In this lecture you will learn:

- Multilayer structures: non-normal incidence

\[ V(z)_{|z<0} = V_{+1} e^{-j k_1 z} + V_{-1} e^{+j k_1 z} \]
\[ V(z)_{|z>0} = V_{+2} e^{-j k_2 z} \]

Boundary conditions:

1. \[ V_{+1} + V_{-1} = V_{+2} \]
2. \[ \frac{V_{+1}}{Z_{o1}} - \frac{V_{-1}}{Z_{o1}} = \frac{V_{+2}}{Z_{o2}} \]

\[ \Gamma = \frac{V_{-1}}{V_{+1}} = \frac{Z_{o2}/Z_{o1} - 1}{Z_{o2}/Z_{o1} + 1} \]

\[ E(z)_{|z<0} = \hat{x} E_{+1} e^{-j k_1 z} + \hat{x} E_{-1} e^{+j k_1 z} \]
\[ E(z)_{|z>0} = \hat{x} E_{+2} e^{-j k_2 z} \]

Boundary conditions:

1. \[ E_{+1} + E_{-1} = E_{+2} \]
2. \[ \frac{E_{+1}}{\eta_1} + \frac{E_{-1}}{\eta_1} = \frac{E_{+2}}{\eta_2} \]

\[ \Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\eta_2/\eta_1 - 1}{\eta_2/\eta_1 + 1} \]
ECE 303 – Fall 2005 – Farhan Rana – Cornell University

Waves at Interfaces: TE Wave - I

$$E(z)_{z<0} = \hat{y} E_{+1} e^{-j(k_{1x} x + k_{1z} z)} + \hat{y} E_{-1} e^{-j(k_{2x} x - k_{2z} z)}$$

$$E(z)_{z>0} = \hat{y} E_{+2} e^{-j(k_{1x} x + k_{1z} z)}$$

Boundary Conditions:

1. $$E_{+1} + E_{-1} = E_{+2}$$
2. $$\left[ \begin{array}{c} E_{+1} \\ \eta_1 \cos(\theta_1) \\ \eta_1 \cos(\theta_1) \end{array} \right] - \left[ \begin{array}{c} E_{-1} \\ \eta_1 \cos(\theta_1) \\ \eta_1 \cos(\theta_1) \end{array} \right] = \left[ \begin{array}{c} E_{+2} \\ \eta_2 \cos(\theta_2) \\ \eta_2 \cos(\theta_2) \end{array} \right]$$

$$T = \frac{E_{+2}}{E_{+1}} = \frac{2 \eta_2 / \cos(\theta_2)}{\eta_1 / \cos(\theta_1) + 1}$$

$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\eta_2 / \cos(\theta_2) - 1}{\eta_1 / \cos(\theta_1) + 1}$$

Waves at Interfaces: TE Wave - II

One may replace the above problem with a “dummy” normal incidence problem:

$$E_{-1} = \hat{y} E_{-1} e^{-j(k_{1x} x + k_{1z} z)} + \hat{y} E_{-1} e^{-j(k_{2x} x - k_{2z} z)}$$

And then calculate the reflection and transmission coefficients for the $E$-field:

$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\eta_2 / \cos(\theta_2)}{\eta_1 / \cos(\theta_1) + 1}$$

$$T = \frac{E_{+2}}{E_{+1}} = \frac{2 \eta_2 / \cos(\theta_2)}{\eta_1 / \cos(\theta_1) + 1}$$
Waves at Interfaces: TE Wave - III

In the “dummy” normal incidence problem:

• The wavevectors are taken to be the z-component of the actual wavevectors in each medium

• The impedances in each medium are taken to be the actual impedances divided by the cosines of the angles of propagation w.r.t. the z-axis

$$\Gamma = \frac{E_{-1}}{E_{+1}} = \frac{\eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)} - 1 \quad \frac{1}{\eta_2/\cos(\theta_2)} + 1$$

$$T = \frac{E_{+2}}{E_{+1}} = \frac{2 \eta_2/\cos(\theta_2)}{\eta_1/\cos(\theta_1)}$$

Warning: The “dummy” normal incidence problem is only meant to be used to calculate reflection and transmission coefficients – don’t use this framework for anything else

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Tri-layer Structure: TE Wave - I

How does one solve a problem like this?

$$\Gamma = \frac{E_{-1}}{E_{+1}} = ?$$
Tri-layer Structure: TE Wave - II

Replace the above problem with a “dummy” normal incidence problem:

And then use the usual methods to solve it (impedance transformations, Smith chart, etc)

Waves at Interfaces: TM Wave - I

Work with the E-field component parallel to the media interface (i.e. the x-component)

Boundary Conditions:

1. \[ E_{x+1x} + E_{-1x} = E_{+2x} \]
2. \[ \left\{ \begin{array}{l} E_{x+1x} = E_{x-1x} e^{-j(k_{1x} x + k_{1z} z)} + E_{-1x} e^{-j(k_{2x} x - k_{2z} z)} \\ E_{x+2x} = E_{x+1x} e^{-j(k_{1x} x - k_{1z} z)} \end{array} \right. \]
One may replace the above problem with a “dummy” normal incidence problem:

And then calculate the reflection and transmission coefficients for the x-component of the E-field:

\[ \Gamma = \frac{E_{-1x}}{E_{+1x}} = \frac{\eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1) + 1} \]

\[ T = \frac{E_{+2x}}{E_{+1x}} = \frac{2 \eta_2 \cos(\theta_2)}{\eta_2 \cos(\theta_1) + 1} \]

Work with the E-field component parallel to the media interface (i.e. the x-component)

\[ E_x(r)_{|z<-l} = E_{+1x} e^{-j(k_{1x} x + k_{1z} (z + l))} + E_{-1x} e^{-j(k_{1x} x - k_{1z} (z + l))} \]

\[ E_x(r)_{|z<0} = E_{+2x} e^{-j(k_{2x} x + k_{2z} z)} + E_{-2x} e^{-j(k_{2x} x - k_{2z} z)} \]

\[ E_x(r)_{|z>0} = E_{+3x} e^{-j(k_{3x} x + k_{3z} z)} \]

How does one solve a problem like this?

\[ \Gamma = \frac{E_{-1x}}{E_{+1x}} = ? \]
Tri-layer Structure: TM Wave - II

Replace the above problem with a "dummy" normal incidence problem:

And then use the usual methods to solve it (impedance transformations, Smith chart, etc)