Old Teachers, Old Ideas, and the Effect of Population Aging on Economic Growth

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Abstract

As populations age, the degree to which workers' human capital reflects the cutting edge of technology falls because education took place further in the past. This "pure vintage" effect of aging is well known. In this paper, we explore a second effect of aging: In an older population, older teachers pass on knowledge that was current further in the past. We show that this "teacher multiplier" can significantly increase the technological backwardness of the labor force. We present both an analytic model that can be solved for steady states and a numerical model that can describe transitions in the average vintage of human capital as population age structure changes over time. We also discuss evidence on the effect of age on the technological up-to-dateness of workers in general and teachers in particular.

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1. Introduction

The population aging that is now underway in almost all of the developed world will have large economic consequences (Bloom et al. 2010). The effect of aging on the government budgets are already starting to be felt. The redistribution of population with respect to the life cycle of capital accumulation is also expected to affect asset markets (Bosworth et al. 2004).

One aspect of aging that is of particular interest is the change in the nature of the labor force. As the overall population ages, we expect that the average age of workers will also rise significantly. The extent to which the aging of the population translates into the aging of the workforce is dependent on age-specific labor force participation rates, but the current projections by the US Bureau of Labor Statistics suggest that we should expect a significant change in the age composition of the labor force. [1] In 1998, workers age 55 and older accounted for 12 percent of the US civilian labor force, but by 2018, older workers are projected to account for almost a quarter of workers (Toossi 2009).

This aging of the labor force is in turn expected to produce a number of results. Older workers tend to have both lower unemployment rates and lower mobility – thus whether an older labor force will have lower average unemployment or simply less flexibility is not clear. The increase in the ratio of more senior to less senior workers may lower the return to seniority and flatten the cross-sectional wage profile. In this paper, we examine another effect of population aging on the nature of the labor force, specifically, the effect of aging in increasing the gap between the technical knowledge embodied in the skills of workers and the cutting edge of technology. One aspect of aging's effect on this gap is well known. Older workers will have been out of school longer, and thus, in their formal education, will have learned about an older technology. We call this the "pure vintage" effect of aging on the technological knowledge or workers. Exacerbating this pure vintage effect of aging is the fact that workers near the end of their careers have less incentive to invest in human capital in order to keep their technological knowledge updated, since they have a shorter horizon over which to reap the returns from such an investment.

In this paper we are concerned with a second effect of population aging on the technological up-to-dateness of workers: that is, in the presence of population aging, the

[1] After decades of decline, the trend in the labor force participation of older male workers has changed and labor force participation increased significantly between 1988 and 2008 (Toosi 2009). Researchers have offered a number of explanations for this change in trend including an increase in life expectancy, the rising labor force participation of women and a desire for shared leisure, a reduction in physically-demanding jobs, changes in the Social Security program, less access to employer provided retiree health insurance, the end of mandatory retirement, and a shift in the structure of pension plans from defined benefit plans which include age-specific retirement incentives to defined contribution plans where pension assets continue to accumulate with additional years of employment (Munnell and Sass 2007, Schirle 2008, Monk and Munnell 2009, Friedberg and Webb 2005). Given the numerous forces pushing older workers to stay employed longer, the US Bureau of Labor Statistics projects that the trend of increasing labor force participation will continue through at least 2018 (Toosi 2009).

average worker will have had older teachers, who would have been teaching older knowledge. This is a second reason why aging will lead to a technologically out-of-date workforce.[2]

Although the model that we present in this paper does not differentiate between different levels of education, our main concern is with tertiary education. The basic skills taught in primary and secondary schools are, for the most part, not closely linked to particular technologies. A student finishing high school today with a vintage 1950 education would be only slightly disadvantaged in the current labor market. By contrast, tertiary education is more closely concerned with conveying skills appropriate to the current technological environment. This is even more the case with post-graduate training. Finally, in the most developed countries, teachers themselves are educated at least through the tertiary level.

In our model, we assume that teachers age at the same rate as the overall population, but there is empirical evidence that teachers, and particularly college professors, may be aging at a faster rate. In Italy, for example, 24 percent of professors are aged 60 or older. The situation is more extreme for physicists - 41 percent of physicists are aged 60 or older (Labini and Zapperi 2007). The job of a university professor is not physically demanding, the life expectancy of educated workers is increasing, and employers have very few tools to encourage retirement. In the US, the mandatory age for faculty was abolished in 1994. This has led faculty to delay retirement, and will exacerbate the general population aging effects (Ashenfelter and Card 2002). Additionally, college teachers in the United States generally have defined contribution pension plans, which do not contain incentives to retire (Friedberg and Turner, 2010). Although many European countries still have mandatory retirement for faculty, the age varies by country. In Italy, the mandatory retirement age can be as high as 75, a fact which likely contributes to the observed distribution.

Having said this, however, there is much about our model that is also applicable to lower levels of education. For example, the use of recent recently developed educational technology by teachers in high and middle school should depend on teachers' own training in that technology (Bebell et al. 2003). And exposure to recent technology will be important. While the age distribution of primary and secondary teachers is likely not at old as extreme as the age distribution of Italian physicists, primary and secondary teachers in the US are also getting older. The modal age for teachers in 1970 was 22 and by 2000 the modal age had increased to 48 (Friedberg and Turner 2010). Unlike college professors, teachers in US primary and secondary schools traditionally receive defined benefit pensions that are structured to provide strong incentives for retirement. However, the ongoing retrenchment in state and local budgets in the US is almost certain to lower the value of such pensions, and to lead many teachers to stay on the job longer than had been expected (Friedberg and Turner 2010).

[2] The literature on the interaction of aging and technological progress is quite large, and we do not mean to suggest that the effects we study in our model are the only important ones. In the analysis of Boucekkine, Raouf, David De la Croix, and Omar Licandro (2002), the ages of both school leaving and retirement are endogenous, as is the rate of technological progress. The model that we present is radically stripped down in the interest of highlighting a single, novel channel.

The rest of this paper is organized as follows. In Section 2, we start by discussing evidence on the pure vintage effect of aging on an individual's distance from the technological frontier, that is, how up to date the technology he or she uses is and the degree to which a teacher's up to date-ness depends on his or her age. In Section 3, we construct a formal model of the gap between average worker knowledge and the technological frontier, and how this is affected by population age structure. This section also presents some illustrative calculations showing the magnitude of the teacher effect for different parameter values, and considers a quantitative comparison of the human capital effects considered in this paper and the more tradition Solow model effect of slowing population growth. Section 4 presents some extensions of the model, and in particular considers the dynamic path of technological backwardness of the labor force in response to a change in population growth rate. Section 5 concludes.

2. Age and Technological Backwardness of Workers and Teachers

A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up familiar with it.

Max Planck, 1918 Nobel Laureate in Physics and Founder of Quantum Theory

In the basic life cycle model of human capital accumulation, individuals will choose to acquire most of their human capital prior to entering the labor market (Ben-Porath 1967). A greater investment in human capital at younger ages is rational because the investments during a later period would produce returns over a shorter period. While a 22-year old may have 40 or more years to reap the rewards of an additional year of study, a 52-year old has a much more limited time horizon. This pattern of early investment generates the age-earnings profile where earnings are relatively low when workers are young, and then rise at individuals move into the labor force and start to capture the returns on their human capital investments.

This tendency to acquire human capital early to maximize the potential years of returns is further reinforced by the opportunity costs of schooling. When workers are young, the opportunity costs of education and training are relatively low. As workers enter the labor force, gain experience, and start to receive the returns on their initial human capital investment, the opportunity cost of additional education and training are much more significant.

If workers do continue their training after the initial stage of investment, most of the training is either informal or employer-provided training. Becker (1962) introduced a distinction between *general training,* which enhances the productivity of the individual for all types of jobs, and *specific training*, which only enhances the worker's productivity for one particular type of job. With a competitive labor market, employers have no incentive to provide general training because an employee could take that training to another employer. If the frontier of knowledge has expanded since a worker finished his period of education, employers will not provide the general training that is necessary for the worker to upgrade their skills and reach the cutting edge. Workers will only choose to invest in this training if the marginal discounted returns of this investment is greater than the cost, and with each additional year of aging, the likelihood of the additional investment will decline.

As workers age, the formal human capital that they had acquired in their primary period of education and training becomes more and more out of date. Their knowledge is of an older vintage that is constantly moving further behind the technological frontier (Rosen 1975). Employers do not have the incentive to provide the training that would allow workers to catch up because workers could take this general training to other firms. Workers have less incentive to update their skills as their opportunity costs of training grow and their potential period to reap the rewards declines.

If this vintage effect of aging is real, we would expect to find that older workers are further from the frontier of knowledge. For concreteness, we focus on doctors. This profession has the advantage of being widely studied, although we acknowledge that it is not clear how the age profile of technological backwardness in this profession compares to other fields. On the one hand, the medical profession is technologically very intensive and the growth rate of technology might be higher than in other areas, leading to more rapid obsolescence. On the other hand, doctors have licensing requirements and other forms of supervision (not to mention legal liability) that might force them to remain more up to date than workers in other fields.

In casual observations at hospitals there are signs that younger doctors are more eager to adopt the latest technologies. Surveys of physicians confirm that younger doctors are significantly more likely to use personal digital assistants (PDAs) to access medical information, track patients, and review treatment protocols and potential drug interaction (Garritty et al. 2006). While there is reason to believe that being at the cutting edge of technology is advantageous, in this case, the impact on medical outcomes is not clear.

In other cases, distance from the frontier of medical knowledge has strong implications for patient outcomes. Choudhry et al. (2005) conducted a systematic review of all medical studies that examine the relationship between clinical experience and quality of care. They assessed studies that looked at differences in knowledge, adherence to standards of care for diagnosis, screening, or prevention, adherence to standards of care for therapy, and mortality outcomes. More than half (52 percent) of the studies reviewed found a negative association between age and quality of care for all outcomes. Another 20 percent found a negative association for some of the outcomes. Only 1 study found a positive relationship between clinical experience and the quality of care.

All of the studies that assessed the knowledge of practicing doctors reported a negative association between knowledge and experience. This is in spite of the fact that experience on its own should be a good. In one study, physicians were surveyed about the advantages of different treatments for acute myocardial infarctions (Ayanian et al. 1994). Randomized controls trials conducted over the past decades have found clear evidence that the use of thrombolytic agents, aspirin, and beta-blockers can increase survival for patients with myocardial infarction, while the use of lidocaine and calcium-channel blockers have no impact on survival. For many doctors, this clear evidence was generated after they had finished their formal training. If doctors fail to appropriately update their knowledge as they age, we would expect that younger doctors had more knowledge of the appropriate treatment protocols. Unfortunately, that is exactly what Ayanian et al. found.

After examining all of the studies that measured the association of clinical experience and outcomes, Choudhry et al. conclude that while skill depreciation could explain some of the findings, the more central concerns are skill obsolescence and the inadequate updating of physician "tool-kits" by older doctors. Practices that involved fundamental changes in theory, as opposed to simply new techniques, such as using less aggressive therapy for early-stage breast cancer, may be particularly difficult for older doctors to adopt.

This backwardness is also observed more broadly among scientists. Older scientists cite older knowledge in their research (Gingras et al. 2008). While the failure of younger authors to acknowledge earlier research may also be problematic, the citation patterns of older researchers suggest they are working from an older knowledge base. These citations patterns are confirmed by other research that finds that as researchers age they spent less time reading new journals and following the latest developments in their field (van Dalen 1998).

Teacher Effects

As the aging of the labor force moves the average worker's human capital further from the technological frontier, there are potentially important externalities to consider. If there are positive externalities from the average stock of human capital, the presence of elderly workers may lower overall productivity (Sala-i-Martin 1995). In this paper we are concerned with a second effect of population aging on the technological up-to-dateness of workers: that is, in the presence of population aging, the average worker will have had older teachers, who would have been teaching older knowledge. This is a second reason why aging will lead to a technologically out-of-date workforce.

In the production of human capital, individuals play a central role. We learn from our teachers, and the same aging doctors and scientists that are making decisions based on an older vintage of knowledge are teaching the innovators of tomorrow. In the diffusion of new technology, information moves from person to person (Keller 2004).[3] If teachers are not at the frontier of knowledge, their students will enter the labor market behind.

The transfer of human capital from teacher to student is going to be dependent on two factors – the knowledge that the teacher has to transmit and the ability of the teacher to transmit knowledge. While the ability of the teacher to transmit knowledge certainly

[3] One might argue that internet technology has allowed knowledge to become a public good with individuals able to access cutting edge science instruction using MIT's OpenCourseWare without a teacher as an intermediary. Yet, these public access sites do not appear to be replacing traditional edition. In 2010, the OpenCourseWare site had 9.6 million visitors with each visitor returning to the site an average of 1.6 times.

increases during the first few years of teaching experience, there is little evidence that the returns to experience continue to grow throughout a teacher's career (Rockoff 2004). Even if a teacher's pedagogical skills do not decline at older ages, the gap between the teacher's knowledge and the cutting edge knowledge will grow each year. The gap in knowledge will be particularly large for fields like computer science and engineering that evolve quickly.

3. Basic Model

We take the growth of technology to be exogenous. Let *A*(t) be the state of technology at time *t*. Technology grows at rate *g*. We assume that the population grows at rate *n*. We normalize the age at which workers enter the labor force to be zero. Workers retire at age *T* and there is no mortality before retirement. We consider a stable population in which *n* has been constant for sufficiently long that the age-structure is unchanging. In this stable population, there will be e^{-ns} workers of age *s* for each worker of age zero (below we discuss transitional dynamics).

Let *b*(*a*,*t*) be the knowledge level of a worker age *a* at time *t*. The average level of knowledge of the labor force, *B*(*t*) is given by the equation

(1) $$B(t) = \int_0^T b(a,t)e^{-na}da \Big/ \int_0^T e^{-ns}ds$$

We assume for simplicity that teachers are a representative sample of the labor force, so that *B*(t) will also be the average level of technological knowledge among teachers.

The knowledge of students who are just finishing their schooling and entering the labor force, that is, *b*(0,*t*) is determined by both the knowledge level of their teachers and the level of technology at the time of their graduation. We model this idea by assuming that

(2) $$b(0,t) = \alpha A(t) + (1-\alpha)B(t)$$

The smaller is the parameter α, the more important are a student's teachers relative to the state of knowledge in the economy in determining her own knowledge at the time of graduation.

In this basic version of the model we assume that once a person has begun working, she no longer acquires any new knowledge, that is, $b(a,t) = b(0,t-a)$. In the extension discussed below we alter this assumption to allow for knowledge acquisition after graduation.[4] In the basic model, the average level of knowledge among workers in the economy will be

(3) $$B(t) = \left(\int_0^T b(0, t-s) e^{-ns} ds) \right) \bigg/ \int_0^T e^{-ns} ds$$

In the steady state of the model, the knowledge level of new graduates and the average knowledge level of the labor force will both grow at the same rate as the aggregate level of knowledge, that is:

(4) $$\frac{\dot{A}(t)}{A(t)} = \frac{\dot{B}(t)}{B(t)} = \frac{\dot{b}(0,t)}{b(0,t)} = g$$

The above equation allows us to re-write equation (3) in terms of the knowledge level of current graduates,

(5) $$B(t) = \left(\int_0^T b(0, t) e^{-(n+g)s} ds) \right) \bigg/ \int_0^T e^{-ns} ds$$

This can be further simplified to

(6) $$B(t) = b(0,t) \frac{n}{n+g} \left(1 - e^{-(n+g)T}\right) \big/ \left(1 - e^{-nT}\right)$$

Equations (2) and (6) are two equations in the two unknowns, $B(t)$ and $b(0,t)$. Substituting the former into the latter and re-arranging, we get an expression for the ratio of the average knowledge of workers to the level of technology at time t :

(7) $$\frac{B(t)}{A(t)} = \frac{1}{1 + \frac{1}{\alpha}\left[\frac{g+n}{n} \left(\frac{1 - e^{-nT}}{1 - e^{-(n+g)T}} \right) - 1 \right]}$$

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[4] In both the basic model and the extension discussed below, it is the case that workers who have just finished their schooling have more knowledge than those who have been in the labor force for some time. On the face of it, this assumption may seem to contradict the evidence discussed above that lifetime profiles are hump-shaped, rising for several decades following entry into the labor force. We don't see these as necessarily contradictory, however. It is possible that while total human capital may rise over the course of a worker's career, the specific aspect of human capital associated with knowledge declines with time since the individual went to school. The gap could be filled with an aspect of human capital related to maturity, judgment, and so on, that is not a function of the level of technology.

This equation shows the effects of population aging, technological progress, and the retirement age on the ratio of workers' knowledge to the cutting edge. A lower value of n will increase the term in square brackets in the denominator, implying a lower ratio of workers' knowledge to the cutting edge. A higher value of either g or T will have the same effect.

Equation (7) also shows the role of teachers in multiplying the effects of population growth or other determinants on technological backwardness. A small value of α, which implies that more of a worker's stock of knowledge results from her schooling, implies that technological backwardness due to slow population growth will be amplified.

Equation (7) can also be interpreted as showing the technological lag of the labor force in a country behind the cutting edge of technology. We designate the gap, measured in years, as G. Since technology grows at rate g, the relation between the gap and the ratio of worker knowledge to the cutting edge is simply $A(t) = B(t)e^{gG}$. This can be rewritten as

$$G \approx \frac{\ln(A(t)) - \ln(B(t))}{g}. \tag{8}$$

3.1 Illustrative calculations

For any values of n, g, T, α, we can use the equations (7) and (8) to calculate B/A and the implied gap in technology measured in years. Our primary interest is in how varying population growth affects the size of the gap, so we consider values of n ranging from 2% to -1%. For the value of g we use 1%. For T, we will use 40. We also calculate the effect of n on the technology gap in the case where teachers are not important (that is, α=1).

Table 1: *B/A* in the Basic Model

	Population Growth (*n*)						
α	**-0.01**	**-0.005**	**0**	**0.005**	**0.01**	**0.015**	**0.02**
0.25	0.521	0.530	0.540	0.549	0.559	0.569	0.579
0.5	0.685	0.693	0.701	0.709	0.717	0.725	0.733
0.75	0.766	0.772	0.779	0.785	0.792	0.798	0.805
1	0.813	0.819	0.824	0.830	0.835	0.841	0.846

To interpret this table, we focus on the difference between the case where α is one (no teacher effect) and other values of α. Consider the case of α =0.5, and consider the case where population growth falls from 2% per year to -1% per year. Although this is a large change is population growth, it is not out of the range of what is being experienced in some of the world's most developed economies over a period of decades (recall that the model here examines only steady states; below we look at transition dynamics). When α=1, the reduction in population growth leads to the ratio of B/A falling by 3.9 percent of its initial value (that is, from .846 to .813). In other words, comparing steady states, in the case where population growth was -1% per year, income per capita would be 3.9% lower than in the case where population was growing by 2% per year, due to the "pure vintage" effect of workers having more out of date human capital. When α=0.5, the decline is B/A is 6.5% (from .733 to .685). This is a combination of the pure vintage and teacher effects of aging. In other words, the teacher effect leads to an increase by a factor of 1.69 (that is, 6.5/3.9) in the impact of aging on the technological backwardness of the labor force.[5] We call this the "teacher multiplier," that is, the extent to which the teacher effect multiplies the general effect of population aging. When α=.75 (giving teachers a less important role) the teacher multiplier goes down. For the values of *g* and *T* that we have been using, and for a reduction of population growth from 2% to -1%, the teacher multiplier in this case is 1.24.

The numbers in Table 1 can also be interpreted as a measure of technological backwardness, in years. For the case where α=1, a reduction in labor force growth from 2% to -1% leads to an increase in average technological backwardness of workers of 3.9 years. In the model without teacher effects, every worker is always as many years behind the technological frontier as the number of years since he/she finished schooling. The increase in the average number of years behind the technological frontier results from a reweighting of the labor force toward older ages. Setting α to 0.75 and 0.5, respectively, leads the average level of technological backwardness to increase to 5.0 years and 6.7 years.

Before going on, it is worth pausing to ask whether the numbers just derived are large. At present, we have no estimate of the magnitude of α, the parameter that determines the teacher effect. A value of 0.5 probably assigns too much importance to teachers. If we take the case, of α=0.75, which might be more reasonable, it says that the "teacher effect" multiplies the vintage effect of population aging by a factor of 1.24, and leads to a large demographic change costing one more year of output growth than it otherwise would have. These numbers may seem small in relation to the overall burden of aging (though the vintage effect as well as other channels) but they are nonetheless large relative. For example, one year's lost output growth (where output is growing at one percent per year) leads to output being one percent per year lower. That is a lot of money. While the teacher effect only makes aging a little bit worse economically than it would otherwise be, it is still paying attention to its effects, and perhaps considering how to ameliorate them.

3.2 Aging Effects vs. Solow Effects

The basic model presented above allows us to quantify the effect of slowing labor force growth on the technological up-to-dateness of the labor force, both through the pure vintage effect and through the schooling effect. In considering the quantitative importance of these

[5] Note that the value of *B/A* is also lower in general when α<1 than when α=1. While this is an interesting phenomenon in its own right, we do not think of it as being an issue of population aging *per se*.

channels, it is natural to compare them to a more established effect of slowing population growth which is the channel of capital deepening in the Solow model.

Consider a standard Solow model with exogenous technological progress. As above, $A(t)$ is the level of technology and $B(t)$ is the level of knowledge embodied in current workers. The latter is what is relevant for production of output, which is given by the Cobb-Douglas production function

$$Y = K^{\rho}(BL)^{1-\rho}, \tag{9}$$

where the time index is suppressed for convenience.

Because technological progress is exogenous while the level of skills of the population is not, it is convenient to express the steady state in terms of output per efficiency unit of $A(t)$ rather than $B(t)$. Thus

$$\frac{Y}{AL} = \frac{B}{A} \times \frac{Y}{BL} = \frac{B}{A} \times \left(\frac{s}{n+g+\delta}\right)^{\frac{\rho}{(1-\rho)}}, \tag{10}$$

where s is the investment rate and δ is the rate of depreciation. The first term is just the effect of population growth on the skills of the labor force, as derived in equation (7). As shown above, lowering population growth lowers the magnitude of the first term, while it is well known that lowering n raises the second term via reduced dilution of the capital stock. In principle, then, it is possible for overall effect of lowering population growth to go in either direction, of for there to be a non-monotone effect of population growth on steady state output per efficiency unit.

In practice, however, the Solow effect dominates for reasonable sets of parameters. For example, consider a relatively standard parameterization of $g = .01$, $\delta = .05$, and $\rho = \frac{1}{3}$ (the value of the saving rate is not relevant). Moving from $n = .02$ to $n = -.01$ raises the value of the second term in equation (10) by 26%. By contrast, as mentioned above, in the case where $\alpha = .5$, a similar change in population growth lowers the value of the first term in equation (10) by 6.5%.

The magnitude of the human capital effect can be increased by choosing a more rapid rate of technological progress. For example, in the case of $\alpha = 0.5$, changing from $n = .02$ to $n = -.01$ lowers the value of B/A by 11.5% when $g = .02$ and by 15.1% when $g = .03$. In a similar fashion, the magnitude of the Solow effect can be decreased by choosing high rates of deprecation or technological progress, as well as by choosing a low value for capital's share ρ. For example, when the value of $(g + \delta)$ is set at .11 (instead of the .06 assumed previously), then a reduction in population growth from 2% to -1% leads to the second term in equation (10) rising by 13.8%.

Thus it not impossible to pick parameters such that the human capital effect dominates Solow effect, but it is somewhat difficult. However, this exercise also makes it clear that for a relatively broad set of parameters, the human capital vintage effects on which this paper focuses can be of at least some quantitative significance in comparison to the Solow effect.

4. Model Extensions

4.1 Learning During Working Years

As mentioned above, our basic model makes the assumption that a worker's level of knowledge is fixed at the time of her entry into the labor force. A more reasonable alternative is to assume that she keeps on learning during her working years.[6] One could think about a number of specifications for this learning process. Presumably, the amount of effort that a worker puts into remaining technologically up to date will depend on both how far behind the technological frontier she currently is, and also on her age. Whether being close to the technological frontier makes assimilating new technologies easier (because they are not very different from one's current knowledge) or harder (because they have not yet been codified into easily digestible form) is not clear. As discussed in Section 2, it is well understood that workers closer to the end of their working lives will have less incentive to work to update their human capital, because they have fewer years over which to recoup their investment. Further, there may be an independent effect of age on an individual's flexibility of mind in assimilating new ideas. Many of these ideas could be incorporated into our model, although at the cost of a good deal of complication.[7]

We take a relatively simple approach to this issue. We assume that that the growth rate of a worker's knowledge following the completion of her schooling is some fixed fraction, $0<\beta<1$, of the growth rate of knowledge in the economy. Using this assumption, the model is relatively straightforward. The equations determining the level of technology among new entrants to the labor force and the average level of technology in the economy remain the same as in the basic model:

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[6] An alternative variation to the basic model would be a model that allows the amount of schooling to be endogenously affected by an individual's distance from the technological frontier. Having older teachers may cause individuals to delay entry into the labor force and continue their education. Jones (2010) has demonstrated that the accumulation of knowledge across generations has forced innovators to spend more time in training to reach the technological frontier.

[7] For example, the idea that knowledge acquisition is faster, the farther behind the technology frontier a person is, could be expressed in the form

$$\dot{b}(a,t) = \gamma\left[A(t) - b(a,t)\right],$$

where the parameter γ determines the speed of an individual's knowledge catch-up to the cutting edge of technology. This setup would parallel literature on technology transfer across countries – see Barro and Sala-i-Martin (1997). The resulting dynamics are quite complex.

(2) $$b(0,t) = \alpha A(t) + (1-\alpha)B(t)$$

(3) $$B(t) = \left(\int_0^T b(0,t-s)e^{-ns} ds) \right) \Big/ \int_0^T e^{-ns} ds$$

Now, instead of assuming that people don´t learn anything after entering the labor force, we assume that:

(9) $$\frac{\dot{b}(a,t)}{b(a,t)} = \beta \frac{\dot{A}(t)}{A(t)} = \beta g$$

This implies that $b(a,t) = b(0,t-a)e^{\beta g a}$. Substituting into (3):

(10)
$$B(t) = \frac{\int_0^T b(0,t-s)e^{\beta g s} e^{-ns} ds}{\int_0^T e^{-ns} ds}$$

As before, in the steady state

(11) $$\frac{\dot{A}(t)}{A(t)} = \frac{\dot{B}(t)}{B(t)} = \frac{\dot{b}(0,t)}{b(0,t)} = g$$

implying that $b(0,t) = b(0,t-s)e^{gs}$

Substituting into (10):

(11)
$$B(t) = \frac{\int_0^T b(0,t)e^{-gs} e^{\beta g s} e^{-ns} ds}{\int_0^T e^{-ns} ds}$$

This can be further simplified to

(12)
$$B(t) = \frac{\int_0^T b(0,t)e^{-[n+(1-\beta)g]s} ds}{1 - e^{-nT} / n} = \left[\frac{n}{[n + (1-\beta)g]} \right] \left[\frac{1 - e^{-[n+(1-\beta)g]T}}{1 - e^{-nT}} \right]$$

And so finally:

(13) $$B(t)/A(t) = \frac{1}{1+\frac{1}{\alpha}\left[\frac{[n+(1-\beta)g]}{n}\right]\left[\frac{1-e^{-nT}}{1-e^{-[n+(1-\beta)g]T}}\right]},$$

where it is easy to see that the basic formulation is a special case $\beta = 0$. Table 2 shows the ratio of average technology to the cutting edge for the same parameters examined in Table 1. For β we use a value of 0.5.

Table 2: *B/A* in the Model with Learning During Working Years

	Population Growth						
A	**-0.01**	**-0.005**	**0**	**0.005**	**0.01**	**0.015**	**0.02**
0.01	0.083	0.085	0.088	0.091	0.094	0.098	0.101
0.25	0.693	0.700	0.708	0.715	0.722	0.730	0.738
0.5	0.819	0.824	0.829	0.834	0.839	0.844	0.849
0.75	0.871	0.875	0.879	0.883	0.886	0.890	0.894
1	0.900	0.903	0.906	0.909	0.912	0.915	0.918

The values of *B/A* in Table 2 are all closer to one than those in Table 1, since in this economy workers are on average less technologically backward than in an economy where there is no learning once schooling is completed. The pure vintage effect of aging is smaller, for the same reason. For example, when α=1 (no teacher effect) a reduction in population growth from 2% to -1% lowers *B/A* by 2.0% percent of its initial value, vs. 3.9% of its initial value when there is no learning during the working years. It is interesting to note that the teacher multiplier (the extent to which the reduction in *B/A* is larger in the presence of teacher effects than in their absence) is actually larger in this case. When α=0.5, the teacher multiplier is 1.80, as compared to 1.69 when there is no learning during working years. When α=0.75, the teacher multiplier is 1.31, vs. 1.24 when there is no learning during working years. The increase in the average level of technological backwardness (measured in years) is 2.0 when there is no teacher effect, 2.6 when α=.75, and 3.6 when α=0.5.

4.2 Transitional Dynamics

The analysis in the two sections above considers only steady states for mathematical tractability. To be clear, the steady states in this model are "steady" in two senses. First, they are, in demographic terms, stable populations, in which the size ratio or entering cohorts has been constant for long enough that the population age structure (the relative sizes of age groups) is constant. Second, they are steady states in the sense that the age profile of relative technological backwardness has stabilized. An individual's relative technological backwardness is a function of both her age and of the average level of technological backwardness at the time she finished school.

Now consider what happens when there is a change in population growth. Dynamics arise from two sources: first, a change in the growth rate of population produces a change in the age distribution of workers that takes T periods to complete. Second, the "teacher multiplier" produces a longer dynamic trajectory: as the labor force ages, the average technological backwardness of workers (and thus teachers) increases; thus over time students complete their schooling having absorbed technology from an earlier vintage; and these students go on to become teachers of the next generation.

To examine transition, we use a discrete time version of the model.[8] We consider an economy which is in steady state, then "shock" it by lowering the growth rate of the labor force. To be specific, we consider a change in the ratio of new entrants to the labor force (zero year olds) to those members from the preceding age cohort (one year olds). Define n as such that the ratio of zero year olds to one year olds is $(1+n)$. In a demographic steady state (i.e. a stable population), n will be the growth rate of the labor force. This will not be true along transition paths, however.

For simplicity, we focus on the basic model, in which there is no acquisition of new technical knowledge after an individual finishes her schooling. In all cases, we examine the path of $B(t)/A(t)$ *relative to the initial steady state*, following a reduction in the growth rate of population. By measuring the technology ratio relative to the initial steady state, we are able to focus attention on how variations in the parameters affect the dynamic response of the economy to a shock – in other words, we can show how incorporating the teacher effect changes the results of the model.

Figure 1 shows the paths of relative technology in response to a decline in labor force growth. We consider a reduction in n from 2% to -1%. We show the time paths for relative technology for three different values of α, the importance of teachers in determining human capital: 1.0 (no teacher effect), 0.75, and 0.5. The values of g and T are 1% and 40 years, respectively.

The figure shows that, as predicted by the steady state analysis above, the total effect of a decline in labor force growth on the relative level of technology is larger, the larger is the importance of teachers in determining human capital. In addition, when teachers are more important, the timing of the effect is changed. When α is one, the complete transition only takes 40 years, which is how long it takes the working age population to assume its new stable configuration. In this case, half the change is accomplished in 20 years. When α is 0.75, half of the change technological backwardness is accomplished in 23 years, and when α is 0.5, it takes 27 years. In both the case of α=.75 and α=.5, technological backwardness is slightly higher than the case of α=1 at a horizon of 40 years, but most of the extra technological backwardness that results from the teacher effect is observed after 40 years have passed.

We can also translate these changes in the level of technological backwardness into an annual growth effect. In the steady state of the model, output per worker grows at rate

[8] The simulation is implemented on a spreadsheet, and is available on request.

g. During a transition, the average level of technology in the country growth more slowly – this is the means by which *B(t)/A(t)* falls. In the case where α=1 (no teacher effect), and using our base case parameters of *g*=.01 and *T*=40, the total reduction in *B/A* as shown in the steady state analysis is 3.9%, and this takes 40 years to come about. Thus the vintage effect lowers growth by one tenth of one percent per year over this period. Adding in the teacher effect in its strong form (α=0.5), income after 40 years is approximately 4.7% lower than the initial steady state. In other words, the teacher effect lowers growth by an additional 0.02% per year over this 40 year period (although as mentioned above, most of the teacher effect comes after the 40 year horizon).

As a second experiment, we explore the extent to which the growth rate of technology, *g*, interacts with the teacher effect. We consider two values of *g*, 1% and 2%, as well as two value of α, 1.0 and 0.75. The paths of relative technology in the four cases are shown in Figure 2. The figure makes two points. First (as can also be shown in the steady state analysis), the more rapid is technological progress, the larger is the pure vintage effect of a reduction population growth. Second, the faster is technological progress, the larger is the *additional* reduction in income that results from the teacher effect.

5. Conclusion

The aging of the labor force will likely have a significant influence on labor market productivity and economic growth. Most of the existing literature has focused on the direct effect of worker aging with research attempting to estimate the productivity of older workers. This literature has considered the productivity tradeoffs from older workers who have an older vintage of knowledge and potential deterioration of human capital but decades of labor market experience. Our paper considers a different, secondary effect of population aging driven by the aging of the teaching force. With older teachers, new entrants to the labor force are further from the technical frontier.

We present a simple model and illustrative calculations that consider the importance of teacher aging and the various factors that may amplify the impact. Our calculations suggest that teacher aging is not an inconsequential concern. Our findings are consistent with evidence of a strong correlation between the age structure of the population and the growth rate of productivity (Feyrer 2007).

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Figure 1

Reduction in *n* from 2% to -1%: paths for relative technology for three different values of α: 1.0 (no teacher effect), 0.75, and 0.5. The values of *g* and *T* are 1% and 40 years, respectively.

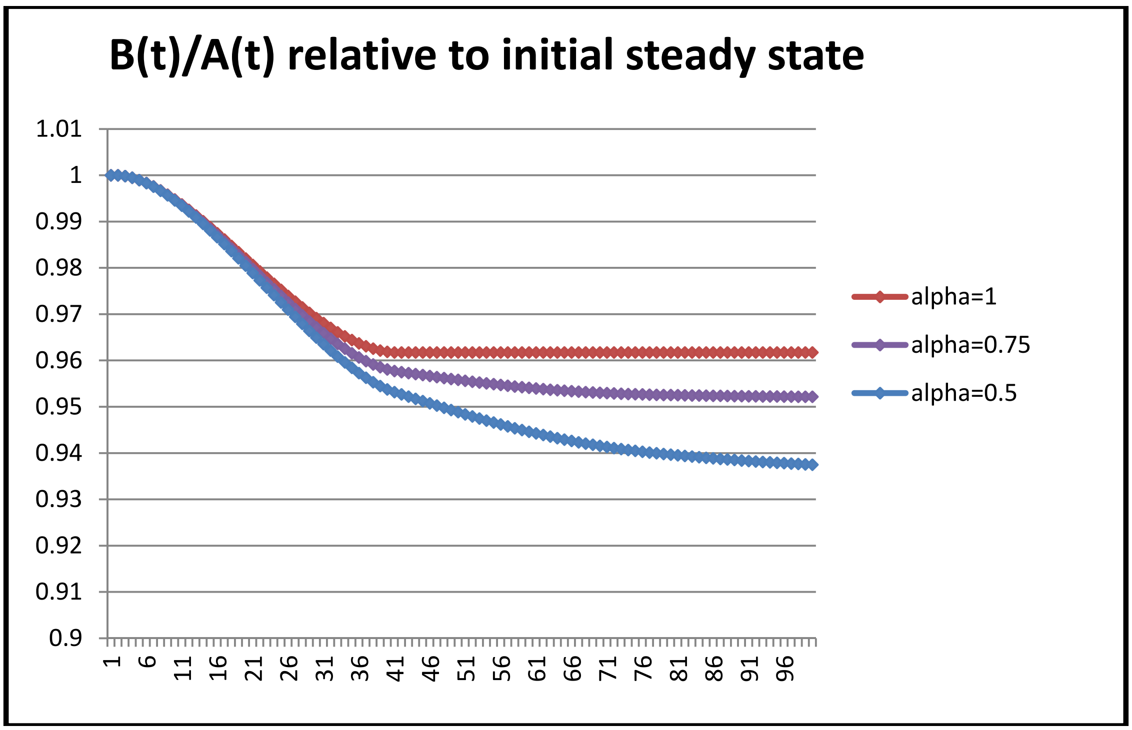

Figure 2

Reduction in *n* from 2% to -1%: we consider two values of *g*, 1% and 2%, as well as two values of alpha, 1.0 and 0.75. The values of *g* and *T* are 1% and 40 years, respectively.