 2015b) tested in previous studies had substantially more experience with decimal notation than the younger children in the current study and may have begun to rely more on decimal notation than fraction notation. Thus, it may be that fraction magnitudes are more critical early in rational number education, whereas decimals may become more critical after more experience with these notations. Fraction notation is notoriously difficult (e.g., Ni & Zhou, 2005) and requires an understanding of the relational aspect of fraction notation as well as an ability to compare magnitudes. Thus, an early understanding of fraction notation in children may be critically important because it reflects children's earliest conceptual learning of proportional magnitudes. Fractions are typically learned first and are particularly novel representations of quantity (i.e., the bipartite structure differs from the place value structure of whole numbers and decimals), so being able to understand this notation in the absence of an alternative (i.e., before substantial practice with decimals) is likely associated with better understanding of rational numbers and/or complex notation, a skill that is critical for algebra. Adults and older children, on the other hand, may be able to circumvent the difficulties of fraction notation by relying on decimal notation. After extensive practice with decimal notation, individual differences in how well older children and adults adapt to thinking about magnitudes using decimal notation may become a better indicator of rational number magnitude understanding and algebra performance. Future research could investigate this question directly by looking at conceptual and procedural understanding of both fractions and decimals longitudinally. This would allow us to investigate how learning alternative notations (decimals) may affect children's understanding of previously learned notations (fractions) in order to better understand the relations between these topics over time.

The fact that data from children in our study differed from those found with adults is not entirely surprising given the difference of years of experience in dealing with rational number notation. However, the contrast between our findings—that children's fraction magnitude processing was most predictive of algebra performance—and those of DeWolf et al. (2015b) obtained with slightly older children may be more striking. However, it should be noted that the magnitude measure employed in DeWolf et al.'s study, the number line task, is distinct in format and content to that of our magnitude comparison task. First, the number line task requires an explicit spatial representation (mapping number onto a line), which may involve a distinct set of abilities in addition to understanding decimal magnitudes that map onto algebra performance. Our magnitude comparison task, on the other hand, requires both an understanding of the components of rational number notation and an ability to actually compare rational number magnitudes. Thus, in line with other findings suggesting that an understanding of the relational structure of fractions is significantly predictive of algebra ability in seventh graders (DeWolf et al., 2015b), it may be that our magnitude comparison task assessed symbolic understanding of the relational structure in addition to magnitude representation. Thus, although the current study does emphasize the importance of symbolic fractions, based on the potential issues with the decimal task in the current study (i.e., ceiling performance levels and potential for whole-number-based responding) and the findings of other recent work (DeWolf et al., 2015b), the current study cannot rule out that understanding decimal notation may also be vitally important. Thus, the differences across magnitude tasks and the specific types of knowledge leading to a relation between performance on these tasks and pre-algebra/algebra are relatively open questions and should be pursued further.

Lastly, given that even children in the current study showed distinct patterns of performance between measures of accuracy and RT on the same tasks, it may be that the relation to algebra also depends on the level of behavior (e.g., accuracy vs. RT) being measured. It should be noted that there was not a significant correlation within the current dataset between average RT on the decimal versus decimal comparisons and algebra ability, similarly to what was reported with accuracy. However, this may be due, in part, to differences when measuring fluency (RT) on the comparison task and proficiency (accuracy) on the algebra assessment. Future research should investigate differences across distinct aspects of magnitude knowledge (e.g., symbolic understanding, mapping between symbols and spatial representations, comparing magnitudes) using various levels of behavior (e.g., fluency, accuracy, strategies) in order to better understand how each of these aspects of symbolic magnitude understanding is related to algebra ability.

It is also worth noting that the current study used a measure of pre-algebra ability that was primarily focused on understanding math equivalence and the functional relation among elements but did not rely on the specific use of rational numbers. This was a purposeful choice in order to look at more indirect relations between pre-algebra and rational numbers, for example working with unknown values and creating or using relations among values in a systematic way. However, there are many other components of pre-algebra and algebra understanding that the current assessment did not directly assess (e.g., Linchevski, 1995). Thus, future work should further investigate the relation between rational number processing and advanced math by specifically investigating and comparing distinct aspects of pre-algebra and algebra learning. For example, it may be that math equivalence is a particularly relevant aspect of pre-algebra in terms of fraction understanding because both require an understanding of the relations among different components of the math symbols.

In sum, the current work suggests that children represent rational number magnitudes in an integrated system across distinct notations by revealing magnitude-dependent responding within notations and when comparing across distinction notations, although integrating fractions and decimals with whole numbers may be more difficult than integrating fractions and decimals with each other. Furthermore, children showed worse magnitude and arithmetic understanding of fractions compared with decimals even early on in the learning of both notations. Lastly, the current data suggest that both rational number magnitude understanding and arithmetic ability are related to pre-algebra in children. However, the specific notations primarily driving these relations may be different from the relations seen during adulthood (Hurst & Cordes, 2017a). Therefore, future research should continue to investigate conceptual and procedural understanding of rational numbers and how these skills and conceptual knowledge are related to pre-algebra and algebra ability. Most notably, however, the current study highlights the importance of investigating children's understanding of rational numbers using both fraction and decimal notation across various educational stages because they may be differentially related to and critical for pre-algebra and algebra understanding.

Appendix

Complete list of questions from the pre-algebra assessment

1) Find the number that goes in each box:

   ☐ + 9 = 8 + 5 + 9   6 – 4 + 3 = ☐ + 3

2) Decide if each number sentence is true or not

89 + 44 = 87 + 46	True	False	Don't Know
7 + 6 = 6 + 6 + 1	True	False	Don't Know

3) In each of these sentences find the value of the letter. In other words, what value for the letter will make the sentence true?

   Find the value of z: z + z + z = z + 8
   Find the value of c: c + c + 4 = 16

For each of these, you have to either figure out the rule used in the table or use a rule give to you.

4) Fill in the missing values in the table using the rule at the top.

Rule: Multiply by 2 and then add 1	
Input	Output
2	5
3	7
4	9
5	
8	

5) Which of the number sentences shows a rule used in the table below?

A	B
3	7
2	5
0	1

   a. B = A – 4
   b. B = A – 1
   c. B = 2 x A – 1
   d. B = 2 x A + 1

For this problem, you'll have to figure out the rule that is used in the table.

6) Fill in the missing values in the table.

Column A	Column B
2	6
3	7
4	8
5	9
6	
14	
	25

7) What is the rule for figuring out what number belongs in the column B?

Complete list of questions for the Fraction Arithmetic Assessment

Problems in the fraction assessment included (in this order):

$\frac{2}{3} + \frac{5}{6}$ $\frac{2}{5} + \frac{3}{4}$ $\frac{1}{2} + \frac{4}{7}$ $\frac{5}{8} + \frac{2}{4}$ $\frac{3}{9} + \frac{2}{3}$

Complete list of questions for the Decimal Arithmetic Assessment

Problems in the decimal assessment included (in this order):

0.5 + 0.38 0.21 + 0.63 0.78 + 0.19 0.45 + 0.8 0.53 + 0.49

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