Lecture 19
High Frequency Analysis of FET Circuits

In this lecture you will learn:

• High Frequency Analysis of FET Circuits
• Miller Effect and the Miller Capacitance
• The Transition Frequency and the Ultimate Performance limits of FET Devices

Phasor Analysis: Complex Impedances

\[ i(t) = \text{Re}\{i(\omega)e^{j\omega t}\} \]
\[ v(t) = \text{Re}\{v(\omega)e^{j\omega t}\} \]
\[ v_r(t) = \text{Re}\{v_r(\omega)e^{j\omega t}\} \]

Capacitive Impedance/Admittance:
\[ Z(\omega) = \frac{1}{Y(\omega)} = \frac{1}{j\omega C} \]

Inductance Impedance/Admittance:
\[ Z(\omega) = \frac{1}{Y(\omega)} = j\omega L \]
One can compute the voltage phasors using the impedances:

\[
v_c(\omega) = \frac{1}{j\omega C} \frac{R}{R + \frac{1}{j\omega C}} v(\omega) = \frac{1}{1 + j\omega RC} v(\omega)
\]

\[
v_r(\omega) = \frac{R}{R + \frac{1}{j\omega C}} v(\omega) = \frac{j\omega RC}{1 + j\omega RC} v(\omega)
\]
Phasor Analysis: Bode Plots

A transfer function with a pole at frequency $1/RC$

$$10\log_{10}|H(\omega)|^2$$

$3$ dB break point

Slope = $-20$ dB/decade

A transfer function with a zero at zero frequency and a pole at frequency $1/RC$

$$10\log_{10}|H(\omega)|^2$$

$3$ dB break point

Slope = $+20$ dB/decade
NFET: Capacitances

Source

\[ y = 0 \]

\[ y = L \]

y

P-Si Substrate (or Bulk)

NFET: High Frequency Small Signal Model

Gate

\[ i_g \]

Source

\[ C_{gb} \]

\[ C_{sb} \]

Base

Drain

\[ i_d \]

\[ v_{gs} \]

\[ v_{gs} \]

\[ C_{gs} \]

\[ C_{gd} \]

\[ C_{db} \]
In Saturation:

\[ C_{gs} = \left. \frac{\partial Q}{\partial V_{GS}} \right|_{V_{GD}, V_{GB}} = \frac{2}{3} W L C_{ox} + W C_{ov} + W C_p \neq \frac{2}{3} W L C_{ox} \]

\[ C_{gd} = \left. \frac{\partial Q}{\partial V_{GD}} \right|_{V_{GS}, V_{GB}} = W C_{ov} + W C_p \neq 0 \]

**NFET: Simplified High Frequency Small Signal Model**

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![Diagram of NFET model](image)
The Common Source Amplifier

\[ v_s(t) = \text{Re} \left\{ v_s(\omega) e^{j\omega t} \right\} \]

A high frequency small signal model can be built as follows:

\[
\begin{align*}
V_{DD} & \quad + \quad R \\
\downarrow & \quad \text{\vphantom{\downarrow} } \\
V_{IN} & \quad \downarrow \quad \text{\vphantom{\downarrow} } \\
+ & \quad \text{\vphantom{\downarrow} } \\
V_s(t) & \quad \downarrow \quad \text{\vphantom{\downarrow} } \\
\downarrow & \quad \text{\vphantom{\downarrow} } \\
\text{\vphantom{\downarrow} } & \quad \text{\vphantom{\downarrow} } \\
V_{OUT} + v_{out}(t) & \quad \text{\vphantom{\downarrow} } \\
\text{\vphantom{\downarrow} } & \quad \text{\vphantom{\downarrow} } \\
\text{\vphantom{\downarrow} } & \quad \text{\vphantom{\downarrow} } \\
\end{align*}
\]

The Common Source Amplifier: Open Circuit Voltage Gain

Need to find:
\[ A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} \quad \text{and} \quad H(\omega) = \frac{v_{out}(\omega)}{v_s(\omega)} \]

KCL at (1) gives:
\[ i_d(\omega) = g_o v_{out}(\omega) + g_m v_{in}(\omega) + \left[ v_{out}(\omega) - v_{in}(\omega) \right] j\omega C_{gd} \]

Also:
\[ v_{out}(\omega) = -i_d(\omega) R \]

The above two give:
\[ A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \frac{g_m R - j\omega R C_{gd}}{1 + g_o R + j\omega R C_{gd}} \]
The Common Source Amplifier: Open Circuit Voltage Gain

\[ A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = \frac{g_mR - j\omega RC_{gd}}{1 + g_gR + j\omega RC_{gd}} = -g_m(R \parallel r_o) \frac{1 - j\omega C_{gd}}{1 + j\omega C_{gd}(R \parallel r_o)} \]

\[ 10\log_{10}|A_v(\omega)|^2 \]

Slope = -20 dB/decade

A relatively large frequency

These poles might not limit the frequency performance……PTO

The input impedance is:

\[ Z_{in}(\omega) = \frac{1}{j\omega C_{gs} + C_{gd}(1 - A_v(\omega))} \]
Looking in from the input terminal the capacitance \( C_{gd} \) seems to be magnified by a factor proportional to the open circuit voltage gain of the amplifier!!

\[
\frac{v_{in}(\omega)}{v_s(\omega)} = \frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)} = \frac{1}{1 + j\omega C_{gs} + C_{gd}(1 - A_v(\omega))R_s}
\]

Looking in from the input terminal the capacitance \( C_{gd} \) seems to be magnified by a factor proportional to the open circuit voltage gain of the amplifier!!

Finally:

\[
H(\omega) = \frac{v_{out}(\omega)}{v_s(\omega)} = \frac{v_{out}(\omega)v_{in}(\omega)}{v_{in}(\omega)v_s(\omega)} = A_v(\omega) \frac{Z_{in}(\omega)}{R_s + Z_{in}(\omega)}
\]

\[
= \frac{1}{1 + j\omega C_{gs} + C_{gd}(1 - A_v(\omega))R_s}
\]

Where:

\[
A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = -g_m(R || r_o) \left[ \frac{1 - j\omega C_{gd}}{g_m} \right] \left[ \frac{1}{1 + j\omega C_{gd}(R || r_o)} \right]
\]

The Common Source Amplifier: Input Impedance

The Common Source Amplifier: Total Gain
The Common Source Amplifier: Poles and Zeros of the Total Gain

\[ H(\omega) = \frac{A_v(\omega)}{1 + j\omega[C_{gs} + C_{gd}(1 - A_v(\omega))]}R_s = \frac{H(0)(1 - j\omega \tau_1)}{(1 + j\omega \tau_1)(1 + j\omega \tau_2)} \]

A little tedious algebra can show that the transfer function above has two poles and a zero

\[ H(0) \approx -g_m (R \parallel r_o) \quad \frac{1}{\tau_3} = \frac{g_m}{C_{gd}} \]

1) Assuming \( g_m R_s \gg 1 \), these poles are:

\[ \frac{1}{\tau_1} \approx \frac{g_m}{C_{gs}} \quad \frac{1}{\tau_2} \approx \frac{1}{C_{gd}(R \parallel r_o)} \]

This pole will likely determine the smallest frequency at which the total gain rolls over

2) Assuming \( g_m R_s << 1 \), these poles are:

\[ \frac{1}{\tau_1} \approx \frac{1}{R_s C_{gs}} \quad \frac{1}{\tau_2} \approx \frac{1}{C_{gd}(R \parallel r_o)} \]

This pole will likely determine the smallest frequency at which the total gain rolls over

The Common Source Amplifier: The Miller Approximation

If the poles and zeros in \( A_v(\omega) \) are at a higher frequency than the poles and zeros associated with the transfer function \( \frac{v_{\text{out}}(\omega)}{v_{\text{in}}(\omega)} \) (which would be the case if \( g_m R_s >> 1 \)) then one may approximate the open circuit gain \( A_v(\omega) \) by its low frequency value:

\[ A_v(\omega) \approx -g_m (R \parallel r_o) \]

Miller approximation

And then:

\[ H(\omega) = \frac{v_{\text{out}}(\omega)}{v_{\text{in}}(\omega)} = \frac{v_{\text{out}}(\omega)}{v_{\text{in}}(\omega)} \frac{v_{\text{in}}(\omega)}{v_{\text{in}}(\omega)} = A_v(\omega) \frac{Z_{\text{in}}(\omega)}{R_s + Z_{\text{in}}(\omega)} \]

\[ \approx \frac{-g_m (R \parallel r_o)}{1 + j\omega[C_{gs} + C_{gd}(1 + g_m (R \parallel r_o))]} \]

Looking in from the input terminal the capacitance \( C_{gd} \) seems to be magnified by a factor proportional to the low frequency open circuit voltage gain of the amplifier!!

And now the single pole is at:

\[ \frac{1}{\tau} \approx \frac{1}{C_{gs} + C_{gd}(1 + g_m (R \parallel r_o))} \]

\[ R_s \]
The Common Source Amplifier: Total Gain Under the Miller Approx

\[ 10 \log_{10} |H(\omega)|^2 \]

-20 dB/dec

\[ 10 \log_{10} \left| -g_m (R \parallel r_o) \right|^2 \]

\[ \frac{v_{out}(\omega)}{v_{in}(\omega)} = H(\omega) \]

\[ R_s C_{gs} + C_{gd} (1 + g_m (R \parallel r_o)) \]

Can be large

The Common Source Amplifier: Input Impedance

This approximate equivalent circuit will also work at not too high frequencies.

Looking in from the input terminal the capacitance \(C_{gd}\) seems to be magnified by a factor proportional to the low frequency open circuit voltage gain of the amplifier!!

\[ Z_{in}(\omega) \approx \frac{1}{j\omega \left[ C_{gs} + C_{gd} (1 + g_m (R \parallel r_o)) \right]} \]
The Miller Effect and the Miller Capacitance

John A. Miller (1920)

Let's generalize a bit: Consider a voltage amplifier:

\[ v_{out}(\omega) = A v_{in}(\omega) = A v_s(\omega) \]

\[ |A| >> 1 \]

Consider now a capacitor sitting at the input of an amplifier:

\[ v_{out}(\omega) = A v_{in}(\omega) \]

\[ = \frac{A}{1 + j\omega R C} v_s(\omega) \]

Input voltage to the amplifier decreases and so does the output voltage of the amplifier at high frequencies (but not too bad.....)

Consider now a capacitor straddling the input and the output of an amplifier:

\[ v_{out}(\omega) = A v_{in}(\omega) \]

\[ = \frac{A}{1 + j\omega R C (1 - A)} v_s(\omega) \]

Input voltage to the amplifier decreases and so does the output voltage of the amplifier at high frequencies

But the effective capacitance seen from the input side now is bigger (compared to the case on the previous slide) by a factor proportional to the gain of the amplifier

The amplifier input voltage, and the output voltage, will now begin to drop-off at a much lower frequency!!

This is the Miller effect and the capacitance positioned this way is called the Miller capacitance.
The Common Source Amplifier: Output Impedance

Need to find:

\[ Z_{out}(\omega) \]

KCL at (2) gives:

\[
\left[ v_t(\omega) - v_{gs}(\omega) \right] j\omega C_{gd} = j\omega C_{gs} v_{gs} + \frac{v_{gs}}{R_s}
\]

This gives:

\[
v_{gs}(\omega) = \frac{j\omega C_{gd} R_s}{j\omega (C_{gs} + C_{gd}) R_s + 1} v_t(\omega)
\]

Not a bad approximation even at moderately high frequencies
The Common Source Amplifier: Short Circuit Current Gain

\[ i_o(\omega) = i_{in}(\omega) \quad \text{\( C_{gd} \)} \]

Need to find:

\[ \frac{i_{out}(\omega)}{i_{in}(\omega)} \]

Start from KCL at (1):

\[ (0 - v_{in}(\omega))j\omega C_{gd} + g_m v_{in}(\omega) + i_{out}(\omega) = 0 \]

\[ \Rightarrow \frac{i_{out}(\omega)}{v_{in}(\omega)} = -(g_m - j\omega C_{gd}) \]

\[ \Rightarrow \frac{i_{out}(\omega)}{i_{in}(\omega)} \cdot \frac{v_{in}(\omega)}{i_{in}(\omega)} = -(g_m - j\omega C_{gd}) Z_{in}(\omega) = -g_m Z_{in}(\omega) \]

\[ = \frac{g_m}{j\omega(C_{gs} + C_{gd})} \]

Short Circuit Current Gain and the Transition Frequency \((f_T \text{ or } \omega_T)\)

For most transistors, the short circuit current gain falls off with frequency with a -20 dB/dec slope (at high enough frequencies)

The frequency at which the short circuit current gain equals unity is called the transition frequency \((f_T \text{ or } \omega_T)\)

The transition frequency expresses the maximum useful frequency of operation of the transistor
The Common Source Amplifier: The Transition Frequency

\[ \frac{i_{\text{out}}(\omega)}{i_{\text{in}}(\omega)} = \frac{g_m}{\omega(C_{gs} + C_{gd})} \]

10\log_{10} \left| \frac{i_{\text{out}}(\omega)}{i_{\text{in}}(\omega)} \right| = -20 \text{ dB/decade}

The transition frequency is:

\[ \omega_T = \frac{g_m}{C_{gs} + C_{gd}} = \frac{g_m}{C_{gs}} \]

This is the highest frequency at which the transistor is still useful.

Q: What is its physical significance?

Therefore the transition frequency is:

\[ \omega_T = \frac{g_m}{C_{gs}} = \frac{3}{2} \frac{\mu n (V_{GS} - V_{TN})}{L^2} \]

Therefore, the electron transit time sets the maximum frequency of operation of the FET!!