In this lecture you will learn:

• The operation of bipolar junction transistors
• Forward and reverse active operations, saturation, cutoff
• Ebers-Moll model
• Small signal models
NPN Bipolar Junction Transistor

PNP Bipolar Junction Transistor
Suppose:
The base-emitter junction is forward biased $V_{BE} > 0$
The base-collector junction is zero biased $V_{CB} = 0$

This biasing scheme will put the device in the “forward active” operation (to be discussed fully later)

Consider the action in the base first ($V_{BE} > 0$ and $V_{CB} = 0$)
• The electrons diffuse from the emitter, cross the depletion region, and enter the base
• In the base, the electrons are the minority carriers
• In the base, the electrons diffuse towards the collector
• As soon as the electrons reach the base-collector depletion region they are immediately swept away into the collector by the strong electric fields in the depletion region
Consider the base first:

In the base, the electron population can be written as:

\[ n(x) = n_{po} + n'(x) \]

Equilibrium electron density \hspace{1cm} Excess electron density

In the base, the excess electron population satisfies the differential equation:

\[ \frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{L_n} = 0 \]

边界条件

\[ n'(x_p) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{kT} - 1 \right) \]

\[ n'(x_p + W_B) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{kT} - 1 \right) = 0 \]

\[ n'(x) = n'(x_p) \left(1 - \frac{x - x_p}{W_B} \right) = n_i^2 \frac{qV_{BE}}{kT} \left(1 - \frac{x - x_p}{W_B} \right) \]

忽略载流子复合（即假设 \( L_n = \infty \))

\[ \frac{\partial^2 n'(x)}{\partial x^2} = 0 \]

边界条件

\[ n'(x_p) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{kT} - 1 \right) \]

\[ n'(x_p + W_B) = \frac{n_i^2}{N_{aB}} \left( \frac{qV_{BE}}{kT} - 1 \right) = 0 \]

解决方案是：

\[ n'(x) = n'(x_p) \left(1 - \frac{x - x_p}{W_B} \right) = n_i^2 \frac{qV_{BE}}{kT} \left(1 - \frac{x - x_p}{W_B} \right) \]
Consider the emitter now:

In the emitter, the hole population can be written as:

\[ p(x) = p_{no} + p'(x) \]

where 
- \( p_{no} \) is the equilibrium hole density
- \( p'(x) \) is the excess hole density

The excess hole density satisfies the differential equation:

\[ \frac{\partial^2 p'(x)}{\partial x^2} = 0 \]

Boundary conditions:
- \( p'(-x_n) = \frac{n_i^2}{N_{dE}} \left( \frac{qV_{BE}}{eKT} - 1 \right) \)
- \( p'(-x_n - W_E) = 0 \)

Solution is:

\[ p'(x) = p'(-x_n) \left( 1 + \frac{x + x_n}{W_E} \right) = \frac{n_i^2}{N_{dE}} \left( \frac{qV_{BE}}{eKT} - 1 \right) \left( 1 + \frac{x + x_n}{W_E} \right) \]
In the base:
• The electron current is:

\[ J_n(x) = q D_n \frac{\partial n(x)}{\partial x} = q n_i^2 \frac{D_n}{N_{aB}W_B} \left( \frac{qV_{BE}}{kT} - 1 \right) \]

In the emitter:
• The hole current is:

\[ J_p(x) = -q D_p \frac{\partial p(x)}{\partial x} = q n_i^2 \frac{D_p}{N_{de}W_E} \left( \frac{qV_{BE}}{kT} - 1 \right) \]

Emitter current:
• The current flowing out of the emitter is the sum of the total electron and total hole currents in the emitter:

\[ I_E = q n_i^2 A \left( \frac{D_p}{N_{de}W_E} + \frac{D_n}{N_{aB}W_B} \left( \frac{qV_{BE}}{kT} - 1 \right) \right) \]
Collector Current:
• The current going into the collector is due to the electrons that got swept from the Base through the Base-Collector depletion region by the electric-fields:

\[ I_C = qn_i^2 A \left( \frac{D_n}{N_{ad}W_B} \frac{qV_{BE}}{e^{VT} - 1} \right) \]

Base Current:
• The current going into the Base is due to the holes that got injected from the base into the emitter:

\[ I_B = qn_i^2 A \left( \frac{D_p}{N_{de}W_E} \frac{qV_{BE}}{e^{VT} - 1} \right) \]
NPN BJT: Circuit Level Parameters

Current gain $\beta_F$:
Current gain of the BJT in the forward active operation is defined as the ratio of the collector and base currents:

$$\beta_F = \frac{I_C}{I_B} = \frac{D_n}{N_{ab}W_B} \frac{N_{de}W_{E}}{D_p}$$

Typical values of $\beta_F$ are between 20-200 and:

$$N_{de} \gg N_{ab} > N_{dc}$$

In the forward active operation $\alpha_F$ is defined as the ratio of the collector and emitter currents:

$$\alpha_F = \frac{I_C}{I_E} = \frac{D_n}{D_p} \frac{N_{ab}W_B}{N_{de}W_{E}}$$

Transistor relation:
$\alpha_F$ and $\beta_F$ are related:

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

NPN BJT: Ebers-Moll Model for Forward Active Operation

Suppose:

- $V_{BE} > 0$
- $V_{CB} = 0$

The circuit level simplified model with an ideal diode and a current-controlled current source models the NPN transistor in the forward active operation.

$$I_F = qn_i^2A \left( \frac{D_p}{N_{de}W_{E}} + \frac{D_n}{N_{ab}W_B} \right) \left( \frac{qV_{BE}}{e^{RT}} - 1 \right)$$

$$= I_{ES} \left( \frac{qV_{BE}}{e^{RT}} - 1 \right)$$
NPN BJT: Forward and Reverse Active Operations

For forward active operation:
\[ \beta_F = \frac{I_C}{I_B} \]
\[ \alpha_F = \frac{I_C}{I_E} \]
\[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \]

In a well designed transistor: \( \beta_F >> \beta_R \)

For reverse active operation:
\[ \beta_R = \frac{I_E}{I_B} \]
\[ \alpha_R = \frac{I_E}{I_C} \]
\[ \beta_R = \frac{\alpha_R}{1 - \alpha_R} \]

NPN BJT: Ebers-Moll Model for Reverse Active Operation

Suppose: \( V_{BC} > 0 \)
\( V_{BE} = 0 \)

\[ I_R = qn_I^2 A \left( \frac{D_p}{N_{dc}W_C} + \frac{D_n}{N_{sb}W_B} \right) \left( \frac{qV_{BC}}{e^{qV_{BC}/kT}} - 1 \right) = I_{CS} \left( \frac{qV_{BC}}{e^{qV_{BC}/kT}} - 1 \right) \]

The circuit level simplified model with an ideal diode and a current-controlled current source models the NPN transistor in the reverse active operation.
NPN BJT: Ebers-Moll Model and Terminal Currents

Terminal currents:

\[ I_R = I_{CS} \left( \frac{qV_{BE}}{e^{KT}} - 1 \right) \]
\[ I_F = I_{ES} \left( \frac{qV_{BE}}{e^{KT}} - 1 \right) \]

And

\[ I_B = (1 - \alpha_F)I_F + (1 - \alpha_R)I_R \]
\[ I_C = \alpha_F I_F - I_R \]
\[ I_E = I_F - \alpha_R I_R \]

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NPN BJT: Regimes of Operation - I

In forward active operation:

- \( I_B > 0 \), \( V_{BE} > 0 \), \( V_{CB} \geq 0 \)

Since:

\[ V_{CE} = V_{CB} + V_{BE} \]

\[ \Rightarrow \text{In forward active operation: } V_{CE} \geq V_{BE} \]

\[ I_C = qn^2A \left( \frac{D_n}{N_{ab}W_B} \right) \left( \frac{qV_{BE}}{e^{KT}} - 1 \right) = \beta_F I_B \]

\[ \Rightarrow \text{Independent of } V_{CE} \]

- **Forward active**:
  - Base-emitter junction forward biased
  - Base-collector junction reversed biased
  - \( I_B > 0 \), \( V_{BE} > 0 \), \( V_{CB} \geq 0 \)

- **Saturation**:
  - Base-emitter junction forward biased
  - Base-collector junction forward biased
  - \( I_B > 0 \), \( V_{BE} > 0 \), \( V_{CB} < 0 \)
Carrier Densities in Different Regimes of Operation

**Forward active:**
- \( V_{BE} > 0 \)
- \( V_{CB} \geq 0 \)
- \( N_{de} \) (holes)
- \( N_{AB} \) (electrons)
- \( N_{dC} \) (holes)
- \( p'(x) \) (holes)

**Saturation:**
- \( V_{BE} > 0 \)
- \( V_{CB} < 0 \)
- \( N_{de} \) (holes)
- \( N_{AB} \) (electrons)
- \( N_{dC} \) (holes)
- \( p'(x) \) (holes)

The forward biased base-collector junction reduces the collector current!

NPN-BJT: Regimes of Operation - II

**Forward active:**
- Base-emitter junction forward biased
- Base-collector junction reversed biased
- \( I_B > 0 \), \( V_{BE} > 0 \), \( V_{CB} \geq 0 \)

**Saturation:**
- Base-emitter junction forward biased
- Base-collector junction forward biased
- \( I_B > 0 \), \( V_{BE} > 0 \), \( V_{CB} < 0 \)

**Cutoff:**
- Base current zero
- \( I_B = 0 \)

**Reverse active:**
- Base-emitter junction reverse biased
- Base-collector junction forward biased
- \( I_B > 0 \), \( V_{BE} \leq 0 \), \( V_{CB} < 0 \)
**NPN BJT: Different Regimes of Operation**

- **Forward Active**
  - $V_{BE} > 0$, $V_{CE} > 0$
  - $I_C = I_E + I_R$

- **Saturation**
  - $V_{CB} < 0$, $V_{BE} > 0$
  - $I_C = eta F I_B$
  - Load line equation:
    \[ V_{CE} = V_{DD} - I_C R \]
    \[ I_C = \frac{V_{DD} - V_{CE}}{R} \]

- **Reverse Active**
  - $V_{CB} < 0$, $V_{BE} < 0$
  - $I_C = I_E$

**NPN BJT: A Simple Amplifier Circuit**

- **Current gain (in forward active regime):**
  \[ \frac{I_{OUT}}{I_{IN}} = \frac{I_C}{I_B} = \beta F \]

- **Load line equation:**
  \[ V_{CE} = V_{DD} - I_C R \]
  \[ I_C = \frac{V_{DD} - V_{CE}}{R} \]

**Lesson:** Don't let the base-collector junction become forward biased
NPN BJT: Voltage Biasing of a Simple Amplifier Circuit

- Approximate analysis of transistor DC biasing:
  - If: \( V_{IN} < V_{BE-ON} \Rightarrow I_B = 0 \) \( \Rightarrow \) Transistor in cut-off
  - If: \( V_{IN} \geq V_{BE-ON} \Rightarrow \)
    - \( V_{IN} = I_B R_S + V_{BE-ON} \)
    - \( I_B = \frac{V_{IN} - V_{BE-ON}}{R_S} \)
  - Assume forward active operation (\( V_{CE} > V_{CE-SAT} \)):
    - \( I_C = \beta_F I_B \)
    - \( V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta_F \left( \frac{V_{IN} - V_{BE-ON}}{R_S} \right) R \)

Final Step - confirm if the assumption of forward active operation was valid:

- \( V_{CE} \geq V_{CE-SAT} \)
- \( \Rightarrow V_{CE} = V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta_F \left( \frac{V_{IN} - V_{BE-ON}}{R_S} \right) R \geq V_{CE-SAT} \)
NPN BJT Amplifier Circuit: Transfer Curve

- $V_{BE-ON} \sim 0.6 \text{ V}$
- $V_{CE-SAT} \sim 0.2 \text{ V}$

Acting as a current source

$V_{DD} \rightarrow V_{OUT}$

If: $V_{IN} < V_{BE-ON}$ \Rightarrow $I_B = 0 \Rightarrow V_{OUT} = V_{DD}$ \quad Transistor in cut-off

If: $V_{IN} \geq V_{BE-ON}$ \Rightarrow $I_B = \frac{V_{IN} - V_{BE-ON}}{R_S}$

$V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta F R S (V_{IN} - V_{BE-ON})$ \quad Transistor in forward active

If: $I_B > 0 \& V_{CE} = V_{OUT} \leq V_{CE-SAT}$ \Rightarrow Transistor in saturation

NPN BJT Common Emitter (CE) Voltage Amplifier

- $V_{BE-ON} \sim 0.6 \text{ V}$
- $V_{CE-SAT} \sim 0.2 \text{ V}$

We need better techniques to calculate the voltage gain of such amplifier circuits

We need small signal models of the BJTs!
NPN BJT: Small Signal Circuit Model

Base current:
\[ I_B = qn^2 A \left( \frac{D_p}{N_{dE} W_E} \right) \left( \frac{qV_{BE}}{KT} - 1 \right) \]
\[ = I_{BS} \left( \frac{qV_{BE}}{KT} - 1 \right) \]
\[ \Rightarrow I_B + i_b = I_{BS} \left( \frac{q(V_{BE} + v_\pi)}{KT} - 1 \right) \]
\[ i_b = \frac{\partial I_B}{\partial V_{BE}} v_\pi = \frac{q}{KT} I_B v_\pi = q \frac{I_B}{KT} v_\pi = g_m v_\pi \]
\[ g_m = \frac{q I_B}{KT} \]

Collector current:
\[ I_C + I_c = \beta_F (I_B + i_b) \]
\[ \Rightarrow i_c = \beta_F i_b = \beta_F g_m v_\pi = g_m v_\pi \]
\[ g_m = \beta_F g_m = \beta_F \frac{q I_B}{KT} \]

Increases linearly with the collector current
NPN BJT: Forward Active Current vs $V_{CE}$

As $V_{CE}$ becomes more positive, the base-collector junction becomes more reverse biased, and the thickness of the depletion increases thereby reducing the thickness $W_B$ of the base. Consequently, the collector current increases. The output conductance $g_o$ is not zero!

The slope of the $I_C$ vs $V_{CE}$ curves are modeled using the early voltage $V_A$:

$$\frac{dI_C}{dV_{CE}} = g_o = \frac{I_C}{V_A} = \lambda n I_C$$

The early voltage is usually in the 50-200 V range.
NPN BJT: Output Conductance

Saturation: $I_B > 0$
- $V_{CE} > V_{BE}$
- $V_{CE} \geq 2V_{BE}$
- $V_{CE} = V_{BE}$
- $I_B = 0$ (cutoff)

Forward active: $I_B > 0$ & $I_C = \beta I_B$

Output conductance:
$$g_o = \frac{1}{r_o} = \frac{\partial I_C}{\partial V_{CE}}$$

NPN BJT Common Emitter (CE) Voltage Amplifier

Voltage gain:
$$A_v = \frac{v_{out}}{v_s} = -g_m (r_o || R) \frac{r_x}{r_x + R_s} v_s$$
NPN BJT CE Amplifier: Limits of Output Voltage Swing

Minimum output voltage and maximum input voltage:
If the output voltage becomes too small (happens when the input voltage becomes too large), the BJT will go into the saturation region (in the saturation region the gain is small).

Maximum output voltage and minimum input voltage:
If the input voltage becomes too small the BJT will go into cut-off.