Pooling or not Pooling: the role of matching cost on mixed mode equilibria and VMT

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Carpooling is often promoted as a mode to reduce traffic congestion and vehicle miles traveled in road networks. This project develops a game-based mode-choice model for the morning commute involving carpooling supported by a ridesharing platform. In the model, commuters travel from homes in the suburb (Origin) to offices in downtown (Destination) either driving alone or carpooling. A ridesharing platform provides ride-matching services to carpoolers. Carpoolers riding in one vehicle can equally share the shareable travel costs among them but will be charged matching fees by the ridesharing platform. We assume all commuters attempt to minimize their own generalized commuting costs, and derive the evolutionarily stable states (ESS) and social optimum (SO) state by using replicator dynamics and by minimizing the average generalized costs, respectively. Under ESS and SO states, we analyzed the effects of high-occupancy-vehicle (HOV) lanes and matching fees on commuters’ mode choice and the ridesharing platform’s profits. We find that the platform’s profit-seeking behavior may increase congestion on general-purpose (GP) lanes and may reduce the efficiency of the overall system. Congestion on GP lanes is more likely to increase when the ratio of HOV capacity to freeway capacity is high, the proportion of shareable travel costs in generalized costs is low, inconvenience costs joining carpools are high, and the platform is profit-seeking. The results of this model show that ridesharing platforms need to be regulated and HOV capacity needs to be carefully allocated in order to make carpool benefit everyone.
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**Key Words:** Carpooling, carpooling platform, HOV lanes, mode choice, congestion, negative impact

1. Introduction

The sharing economy, which is supposed to reduce resource usage and improve social welfare, is a potential pathway to a sustainable society (Sundararajan, 2016; Heinrichs et al., 2013). In the transportation sector, carpooling (also called ridesharing) can reduce each commuter’s travel costs such as vehicle operating costs, fuel, tolls and parking fees by having more persons traveling in a vehicle in one journey (Ben-Akiva and Atherton, 1977; Yang and Huang, 1999). Because of the potential of decreasing the number of vehicle miles traveled (VMT) and alleviating congestion (Concas and Winters, 2007), carpooling is also environmentally friendly to reduce air pollution, greenhouse gas emissions and carbon footprints. Therefore, since the advent of carpooling during the periods of fuel shortage in the 1970s (Ferguson, 1997),
encouraging more solo drivers to join carpooling has been taken as an appealing way to develop sustainable transportation.

Nevertheless, in the U.S., despite the much effort put into promoting carpooling, its modal share has declined since the 1970s. According to the US Census, 20.4% of American workers commuted to work by carpool in 1970 (Chan and Shaheen, 2012). However, in 2016, only approximately 9% of commuters prefer to carpool with others, and over 76% of commuters choose to drive alone (Tomer, 2017). Similar to the U.S., the usage of carpooling in the English speaking countries such as the UK, Canada, and Australia is also relatively low in comparison with our expectations (Huang et al., 2000; Chan and Shaheen, 2012). The reasons why carpooling is not performing as billed include the proximity of carpool matches, the household car ownership, incompatible work schedules, the loss of independence and privacy, lack of convenience, etc (Buliung et al., 2009; Baldassare et al., 1998). Among these reasons, one major impedance to carpooling is the difficulty in finding partners in a timely manner (anecdotal evidence indicates that carpool pairing are formed over days and lasted for months or years). Causal carpooling, where pairing can happen in real time (e.g. within 24 hours), was not prevalent or feasible before the advent of smart phones and the rapid development of smart phone apps. Fortunately, the emergence of new mobile phone apps (e.g., Waze Carpool and Scoop) can overcome the barriers by offering a community-based online platform that automates the process of finding suitable carpool matches with short detours and compatible travel times (Quinn, 2017). Although these carpooling platforms are still in their infancy and only available in limited geographical areas, they clearly demonstrate the potential to increase carpooling (Ostrovsky and Schwarz, 2018). It is foreseeable that carpooling’s modal share can grow substantially as these platforms mature and the group of commuters who use them grows.

For decades, a substantial number of analytical and simulation studies has been conducted to overcome the challenges to widespread use of ridesharing (Furuhata et al., 2013). The challenges in ridesharing include ride-matching optimization (such as matching/routing/scheduling problems in dynamic ridesharing systems) (Agatz et al., 2012; Mourad et al., 2019; Agatz et al., 2011; Stiglic et al., 2015; Lee and Savelsbergh, 2015; Pelzer et al., 2015; Masoud and Jayakrishnan, 2017; Alonso-Mora et al., 2017; Wang et al., 2018, to name a few), pricing mechanisms (such as double auction, the Vickrey-Clarke-Groves pricing policy, and surge pricing strategy) (Kamar and Horvitz, 2009; Kleiner et al., 2011; Zhang et al., 2015; Bian and Liu, 2019; Bian et al., 2020; Li et al., 2020; Ma et al., 2020, to name a few), and trust and reputation management (Chaube et al., 2010; Sánchez et al., 2016; Baza et al., 2019). Some survey studies investigated the factors related to carpool willingness and shed light on the negative impacts of carpooling, such as loss of flexibility, convenience, and privacy (Chan and Shaheen, 2012; Delhomme and Gheorghiu, 2016; Hou et al., 2020). Other issues, including the potential benefits of carpooling, such as vehicle miles traveled, traffic congestion and air pollution reduction (Dewan and Ahmad, 2007; Yu et al., 2017; Li et al., 2018), and planning decisions, such as the construction of high occupancy vehicle (HOV) lanes and high-occupancy and toll (HOT) lanes (Li et al., 2007; Menendez and Daganzo, 2007; Konishi and Mun, 2010; Chu et al., 2012; Zhong et al., 2020), and
congestion pricing involving carpooling (Yang and Huang, 1999), have been discussed.

Many studies were concerned with the impact of carpooling on transportation by exploring the interactions between congestion, mode choices, and planning decisions. For example, Yang and Huang (1999) studied the carpooling and congestion pricing problem in the presence or absence of HOV lanes in a multi-lane highway based on a deterministic equilibrium model, in which commuters can choose to be carpoolers or solo drivers. Huang et al. (2000) investigated the mode choice behaviors under the deterministic and stochastic scenarios, and solved the no-toll equilibrium and social optimum. Konishi and Mun (2010) studied the welfare effects of HOV and HOT lanes and found that converting HOV lanes to HOT lanes may reduce the social welfare under certain parameters and road conditions. Lou et al. (2011) proposed a self-learning approach to identify optimal pricing strategies for HOT operations as well as efficiently utilize the capacities. Song et al. (2015) investigated the deployment of HOV or HOT lanes in general networks with the assumption that travelers’ mode choices follow a logit model. Xu et al. (2015) extended the static ridesharing problem for general networks with multiple origin-destination pairs and solved the unique ridesharing user equilibrium in the form of a nonlinear complementarity problem. Based on Xu et al. (2015), Di et al. (2018) investigated the deployment of HOT lanes in the network design problem for ridesharing and found that carefully selecting the deployment of HOT lanes can improve traffic efficiency. Li et al. (2019) unified the route choices and mode choices into a restricted path-based ridesharing model to determine the relationship between ridesharing activities and traffic congestion. Ban et al. (2019) proposed a general economic equilibrium model at the macroscopic level to evaluate the impacts of e-hailing services on deadhead miles and traffic congestion. Di and Ban (2019) modeled an analytical framework to investigate congestion effects from multiple travel modes and estimate the impact of ridesharing and e-hailing on transportation system performance.

Besides the static ridesharing models, many studies studied the dynamic ridesharing mode-choice problem in the morning commute. Qian and Zhang (2011) studied the morning commute problem with three modes, including carpooling, transit and solo driving, in a network with a freeway, an arterial road, and one dedicated transit route. Xiao et al. (2016) investigated a morning commute problem with carpooling and HOV lanes considering the parking space constraint at destination. Liu and Li (2017) modeled the morning commute problem in the presence of ridesharing. They found that carpooling could reduce congestion and increase all commuters’ benefits, and these benefits would be further enlarged when levying a time-varying toll. Ma and Zhang (2017) studied the combination of dynamic ridesharing and parking charges and found dynamic parking charges with appropriately ridesharing fees can improve the system performance in reducing vehicle miles and hours traveled. Long et al. (2018) proposed a stochastic ridesharing model by assuming travel time follows a time-independent general distribution that has a positive lower bound to discuss the influence of time uncertainty on carpooling. Yu et al. (2019) considered the effects of heterogeneous commuters on carpooling in the morning commute. Three policy scenarios, no-toll, first-best pricing, and subsidization of carpooling, are studied. They found that first-
best tolling and second-best subsidization could enhance carpooling. Zhong et al. (2020) investigated the effects of HOV lanes and HOT lanes on commuters’ benefits in the morning commute. Their simulation results indicated that HOV lanes promote carpool and boost welfare, and the benefits resulting from HOV lanes increase as the route capacity assigned to HOV lanes.

The literature mentioned above offers managerial insights into the effects of carpooling (ridesharing) on traffic congestion and social welfare. However, relatively less attention has been given to how a platform’s pricing strategies affect carpool ridership and the resulting commuting flow patterns. Furthermore, while the potential benefits of carpooling for commuters are well understood, the introduction of potential negative consequences from carpooling has not been widely discussed. This paper proposed a game-based mode-choice model supported by a carpooling platform to investigate commuters’ choice behaviors to understand the potential benefits and negative impacts of carpooling influenced by HOV lanes and the carpooling platform’s pricing strategy. In the model, we only consider one carpooling platform for providing carpool match services. Although a greater number of carpooling platforms could help develop technologies and reduce costs, it also leads to a fragmentation effect that can cause a failure for matching for a particular platform (Furuhata et al., 2013). Commuters have two options in the model: solo driving and carpooling, in which carpooling is motivated by cost-sharing. The matching decisions are made by the carpooling platform while participants only decide whether or not to partake. We do not distinguish drivers and passengers in the commuting model for two reasons. First, the paper mainly focuses on the behavior analysis and economic discussion in carpooling rather than the matching mechanism. Second, with the rapid development of autonomous vehicles, drivers will not be required in the future. Successful carpooling usually includes the specification of a pick-up and drop-off of a ridesharing participant (i.e., carpooler) and the experiences staying with strangers, which require some amount of slack time and inevitably generate extra travel costs (hereafter called inconvenience costs). Additionally, this model only investigates organized ridesharing operated by a carpooling platform, which provides ride-matching opportunities for commuters without any previous historical involvement (Dailey et al., 1999). Unorganized ridesharing, involving family, colleagues, neighbors, and friends, is not considered in the model. We investigate the effects of HOV lanes and the platform’s pricing on commuters’ choice behaviors and congestion on GP lanes and reveal the potential negative impacts of carpooling on congestion under certain platform pricing scenarios.

The remainder of this report is organized as follows. Section 2 proposes and analyzes the mode-choice model with a carpooling platform. Section 3 investigates the potential benefits and negative impacts of carpooling under this setting. Finally, Section 4 presents the conclusions of the paper with a discussion of future research.

2. The game-based carpooling model

In this section, we develop a game-based mode-choice model supported by a carpooling platform to investigate the effects of the carpooling platform’s pricing strategy and HOV lanes’ capacity on commuters’ carpooling behaviors. Suppose there is a multilane
freeway connecting one origin (O) to one destination (D), and a fixed number of N commuters travel from O to D to work every day. Commuters are treated as a continuum and their measure is fixed at N > 0. Fixed demand is a reasonable assumption for a commuting corridor because the number of people commuting to work is usually relatively constant in a short time period. Furthermore, we suppose there are two available modes traveling from O to D: commuters can either drive a vehicle alone (solo driving) or share a car with others (carpooling). Let n denote the number of carpoolers riding in one vehicle. The two modes are assumed to be perfect substitutes. A carpooling platform provides ride-matching services and charges commuters joining carpools matching fees. Let λ denote the matching fees charged by the carpooling platform.

A commuter’s generalized costs traveling from O to D are usually composed of multiple types of costs, such as fuel costs, travel-time costs, vehicle operating costs, tolls, parking fees, etc. Some costs can be shared by commuters riding in one vehicle, such as fuel costs, vehicle operating costs, tolls, and parking fees. In contrast, some costs must be taken by every commuter independently, such as travel-time costs. Without loss of generality, we divide the generalized costs into two types of costs: shareable travel costs and non-shareable travel costs. A fair cost-sharing mechanism is applied to shareable travel costs, and carpoolers riding in one vehicle can share these costs equally among them (Frisk et al., 2010). Let \( C^S(N_v) \) and \( C^N(N_v) \) denote the shareable and non-shareable travel costs, respectively, in which \( N_v \) is the number of vehicles traveling on the freeway. The two types of costs are both strictly increasing functions of \( N_v \). Additionally, let \( \theta \) denote the inconvenience costs generating from picking up each additional carpooler. Also, these inconvenience costs are assumed to share equally among carpoolers riding in one vehicle. High-occupancy-vehicle (HOV) lanes are reserved for vehicles with carpoolers. The effects of HOV lanes on commuters’ choice behaviors are also taken into consideration in the model. Accordingly, we study two scenarios, i.e., Scenario I without HOV lanes and Scenario II with HOV lanes.

2.1. Scenario I: no HOV lanes

Let \( p_s \) and \( \hat{p}_c \) denote the proportions of solo drivers and carpoolers, respectively, in which \( p_s + \hat{p}_c = 1 \). The number of vehicles traveling on the freeway is \( N_v(p_s) = N/n + N(1 - 1/n)p_s \), in which \( n (n \geq 2) \) is the number of carpoolers riding in one vehicle. The generalized cost functions of solo drivers and carpoolers can be described as

\[
\begin{align*}
C_s(p_s) &= (a_1 + a_2) + (b_1 + b_2)[N/n + N(1 - 1/n)p_s]d \\
\hat{C}_c(p_s) &= (a_1 + a_2/n) + (b_1 + b_2/n)[N/n + N(1 - 1/n)p_s]d + (1 - 1/n)\theta + \lambda
\end{align*}
\]

in which the total inconvenience cost is assumed to share equally by the carpoolers in one vehicle. In Scenario I, there are three possible states that all commuters have the same generalized costs, which are summarized as follows.

2.1.1. Evolutionarily stable state

Although replicator dynamics and evolutionarily stable state (ESS) have been
investigated and applied in many fields such as game theory, economy, and evolutionary biology, they have not received wide attention in transportation. One classical way of predicting a system’s state in transportation is to derive its user equilibrium (UE) (van Essen et al., 2016). ESS is a refinement of UE: an ESS derived from replicator dynamics is also a UE, but a UE may not be an ESS (Iryo, 2019). Furthermore, the trajectories of converging to an ESS from different initial states can be observed and predicted via replicator dynamics. The basic idea in replicator dynamics is that replicators whose fitness is higher than the average fitness of the population will increase their share in the population (Sandholm, 2010). In our model, higher fitness corresponds to lower generalized costs. Therefore, the travel mode (solo driving or carpooling) with lower generalized costs is more likely to increase its share of the population. The replicator dynamics without HOV lanes can be described by one dynamic equation as follows:

\[
\frac{dp_s}{dt} = p_s(C - C_s)
\]  

in which \( \bar{C} \) is the average generalized costs, and \( \bar{C} = p_s C_s + (1 - p_s) \bar{C}_c \). Let \( f(p_s) = p_s \bar{C} - C_s \). The fixed points in the replicator dynamics are given by \( f(p_s) = 0 \), i.e., \( p_s = 0 \), \( p_s = 1 \), and \( p_s = p^*_s \), in which \( p^*_s \) makes \( C_s(p^*_s) = \bar{C}_c(p^*_s) \) hold. According to the values of the first partial derivative at the possible fixed points, we obtain the following thresholds that separate three phases:

\[
\begin{align*}
\lambda^I_{1.2} &= (1 - 1/n) (a_2 - \theta + b (N/n))^d) \\
\lambda^I_{2.3} &= (1 - 1/n) (a_2 - \theta + b (N/n))^d)
\end{align*}
\]

in which \( \lambda^I_{1.2} \) and \( \lambda^I_{2.3} \) are irrelevant with the non-shareable travel costs and only depend on the inconvenience costs and the shareable travel costs. If \( \lambda \leq \lambda^I_{1.2} \), the solution of all commuters joining carpools is an ESS. If \( \lambda \geq \lambda^I_{2.3} \), the solution of all commuters driving alone is an ESS. If \( \lambda^I_{1.2} < \lambda < \lambda^I_{2.3} \), the solution of \( p_s = p^*_s \) is an ESS.

### 2.1.2. Social optimum

Social optimum (SO) is another important solution that can be a benchmark for measuring the efficiency of the ESS. In the model, SO is a state that the average generalized costs (AGC) are minimal. We have two critical points of \( \lambda \) that separate the phases under the optimal flow patterns are

\[
\begin{align*}
\lambda^H_{1.2} &= (1 - 1/n) (a_2 - \theta + [b_2 + d(nb_1 + b_2)]/(N/n))^d) \\
\lambda^H_{2.3} &= (1 - 1/n) (a_2 - \theta + [b_2 + d(b_1 + b_2)]/N)^d)
\end{align*}
\]

in which the two critical points satisfy \( \lambda^H_{2.3} > \lambda^H_{1.2} \). If \( \lambda \leq \lambda^H_{1.2} \), then the number of vehicles is \( N_v(0) = N/n \) and the optimal AGC is \( \bar{C}(0) \); if \( \lambda \geq \lambda^H_{2.3} \), then the number of vehicles is \( N_v(1) = N \) and the optimal AGC is \( \bar{C}(1) \); if \( \lambda^H_{1.2} < \lambda < \lambda^H_{2.3} \), then we
can find a $p^*_s$ for which $\frac{\partial \tilde{C}(p_s)}{\partial p_s} \bigg|_{p_s=p^*_s} = 0$ holds, and the optimal AGC under the optimal flow pattern is $\tilde{C}(p^*_s) = p^*_s \bar{C}_s(p^*_s) + (1-p^*_s)\hat{C}_c(p^*_s)$ and the corresponding number of vehicle is $N_v(p^*_s) = N/n + N(1-1/n)p^*_s$.

2.1.3. The properties of ESS and SO

The following proposition reveals the property between ESS and SO.

**Property 1:** When $\lambda_{1,2}^1 < \lambda < \lambda_{2,3}^1$, solo driving is overused at ESS relative to SO.

Furthermore, because carpooling can reduce the number of vehicles traveling from O to D, it is beneficial to ease congestion and reduce the travel costs of the commuters. Suppose all commuters join carpools, then the number of vehicles traveling from O to D at least halves. Therefore, carpooling can be seen as ‘cooperative’ in the game because it helps reduce congestion and improve traffic efficiency. In corresponding to carpooling, solo driving has many negative impacts, such as increasing congestion and fuel consumption, and can be regarded as a ‘defect’. In games, an N-prisoner’s dilemma (NPD) is a paradox in decisions in which all players act in their own self-interests and do not achieve the optimal outcome (Hamburger, 1973). In an NPD, defecting is the dominant strategy (i.e. defecting is the best strategy no matter how many other players choose to cooperate). If all players choose to defect, the outcome is worse than if all players choose to cooperate. To reveal the value range of $\lambda$ in which an NPD emerges, we introduce $\lambda_{AC}^1$ that makes the generalized costs of all commuters driving alone and all commuters joining carpools equal. The following property reveal the NPD in the absence of HOV lanes.

**Property 2:** When $\lambda_{2,3}^1 < \lambda < \lambda_{AC}^1$, the N-person prisoner’s dilemma emerges. Although the generalized costs of all commuters joining carpools are lower than all commuters driving alone, all commuters choose to drive alone.

2.2. Scenario II: with HOV lanes

In Scenario II, HOV lanes are provided to specially serve carpoolers. Without loss of generality, the congestion parameters can be denoted by route capacity $k$ and a constant $\beta_i$, i.e., $b_i = \beta_i k^{-d}$, where $i = 1,2$. In the scenario, the route has two different types of lanes, that is, the HOV lanes and the GP lanes. The capacity of HOV lanes is set as $k_c = \omega k$, and then the capacity of GP lanes is $k_g = (1-\omega)k$. The number of vehicles traveling on the GP lanes is $N_g(p_s, \hat{p}_c) = N(p_s + \hat{p}_c/n)$ and the number of vehicles traveling on the HOV lanes is $\tilde{N}_c(p_s, \hat{p}_c) = N(1-p_s - \hat{p}_c)/n$. In total, the formula of the number of vehicles traveling on the route is the same as Scenario I, that is, $N_v(p_s) = N/n + N(1-1/n)p_s$. The generalized cost functions of the three types of commuters can be denoted as,
\[
\begin{align*}
    C_s(p_s, \hat{p}_c) &= (a_1 + a_2) + (b_1 + b_2) \left( \frac{N}{1-\omega} \right)^d \left( p_s + \frac{\hat{p}_c}{n} \right)^d \\
    \hat{C}_c(p_s, \hat{p}_c) &= (a_1 + \frac{a_2}{n}) + (b_1 + \frac{b_2}{n}) \left( \frac{N}{1-\omega} \right)^d \left( p_s + \frac{\hat{p}_c}{n} \right)^d + \left( 1 - \frac{1}{n} \right) \theta + \lambda \\
    \hat{C}_c(p_s, \hat{p}_c) &= (a_1 + \frac{a_2}{n}) + (b_1 + \frac{b_2}{n}) \left( \frac{N}{1-\omega} \right)^d (1 - p_s - \hat{p}_c)^d + \left( 1 - \frac{1}{n} \right) \theta + \lambda
\end{align*}
\]

In which \( C_s \) and \( \hat{C}_c \) are increasing functions of \( p_s \) and \( \hat{p}_c \), whereas \( \hat{C}_c \) is a decreasing function \( p_s \) and \( \hat{p}_c \).

### 2.2.1. Evolutionarily stable state

The replicator dynamics in the presence of HOV lanes can be formulated as follows:

\[
\begin{align*}
    \frac{dp_s}{dt} &= p_s(\bar{C} - C_s) \\
    \frac{d\hat{p}_c}{dt} &= \hat{p}_c(\bar{C} - \hat{C}_c)
\end{align*}
\]

in which \( \bar{C} = p_s C_s + \hat{p}_c \hat{C}_c + (1 - p_s - \hat{p}_c)\hat{C}_c \). For convenience, let \( (p_s, \hat{p}_c, \bar{p}_c) \) denote the set of proportions of the three types of commuters. Letting \( \frac{dp_s}{dt} = 0 \) and \( \frac{d\hat{p}_c}{dt} = 0 \), we have the possible fixed points in the replicator dynamics as follows:

1) \((1,0,0)\): All commuters are solo drivers and travel on the GP lanes.
2) \((0,1,0)\): All commuters are carpoolers and travel on the GP lanes.
3) \((0,0,1)\): All commuters are carpoolers and travel on the HOV lanes.
4) \((0,1-\omega,\omega)\): All commuters are carpoolers and travel on the HOV lanes and GP lanes uniformly.
5) \((p_{5,5}^*, 0, 1 - p_{5,5}^*)\): Carpoolers only travel on the HOV lanes and all commuters traveling on the GP lanes are solo drivers.
6) \((p_{5,6}^*, 1 - p_{5,6}^*, 0)\): Commuters only travel on the GP lanes as carpoolers or solo drivers.
7) \((p_{5,7}^*, 1 - p_{5,7}^*, \bar{p}_{5,7}^*)\): There are three types of commuters, i.e, carpoolers traveling on the HOV lanes, carpoolers traveling on the GP lanes and solo drivers.

By analyzing the stability of the seven possible fixed points, we obtain the following thresholds that separate four phases:

\[
\begin{align*}
    \lambda_{1,2}^{III} &= \left( 1 - \frac{1}{n} \right) \left( a_2 - \theta + b_2 \left( \frac{N}{n} \right)^d \right) \\
    \lambda_{2,3}^{III} &= \left( 1 - \frac{1}{n} \right) \left( a_2 - \theta + b_2 \left( \frac{N}{n+1-\omega} \right)^d \right) \\
    \lambda_{3,4}^{III} &= (b_1 + b_2) N^d - \left( b_1 + \frac{b_2}{n} \left( \frac{N}{n-1} \right)^d + \left( 1 - \frac{1}{n} \right) (a_2 - \theta) \right)
\end{align*}
\]

in which the four critical points satisfy \( \lambda_{3,4}^{III} > \lambda_{2,3}^{III} > \lambda_{1,2}^{III} \).
Figure 1 shows the trajectory phases of converging to ESS under different matching fees in the presence of HOV lanes. When $\lambda < \lambda_{1,2}^{II}$, HOV lanes are not necessary because all commuters are carpoolers and travel on HOV lanes and GP lanes uniformly. Then, as $\lambda$ increases, a fixed point $R$ emerges, and it is the ESS. If $\lambda$ increases beyond $\lambda_{2,3}^{II}$, the fixed point $R$ moves to the SD-CH edge, and the commuters traveling on GP lanes are solo drivers. As $\lambda$ increases, the fixed point $R$ moves to SD along with SD-CH edge, and disappears until $\lambda \geq \lambda_{3,4}^{II}$. When $\lambda \geq \lambda_{3,4}^{II}$, all commuters travel on GP lanes as solo drivers, and HOV lanes will be out of use because of the extremely large matching fees.

2.2.2. Social optimum

Next, we calculate the optimal AGC and the corresponding number of vehicles. According to the values of $\lambda$, we have the critical points of $\lambda$ that separate the phases under optimal flow patterns as follows,

$$
\begin{align*}
\lambda_{1,2}^{II} &= (1 - \frac{1}{n}) \{ a_2 - \theta + [b_2 + d(nb_1 + b_2)](\frac{N}{n})^d \} \\
\lambda_{2,3}^{II} &= (1 - \frac{1}{n})(a_2 - \theta) + (d + 1)[(b_1 + b_2)(\frac{N}{1 - \omega})^d \left(\frac{\eta}{1 + \eta}\right)^d - \left(\frac{b_1 + b_2}{n}\right)(\frac{N}{n\omega})(\frac{1}{1 + \eta})^d \\
\lambda_{3,4}^{II} &= (1 - \frac{1}{n})(a_2 - \theta) + (d + 1)(b_1 + b_2)(\frac{N}{1 - \omega})^d
\end{align*}
$$

where $\eta = \left\{ \frac{(d+1)b_1 + b_2}{(d+1)b_1 + b_2} \right\} \frac{1 - \omega}{n\omega}$ and $\lambda_{3,4}^{II}$ is strictly greater than the other two critical points. Unlike the optimal flow patterns in Scenario I, there are three critical points in Scenario II. Moreover, the relative difference between $\lambda_{1,2}^{II}$ and $\lambda_{2,3}^{II}$ is not identified. Because $\lambda_{2,3}^{II}$ is an decreasing function of $\omega$, we can obtain the value of $\omega^*$ according to $\lambda_{1,2}^{II} = \lambda_{2,3}^{II}$. If $\omega < \omega^*$, there are four phases under the optimal flow.
patterns in Scenario II; if $\omega \geq \omega^*$, then the phase that three types of commuters all exist disappears. In this case, we need to find a new critical point $\lambda_{1,3}^{II}$ by comparing the magnitude between $C(0,1-\omega)$ and $\min \tilde{C}(p_s,0)$. Nevertheless, the value of $\omega$ that satisfies the case is usually relatively large, that is, most of the capacity of the route need to assign to HOV lanes. As a consequence, we only consider the situation where $\lambda_{1,2}^{II} < \lambda < \lambda_{2,3}^{II}$ in the following. If $\lambda \leq \lambda_{1,2}^{II}$, then the optimal AGC is $C(0,1-\omega) = (a_1 + a_2/n) + (b_1 + b_2/n)(N/n)d + (1 - 1/n)\theta + \lambda$ and the corresponding number of vehicles is $N_v(0) = N/n$; If $\lambda \geq \lambda_{3,4}^{II}$, then the optimal AGC is $\tilde{C}(1,0) = (a_1 + a) + (b_1 + b_2)/(N/n)d + (1 - 1/n)\theta + \lambda$ and the number of vehicles is $N_v(1) = N$.

2.2.3. The properties of ESS and SO

The following property reveals the property between ESS and SO in the presence of HOV lanes.

**Property 3**: When $\lambda_{1,2}^{II} < \lambda < \lambda_{2,3}^{II}$, solo driving is overused at ESS relative to SO.

Similar to Scenario I, commuters can choose carpool (‘cooperate’) or driving alone (‘defect’) in Scenario II. Is there an NPD in the presence of HOV lanes? By solving $C_s(1,0) = \tilde{C}_c(0,1-\omega)$, we have $\lambda_{AC}^{II}$ that makes the generalized costs of all commuters driving alone and all commuters joining carpools equal. It is evident that $\lambda_{AC}^{II} < \lambda_{3,4}^{II}$. Therefore, there is no NPD in the presence of HOV lanes. In other words, HOV lanes assigned to carpoolers can avoid the NPD emerged from the scenario with no HOV lanes.

3. The effect of carpooling on congestion
The existence of the carpooling platform and HOV lanes can encourage more solo drivers to be carpoolers and reduce traffic congestion when the inconvenience costs and matching fees are not very high. The total generalized costs (TGC) are composed of three components, i.e., total travel costs (TTC), total matching fees (TMF), total inconvenience costs (TIC):

\[
TGC = TTC + TMF + TIC \quad (9)
\]
in which TMF and TIC can be obtained from \( TMF = N(\hat{p}_c + \hat{p}_c)\lambda \) and \( TIC = N(\hat{p}_c + \hat{p}_c)(1 - 1/n)\theta \), respectively.

Next, we analyze the factors that are related to the commuters’ choice behaviors and the carpooling platform’s profits. BPR functions are used in the numerical examples. Unless otherwise specified, the parameters are set as: \( N = 20000 \), \( a = 4 \), \( b = 0.0002 \), \( d = 1 \), \( n = 2 \), \( a_2/a = 0.5 \), \( b_2/b = 0.5 \). Furthermore, to measure the congestion on GP lanes, we define the congestion index as follows:

\[
\xi_{GP} = (p_s + \hat{p}_c/n)/(1 - \omega) \quad (10)
\]
If \( \xi_{GP} > 1 \), then there is more congestion on GP lanes induced by carpool relative to the base scenario; if \( \xi_{GP} < 1 \), then carpooling reduces the congestion not only on HOV lanes but also on GP lanes.

![Figure 2](image)

Figure 2: Matching fees, proportions of solo drivers, and TMF for different \( \omega \) when maximizing TMF.

Figure 2 shows the congestion on GP lanes and the platform’s profits at ESS (SO) for different \( \lambda \) and \( \omega \). As shown in Figure 2 (a), when \( \lambda \) is small (i.e., \( \lambda/C_0 < 0.156 \)), all commuters are carpoolers at ESS, and the number of vehicles on GP lanes will be halved. Then, the congestion on GP lanes increases as \( \lambda \) increases because some commuters drive alone. However, when \( \lambda \) is still relatively small (e.g., \( \lambda/C_0 < 0.2 \),
HOV lanes help reduce congestion on GP lanes. However, when $\lambda$ is relatively large (e.g., $\lambda/C_0 > 0.41$), the existence of HOV lanes will induce more congestion on GP lanes. In this case, the severity of congestion on GP lanes increases as $\omega$ increases. The tendency of congestion increase on GP lanes at SO is similar to that at ESS, but there is less congestion on GP lanes at SO relative to ESS at given $\lambda$ and $\omega$ (see Figure 2 (b)). Unlike the monotonic increase in congestion on GP lanes, the TMF has many patterns as $\lambda$ increases. As shown in Figure 2 (c), the TMF at ESS first increases and then decreases as $\lambda$ increases in the absence of HOV lanes (i.e., $\omega = 0$). The TMF reaches the peak value when $\lambda = \lambda_{1,2}^H$. However, when there are HOV lanes, the TMF decreases as $\lambda$ increases from $\lambda_{1,2}^H$ to $\lambda_{2,3}^H$. In this case, solo drivers and carpoolers both exist on the GP lanes. When $\lambda > \lambda_{2,3}^H$, carpoolers only travel on HOV lanes. The TMF first increases and then decreases as $\lambda$ increases from $\lambda_{2,3}^H$ to $\lambda_{3,4}^H$. Therefore, two maximum points for TMF exist when HOV lanes exist: one is at $\lambda = \lambda_{1,2}^H$ and the other is located between $\lambda_{2,3}^H$ and $\lambda_{3,4}^H$. As shown in Figure 2 (d), the pattern of TMF at SO is similar as that at ESS, and the TMF at SO is larger than that at ESS at given $\lambda$ and $\omega$ because more commuters join carpoolers at SO relative to ESS.

As Milton Friedman once said, “The social responsibility of business is to increase its profits.” (Friedman, 2007). In our model, the carpooling platform’s profits are associated with TMF. Let $\lambda^*$ denote the matching fees that make TMF maximal (i.e., max{TMF}). Figure 3 shows $\lambda^*$, congestion on GP lanes and the carpooling platform’ profits when maximizing TMF at ESS (SO) for different $\omega$. As shown in Figure 3 (a), the $\lambda^*$ is a piecewise function of $\omega$ no matter at ESS or SO. When $\omega$ is small, encouraging as many commuters as possible to join carpools through a relative small matching fee is beneficial for the carpooling platform (see Figure 3 (b)). In this case, $\lambda^* = \lambda_{1,2}^H$ at ESS, and $\lambda^* = \lambda_{1,2}^H$ at SO. Accordingly, the TMF at ESS (SO) is constant (Figure 3 (c)). We can see that $\lambda^*$ at ESS and SO almost triples in the vicinity of $\omega = 0.39$ and $\omega = 0.45$, respectively. Then, $\lambda^*$ increases as $\omega$ increases, and commuters traveling on GP lanes are solo drivers. The congestion on GP lanes becomes more severe as $\omega$ increases. Meanwhile, the carpooling platform can gain more profits as $\omega$ increases. Furthermore, as shown in Figure 3 (c), the carpooling platform can benefit more at SO relative to ESS at a given $\omega$ with different optimal matching fees.

Next, we discuss the impact of shareable travel costs on the congestion on GP
lanes and the carpooling platform’s profits when maximizing TMF. Figure 4 (a-c) show the results at ESS. As shown in Figure 4 (a), the carpooling platform is more likely to charge a high matching fee (i.e., $\lambda^*$) when the proportion of shareable travel costs in generalized costs is low. However, when the proportion of shareable travel costs in generalized costs is high, the carpooling platform is more likely to lower $\lambda^*$ to attract more commuters to join carpools. The pricing strategy of maximizing profits significantly affects congestion on GP lanes (see Figure 4 (b)). When the proportion of shareable travel costs in generalized costs is low, the pricing strategy of maximizing profits is more likely to increase congestion on GP lanes. Furthermore, the severity of congestion on GP lanes increases as the proportion of shareable travel costs in generalized costs decreases. However, when the proportion of shareable travel costs in generalized costs is high, the number of vehicles on GP lanes halves because of carpooling.

Figure 4: Matching fees, congestion on GP lanes, and TMF for different $a_2/a$ and $b_2/b$ when maximizing TMF. The ratio of HOV capacity to freeway capacity is set as $\omega = 1/2$.

Unlike $\lambda^*$ and congestion on GP lanes, as shown in Figure 4 (c), the TMF increases as the proportion of shareable travel costs in generalized costs increases, indicating that the carpooling platform gains more profits when more travel costs can be shared among carpoolers. Figure 4 (d-f) show the results at SO. As shown in Figure 4 (d), the carpooling platform is more likely to charge a high matching fee when $a_2/a$ is low and $b_2/b$ is high. However, when $a_2/a$ is high, the carpooling platform is more likely to charge a relatively low matching fee to attract more commuters to join carpools. Accordingly, the number of vehicles traveling on GP lanes halves when $a_2/a$ is high. Assuming all generalized costs can be shared equally among carpoolers riding in one vehicle (i.e., $a_2/a = 1$ and $b_2/b = 1$). The number of vehicles traveling on GP lanes at ESS halves while congestion on GP lanes becomes more severe at SO. Therefore, optimal congestion pricing leading the system from ESS to SO may increase congestion on GP lanes. As shown in Figure 4 (f), the TMF at SO also increases as the
Finally, we investigate the effects of inconvenience costs on GP lanes’ congestion and the carpooling platform’s profits. As shown in Figure 5 (a), the $\lambda^*$ decreases as $\theta$ increases, indicating that more inconvenience can make the carpooling platform reduce the matching fees. As a result, as shown in Figure 5 (c), the TMF also decreases as $\theta$ increases, indicating that inconvenience costs negatively influence the carpooling platform’s profits. Furthermore, as shown in Figure 5 (b), congestion on GP lanes increases as $\theta$ increases because more commuters choose to drive alone when joining carpools becomes more inconvenient. When the carpooling platform is driven by revenue maximization, the matching fees at SO are larger than those at ESS (see Figure 5 (a)). Furthermore, as shown in Figure 5 (b), congestion on GP lanes at SO may be more severe than at ESS when inconvenience costs are not very high.

4. Conclusions and discussion

In this paper, we have proposed a game-based mode-choice model to investigate the effects of HOV lanes’ capacity and a carpooling platform’s pricing strategy on commuters’ choice behaviors. Commuters can choose to drive alone or carpool, and a carpooling platform provides ride-matching services. Carpoolers riding in one vehicle can share the shareable travel costs equally among them but are required to pay matching fees to the carpooling platform and bear the inconvenient costs resulting from carpooling. We obtain the evolutionarily stable state (ESS) by applying replicator dynamics and derive the social optimum (SO) by minimizing the average generalized costs. Two different scenarios concerning HOV lanes, i.e., Scenario I without HOV lanes and Scenario II with HOV lanes, are studied. We assume the carpooling platform is driven by revenue maximization and investigate the factors related to congestion on GP lanes and the carpooling platform’s revenues. Our numerical examples indicate that the platform’s profit-seeking behavior may increase GP lanes’ congestion, and even reduce overall traffic efficiency.

When the carpooling platform is driven by revenue maximization, the ratio of HOV capacity to freeway capacity is high, the proportion of shareable travel costs in generalized costs is low, and the inconvenience costs joining carpools are high, congestion on GP lanes is more likely to become more severe relative to no carpooling.
and no HOV lanes. Our results reveal a negative side of platform supported carpooling, which should not be neglected in promoting ride-matching services, and support the regulation of carpooling platform. They also point to the importance of property allocating HOV capacity.

Our study has some limitations that could provide opportunities for future research. First, our model mainly focuses on the mode choice behaviors and the potential negative impacts of carpooling supported by a carpooling platform. Although some policy measures, such as regulating the carpooling platform’s matching fees, can be easily obtained from the above analysis, a comprehensive analysis of different policy measures to maintain commuters’ benefits and improve carpooling rate should be conducted. Second, we only consider two modes, solo driving and carpooling; however, commuters usually face multiple mode choices, such as public transport, vanpooling, solo driving, and carpooling, as well as many other choices, such as departure time, route, and destination. Investigating travel choice behaviors when facing multiple objectives can help us further understand the potential benefits and potential pitfalls of carpooling supported by a carpooling platform. Third, we assume there is only one carpooling platform providing ride-matching services in the model. However, in reality, commuters can choose from multiple platforms, such as Uber and Lyft in the U.S. A carpooling platform usually needs to compete with the other platforms to attract more users, and such competition will inevitably impact a platform’s pricing strategy and should be further investigated.

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