Biases and Benefits of Number Lines and Pie Charts in Proportion Representation

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Abstract

In two experiments, we investigate how adults think about proportion across different symbolic and spatial representations in a comparison task (Experiment 1) and a translation task (Experiment 2). Both experiments show response patterns suggesting that decimal notation provides a symbolic advantage in precision when representing numerical magnitude, whereas fraction notation does not. In addition, pie charts may show some advantages above number lines when translating between representations. Lastly, our findings suggest that the translation between number lines and fractions may be particularly error-prone. We discuss what these performance patterns suggest in terms of how adults represent proportional information across these different formats and some potential avenues through which these advantages and disadvantages may arise, suggesting new questions for future work.

Keywords: Fractions; Decimals; Number lines; Pie charts Rational numbers; Proportion

Introduction

How we learn and understand the relationship between numerical symbols (i.e., number words or Arabic numerals) and the quantities they represent is a critical component of numerical cognition research. While the acquisition of symbols representing discrete, countable sets (e.g., 5 ducks) is well-studied, less is known about the acquisition of symbols representing proportional information in our environment, namely, fractions and decimals. The mapping between symbolic fractions and the underlying quantity they represent seems to be particularly difficult for both children and adults (Hurst & Cordes, 2015; Ni & Zhou, 2005), potentially due to the complicated nature of the symbolic representation (e.g., bipartite structure of fractions, etc.), as well as the variety of ways in which proportional information is spatially depicted (e.g., number lines, pie charts). In the current study, we investigated how adults map between symbolic (fractions, decimals) and spatial (number lines, pie charts) representations of proportional quantity.

Evidence from infants (e.g., Denison & Xu, 2009; McCrink & Wynn, 2007), children (e.g., Boyer, Levine, & Huttenlocher, 2008), and adults (Fabbri, Caviola, Tang, Zorzi, & Butterworth, 2012; Matthews & Chesney, 2015) suggests that even by a young age, we can understand proportional information when presented non-symbolically. However, mapping between these non-symbolic representations and symbolic representations (fractions, decimals) is not a trivial task. For example, the bipartite structure of fraction notation can lead people to treat fractions as two distinct whole numbers, rather than a coherent unit (e.g., Ni & Zhou, 2005) and superficial similarities between whole number and decimal notation can lead children to make place-value errors, like “longer decimal = larger value” (e.g., 0.313>0.43; Desmet, Gregoire, & Mussolin, 2010). Despite the presence of whole number biases in both fraction and decimal notation, differences have been noted in the affordances of these distinct representations. For example, the bipartite format of fraction notation has been shown to better convey discrete, part-whole information (DeWolf, Grounds, Bassok, & Holyoak, 2014; Rapp, Bassok, DeWolf, & Holyoak, 2015). Decimal notation, on the other hand, has been shown to better convey continuous numerical magnitude information (DeWolf et al., 2014; Hurst & Cordes, 2015), making it more closely align with continuous quantities (Rapp, et al., 2015). However, how these properties of the symbolic representations (i.e., continuous magnitude versus discrete part/whole information) align with conventional spatial representations and in turn impact adults’ ability to use and manipulate proportional information in both symbolic and spatial forms is an open question.

Spatial representations, particularly pie charts and number lines, are commonly used in educational instruction and in every day communication of proportional information (e.g., pie charts in investment portfolios). Thus, investigating how these spatial, non-symbolic representations are interpreted and manipulated could shed light on numerical representation and educational practices. Although magnitude information in both pie charts and number lines can be represented in a continuous fashion (meaning, not broken up into unit pieces), these spatial representations are perceptually distinct and may offer different advantages and disadvantages for relating to decimal and fraction notation. For example, substantial research suggests that number lines may be best for representing continuous magnitude as it aligns with the manner in which we are posited to represent number – along a mental number line (e.g., Wang & Siegler, 2013). If so, rational number magnitudes presented in decimal notation may more naturally translate to number lines than those presented in fraction notation. On the other hand, pie charts may be better at conveying part-whole structure, since the "whole" refers to the complete circle ("whole" may not be as spatially defined in a number line), highlighting an alignment between fractions and pie charts.

In the current study we address how adults map between symbolic and spatial representations of proportional magnitude using a magnitude comparison task (Experiment 1) and a direct translation task (Experiment 2). In particular, we address the question of rational number magnitude representation in two ways: (1) the ease of magnitude access in various forms and (2) the representational flexibility offered by particular representations.

Experiment 1

Methods

Participants Fifty-four Boston College students (18 to 24 years, M=19.2 years, 39 Female) participated in exchange for course credit. An additional nine adults were excluded based on our exclusion criteria (see Data Analysis).

Stimuli The magnitude comparison task stimuli were fractions, decimals, pie charts, and number lines representing proportions between 0 and 1. On each trial, participants were presented with two proportion stimuli: Fraction vs. Fraction (FvF), Fraction vs. Pie Chart (FvP), Fraction vs. Number line (FvL), Decimal vs. Decimal (DvD), Decimal vs. Pie Chart (DvP), and Decimal vs. Number line (DvL). Each type of trial was presented in a separate block. The two proportion stimuli presented on each trial differed by one of two approximate ratios: Small (approximately 1.125, ranging from 1.08 to 1.14) and Large (approximately 1.5, ranging from 1.43 to 1.52).

The proportion magnitudes used in the fraction and decimal trials were approximately matched (e.g., 1/3 would be converted to 0.33). The magnitudes used in the symbolic comparisons (FvF and DvD) were identical to the magnitudes in the symbolic versus non-symbolic comparisons (FvP, FvL, DvP, and DvL). However, in the pie chart (FvP and DvP) and number line (FvL and DvL) blocks, one of the values in each stimulus pair was represented using a pie chart or number line (respectively) instead of a symbol. The choice of which stimuli were represented using a spatial representation was determined so that the spatial representation conveyed the larger magnitude on half the trials.

The symbolic fraction comparisons were created so that on the FvF trials, the two symbolic fractions (4.7 cm high x 3.1 cm wide) were made up of four distinct positive integers (e.g., 2/3 vs. 3/4 would not occur), in order to avoid the use of denominator or numerator based strategies. On the DvD trials one decimal value included digits to the thousandth position (i.e., three digits after the decimal point; e.g., 0.635; 5.5 cm wide) and one decimal value included digits to the hundredth position (i.e., two digits after the decimal point; e.g., 0.76; 4cm wide). The longer decimal was larger on half the trials in order to make length an unreliable strategy. Pie chart stimuli were white circles (radius=3.4cm) with the corresponding proportion filled in black (clock-wise). The number line stimuli were 8 cm lines extending from the end points of 0 to 1 (labeled under the left and right end points, respectively) with a location on the line indicated by a 0.7 cm vertical line.

Procedure All participants completed a magnitude comparison task in which they were shown two values and asked to choose which was larger as accurately and quickly as possible. There were 8 set orders of the six blocks and the order was counterbalanced across participants with an approximately equal distribution of participants in each order. Each block contained four unique trials from each ratio bin shown twice (once with the largest on the right and once with the largest on the left) in a random order, resulting in 96 total trials (4 comparisons x 2 shown twice x 2 ratios x 6 blocks).

Each stimulus remained on the screen until the participant selected an answer by pressing the left or right arrow on the keyboard corresponding to their response (left or right quantity, respectively). Between each stimulus presentation a fixation cross (0.5cm x 0.5cm) was presented in the center of the screen for 1000ms. Each block started with an instruction screen and two practice trials with feedback. The experimenter remained quietly in the room with the participant throughout the task and answered any questions about the procedure prior to each block.

Data Analysis Reaction time (RT) was the primary dependent variable because accuracy was fairly high with low variability. Only RTs from correct trials and those that were within 3 standard deviations of the individual's mean RT of that trial type were included in analyses. Only data from those blocks in which the participant scored at or above chance (4/8 questions correct) were included. Participants who had missing data based on these criteria were excluded from all analyses (N=9). At the group level, average RTs for each cell that were more than 3 standard deviations away from the group mean were replaced with the next highest value that was not considered an outlier. This resulted in 18/1080 data points being replaced (~1.7%).

Results and Discussion

Comparison Performance First, we used a Repeated Measures ANOVA to investigate whether RT differed across Symbolic Notation (2: Fractions, Decimals), Comparison Type (3: Same Symbol, Pie, Line), and Ratio

(2: Small, Large). See Figure 1 and Table 1 for the descriptive statistics.

Table 1: RT in milliseconds (standard error) for each type of trial in fraction (top) and decimal comparisons (bottom)

	FvF	FvP	FvL	Avg
Small Ratio	3479 (177.8)	3186 (159.1)	3424 (161.9)	3363 (138.0)
Large Ratio	2609 (134.1)	2490 (130.0)	2050 (89.7)	2383 (96.5)
Avg	3044 (145.6)	2837 (125.8)	2737 (114.2)	2873 (109.6)

	DvD	DvP	DvL	Avg
Small Ratio	1098 (30.6)	2446 (114.1)	2792 (136.2)	2112 (76.5)
Large Ratio	976 (23.3)	1586 (45.7)	1618 (43.2)	1394 (30.2)
Avg	1038 (25.9)	2016 (71.3)	2205 (80.9)	1753 (48.9)

There was a main effect of Ratio $F(1,53)=161.8$, $p<0.001$, *partial* $\eta^2=0.75$, Ratio X Symbol interaction, $p=0.003$, *partial* $\eta^2=0.15$, Ratio X Comparison interaction, $p<0.001$, *partial* $\eta^2=0.4$, and Ratio X Symbol X Comparison interaction $p<0.001$, *partial* $\eta^2=0.15$. Pair-wise comparisons investigating whether there were ratio effects (significantly slower RTs for Small ratio trials than Large ratio trials) in each block separately, found significant ratio effects in each of the six blocks (all *p*'s<0.001). However, comparisons involving number lines (DvL and FvL) had significantly larger ratio effects than those involving pie charts (DvP and FvP; *p*'s<0.007) and than those involving only symbols (DvD and FvF; *p*'s<0.03).

Thus, the existence of ratio effects in our data suggests that adults did access the approximate magnitudes of proportional values represented in decimal, fraction, pie chart, and number line form. However, the size of the ratio effects varied depending on the representational form. In particular, those comparisons involving number lines had the highest ratio effects, above those involving pie charts or only symbols. This supports the general idea that number lines are thought to communicate magnitude information better than other representations (Cramer, Post, & DelMas, 2002; Wang & Siegler, 2013). Since number lines are continuous, ordered, and approximate, adults may have been more inclined to use magnitude-based strategies, leading to those comparisons being more dependent upon the particular magnitudes (i.e., higher ratio effects).

There was an overall main effect of Symbol, $F(1,53)=158.8$, $p<0.001$, *partial* $\eta^2=0.75$, with Fraction comparisons taking longer than Decimal comparisons. This finding is consistent with other work suggesting that magnitudes represented in fraction notation take longer to access (Hurst & Cordes, 2015). However, there was also a Symbol X Comparison interaction, $F(2,106)=55.8$, $p<0.001$, *partial* $\eta^2=0.5$. Follow up tests indicated that participants were much faster when comparing two decimals than comparing a decimal with either a pie chart or a number line (*p*'s<0.001). Conversely, comparisons involving two fractions were slower than those involving a fraction and a number line ($p=0.024$) or a fraction and a pie chart ($p=0.07$, *marginal*). Thus, while decimal notation seemed to offer an advantage over spatial representations (that is, performance on trials involving two decimals was better than when a spatial representation was involved), fraction notation appeared to present a symbolic *disadvantage* when processing numerical magnitude. Processing proportional information in fraction notation may not only be more difficult than decimal notation, but it may also be more difficult than processing proportional information via conventional, analog spatial representations. Given that the purpose of numerical symbols is to provide a precise way to communicate numerical magnitudes, it is counter-intuitive that discrete fraction notation does not provide more precise magnitude information than analog spatial representations.

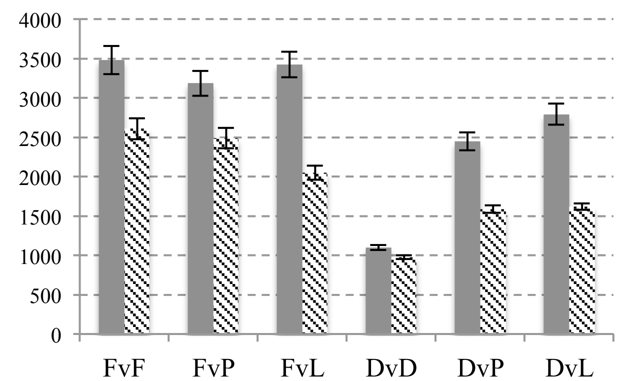

Figure 1: RT on correct trials for the Small (left, grey bars) and Large (right, striped bars) Ratio across each of the six comparisons

Comparison Biases Next, we were interested in whether these distinct symbolic and spatial formats may give rise to biases in the representation of proportional information. That is, did performance differ depending on if the larger value (i.e., the correct response) was symbolic or spatial? We conducted a 2 (Spatial: Pie Chart (PC), Number line (NL)) X 2 (Symbol: Fraction, Decimal) X 2 (Format of Largest: Spatial, Symbolic) repeated measures ANOVA on average RT. There was a main effect of Symbol, $F(1,53)=57.1$, $p<0.001$, *partial* $\eta^2=0.5$ and a Symbol X Spatial interaction, $F(1,53)=10.0$, $p=0.003$, *partial* $\eta^2=0.16$, showing the same pattern of findings reported earlier. However, the critical tests were those involving the format of the largest value. There was a main effect of Format,

$F(1,53)=29.6$, $p<0.001$, *partial* $\eta^2=0.4$, suggesting a bias toward indicating that the symbol was smaller than the spatial representation. However, this was was qualified by two further interactions. A Symbol X Format interaction, $F(1,53)=4.1$, $p=0.048$, *partial* $\eta^2=0.07$, indicated that this bias (faster responses when the symbol was smaller) was greater when comparisons involved fractions (when fraction symbol was the largest value: 2934ms vs. when the fraction was smallest: 2579ms, $p<0.001$) than when the comparisons involved decimals (when decimal symbol was largest: 2136ms; vs. when decimal was smallest: 1973ms, $p<0.005$). Second, there was a Spatial X Format interaction, $F(1,53)=97.6$, $p<0.001$, *partial* $\eta^2=0.65$, which indicated that this bias only held when comparing a symbol to a pie chart. That is, participants were quicker to respond when the pie chart represented the larger value (2097ms) than when it was smaller (2747ms; $p<0.001$); but the reverse pattern was found for number lines, where participants were quicker to respond when the number line depicted a smaller value than the symbol (2323ms vs. 2455ms; $p=0.028$). Thus, adults may be biased toward thinking that pie charts are larger than fractions and decimals, but that number lines are smaller than fractions and decimals. Moreover, given that the symbol bias was greater for fractions, it may be that adults have a tendency to think a fraction represents a smaller value than the equivalent decimal. However, previous studies that had adults directly compare fractions and decimals did not find evidence of this pattern (Hurst & Cordes, 2015). Thus, it may be that these biases only arise when adults are directly thinking about number lines and pie charts in relation to the fractions and decimals.

Thus, in Experiment 1, we show that although adults accessed magnitude information in all comparisons (as evidenced by ratio effects), number lines in particular may encourage magnitude-based comparison strategies (as evidenced by largest ratio effects). Most notably, comparisons involving only symbols were privileged (i.e., faster) over those involving spatial representations, but only for comparisons involving decimal notation; that is, decimals provided a symbolic advantage for conveying magnitude, but fractions did not. Furthermore, although participants were asked to compare across formats, it is unclear how they went about performing these comparisons. Did they explicitly translate magnitude information across representations in order to make the comparison? In Experiment 2, we investigate performance when translating across representational formats (spatial to symbolic and vice versa) in order to shed light on the accuracy with which proportional information is represented in each form.

Experiment 2

Methods

Participants Forty-one adults (18 to 21 years, M=19.2 years, 36 Female) were included in all analyses. Three additional adults participated but were excluded for not following instructions properly. All participants received course credit and none participated in Experiment 1.

Stimuli The same pie chart and number line stimuli from Experiment 1 were used. The fractions and decimal values were equivalent to the pie chart and number line magnitudes so that the magnitudes used in all eight blocks were the same (up to rounding error for decimal magnitudes). All other aspects of the stimuli were identical to Experiment 1.

In order to input their answer, a text box was provided for fractions and decimals. When translating to a number line, a number line (7cm) from 0 to 1 (marked under the left and right endpoints respectively) was shown and participants could move a small vertical line (0.5cm) along the line to select their response. When translating to a pie chart, an empty circle (radius=3cm) was shown on the screen with a 3cm line extending from the top of the circle to the center. Another line extended from the center of the circle to the edge of the circle and moved around the circle corresponding to the location of the participant's cursor. When the participant clicked with the mouse, the pie chart filled in the portion between the top vertical line and the participant's adjustable line black.

Procedure Participants completed a Translation task in which they were given a quantity represented using either a fraction, decimal, pie chart, or number line, and asked to estimate that quantity using a different representation. The task included eight distinct blocks: Pie to Fraction (PtoF), Fraction to Pie (FtoP), Line to Fraction (LtoF), Fraction to Line (FtoL), Pie to Decimal (PtoD), Decimal to Pie (DtoP), Line to Decimal (LtoD), and Decimal to Line (DtoL). There were 8 trials per block, making the task 64 trials (8 trials per block x 8 blocks). The blocks were presented in 8 set orders, counterbalancing across participants.

Prior to each block, participants were shown an instruction screen and the experimenter showed them how to input their response in the correct format. On each trial, the target value was displayed on the left and the empty response (text box, empty line, or empty pie chart) was displayed on the right. Participants could fill in their answer by clicking in a location (number line and pie chart) or typing in a response (decimal and fraction). Participants pressed a button to move on to the next question. The experimenter remained quietly in the room the entire time.

Data Analysis Mean Absolute Error (MAE), calculated as the absolute value of the difference between the correct proportion and the response, was the primary dependent variable.

Exclusion criteria and outlier treatment was identical to that of Experiment 1. At the group level, 8/328 data-points (~2.4% of the data) were considered outliers and replaced with the next highest non-outlier.

Results and Discussion

We conducted a 2 (Direction: Symbol to Spatial vs. Spatial to Symbol) X 2 (Symbol Type: Fraction (F) vs. Decimal (D)) X 2 (Spatial Type: Pie Chart (PC) vs. Number Line (NL)) repeated measures ANOVA on MAE (see Table 2 for descriptive statistics involving PCs (top) and NLs (bottom)).

Table 2: MAE (standard error)

	Translating		
	to PC	*from PC*	Avg
F	0.039 (0.004)	0.035 (0.006)	0.033 (0.003)
D	0.033 (0.002)	0.032 (0.003)	0.036 (0.002)
Avg	0.036 (0.002)	0.033 (0.003)	0.035 0.003

	Translating		
	to NL	*from NL*	Avg
F	0.044 (0.002)	0.050 (0.008)	0.041 (0.004)
D	0.044 (0.003)	0.031 (0.002)	0.044 (0.002)
Avg	0.044 (0.002)	0.041 (0.004)	0.042 0.003

First, there was a main effect of Symbol Type, $F(1,40)=4.5$, $p<0.05$, *partial* $\eta^2=0.1$, revealing adults were more accurate when translations involved a decimal (compared to those involving a fraction). This finding aligns with those of Experiment 1 and previous work (e.g., Hurst & Cordes, 2015) suggesting that decimals provide more accurate magnitude information than fractions. There was also a main effect of Spatial Type, $F(1,40)=9.76$, $p<0.004$, *partial* $\eta^2=0.2$, revealing, in contrast to previous research highlighting the benefits of number lines (Wang & Siegler, 2013), that adults were more accurate when translations involved a pie chart compared to a NL.

Furthermore, there was a three-way interaction between Direction, Symbol, and Spatial Type, $F(1,40)=5.0$, $p<0.05$, *partial* $\eta^2=0.1$. We investigated this three-way interaction further by conducting two 2 X 2 repeated measures ANOVAs, looking at the effect of Direction (2) and Symbol Type (2) separately for PCs and NLs.

The 2 x 2 ANOVA on data from trials involving PCs revealed no main effects or interactions (p's>0.2) suggesting that performance was very similar regardless of whether the translation involved a fraction or a decimal or whether the PC was the target or the initial value.

However, the pattern was not the same when we looked at the data from trials involving NLs. The 2 x 2 ANOVA on data from NL trials revealed a main effect of Symbol, $F(1,40)=4.23$, $p<0.05$, *partial* $\eta^2=0.1$, again suggesting that trials involving decimals resulted in lower error than those involving fractions. There was no main effect of Direction, however, there was a significant Symbol X Direction interaction, $F(1,40)=6.01$, $p<0.02$, *partial* $\eta^2=0.1$. Follow up t-tests revealed that when translating proportional information *to* a NL, performance was equally accurate regardless of whether the starting value was a fraction or a decimal ($p=0.8$). However, when translating proportional information *from* a NL, performance was significantly better when converting into a decimal compared to a fraction ($p=0.023$). That is, adults were particularly inaccurate when converting from a NL into a fraction, a finding that may be attributed to two factors: (1) the fact that fractions are particularly inaccurate for representing magnitude information (Hurst & Cordes, 2015) and (2) a mismatch in the way magnitude information is represented in fraction form (part-whole) and in NL form (linear, continuous).

General Discussion

In this study, we investigated adults' representation of proportional magnitudes across common symbolic (fractions, decimals) and spatial (number lines, pie charts) formats. In line with the exact precision offered by symbols, decimal notation provided adults with the greatest level of precision when comparing magnitudes in Experiment 1 and when translating between different representations of magnitude in Experiment 2. Thus, decimals seemed to offer the symbolic advantage that is expected. In contrast, fractions did not; comparisons involving exclusively fraction notation took the longest, even compared to those involving spatial, non-symbolic representations. These findings of a symbolic magnitude advantage for decimals adds to a growing literature (DeWolf et al., 2014; Hurst & Cordes, 2015) by further suggesting that decimals are also more accurate than analog spatial representations of proportion and, conversely that fractions are potentially less precise at conveying magnitude information than spatial representations.

In addition, Experiment 2 suggested some advantage for estimating proportional magnitude using pie charts over using number lines. Given the literature suggesting that teaching fractions using number lines is beneficial, this finding is counterintuitive (e.g., Cramer et al., 2002; Wang & Siegler, 2014). However, it is important to note that these adults were not receiving instruction on number lines and pie charts, and what's more, in line with instructional practices in the U.S. over the past 15 years, these adults likely received a curriculum that relied heavily on pie charts. Keeping this in mind, there are at least two potential explanations for these findings. On the one hand, adults may have attempted to engage in a partitioning strategy (i.e., dividing the image into the total number of parts) for both pie charts and number lines, but executing that strategy may have been easier with a pie chart. For example, since both pie charts and number lines were presented continuously (i.e., un-partitioned), it may be easier to visibly estimate partitions in a pie chart because it is symmetric through the center of the circle. Thus, the same partitioning strategy may not be equally accurate across the two representations. On the other hand, adults may have opted not to engage in partitioning with number lines but instead invoked an altogether different strategy when faced with a number line trial. For example, given its continuous nature, participants may have attempted to estimate the proportion of the line that was marked using a magnitude-based strategy. This hypothesis (that number lines evoke more approximate strategies than pie charts) is also consistent with the relatively high ratio effects found in Experiment 1 when comparing number line magnitudes. Given that fractions are particularly poor conveyors of magnitude information (Hurst & Cordes, 2015), this poor strategy selection may have led to lower response precision particularly when translating between number lines and fractions. Since fractions are more aligned to discrete representations (Rapp et al., 2015), it may be that the approximate strategy evoked by the number line is particularly difficult to translate into fraction form. Further research could investigate how these performance differences arise by investigating specific strategies invoked for different representations (i.e., pie charts and number lines) as well as in different tasks. Although we found that number lines and fractions may be misaligned in some translation contexts in Experiment 2, this response penalty (slower RTs) was only found when translating *into* a fraction. This finding suggests that adults in Experiment 1 may not have mentally converted spatial representations into fractions when comparing across formats, but instead did the reverse - converting fractions into spatial representations - to make the comparison.

In conclusion, the current study adds to the growing literature investigating the advantages and limitations of proportion representation in various forms. Results suggest that the mapping between symbolic and spatial representations of proportion may heavily depend on the specific symbolic notation involved and on the type of spatial representation of quantity. Future work should attempt to isolate how these different mappings are learned in young children and what impact these distinct representational formats may have on their understanding of proportional information.

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