

Electric Vehicle Sharing Planning and Operations

Center for Transportation, Environment, and Community Health
Final Report



by

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16. Abstract Project Abstract: This project includes literature review and proposing a model for EV planning and operations. We first conducted a literature review on the relevant EV planning and operations. Then we propose a model that integrates both electric vehicle technologies and car sharing operations. Based on the simulation model, a dynamic and time-continuous optimization model seeking the optimal design of charging station location and electric vehicle deployment is developed. By discretizing the model, we proposed a Monte Carlo simulation model to evaluate the total system cost for a given location and vehicle deployment design. A heuristic algorithm is developed to solve the optimal system design of station location and vehicle deployment. A numerical test in Yantai city, China is conducted to illustrate the effectiveness of the proposed model and draw managerial insights into how the key parameters affect the optimal system design.					
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1. Introduction

Private vehicle ownership, though granting a traveler flexible access to fast transportation service, is a leading contributor to increasing highway congestion, intensifying parking burden, escalating transportation cost and deteriorating environment across the world. Private vehicles largely contribute to 17 percent household expenses on transportation, around 70 percent petroleum consumption, and around 30 percent of greenhouse gas emission in the US (Bureau of Transportation Statistics, 2014 Pocket guide of transportation). In addition, private vehicles are parked 23 hours a day on average while public transportation usually has fixed routes and schedules serving limited areas and does not always provide a rapid service. To bridge private and public transportation modes, car sharing was introduced as a new transportation mode that combines the flexibility and mobility of private vehicles and the economic and environmental benefits of public transit. With car sharing services, travelers have flexible access to a fleet of spatially distributed vehicles any time, so that they can make considerable savings without purchasing, storing and maintaining individual vehicles (Millard-Ball 2005). Due to these advantages, commercial car-sharing services are booming all over the world in recent years. For example, over the past decade in the US, the number of vehicles participating in car sharing has grown from under 700 to over 15,000, and the population benefiting from this service increases from 16,000 to over a million (Shaheen, P and Cohen, A 2014).

As the concept of car sharing changes transportation service paradigms, technological advances, such as electrical vehicles (EV), further help make transportation systems greener. Compared with traditional internal combustion engine vehicles (ICEV), EVs can completely eliminate tailpipe pollutants along city streets that can directly inhaled by urban travelers and residents. Even if the energy sources at electric plants are traced, due to the concentration effects and electric generation scales, the overall life-cycle energy consumption and emissions from EVs are still much less than those from ICEVs. As a result, the unit-distance cost and related social impacts from an EV are significantly lower than those from an ICEV. These costs will continue to decrease as the battery technologies evolve and massive productions bring economies of scale. With these salient environmental benefits, replacing a portion of ICEVs with EVs surely helps alleviate the unrepresented energy and environmental pressures across the world, particularly in cities with high population density and low air quality.

The benefits of car sharing and EVs can be mutually enhanced when they are integrated into EV sharing services. The EV technology has a lower unit-distance cost and thus makes EV sharing more affordable. Distributed car-sharing infrastructure including charging stations further reduces people's anxiety on EV range, which is a major barrier against purchasing privately owned EVs (King, Christopher and Griggs, Wynita and Wirth, Fabian and Shorten, Robert, "Using A Car Sharing Model To Alleviate Electric Vehicle Range Anxiety", in *the 16th Yale Workshop on Adaptive and Learning Systems* (2013).). Together, EV sharing can further enhance transportation accessibility and mobility, reduce travel cost, alleviate the energy dependence and improve the environment. Given the high expectation that most EV companies will replace their internal combustion engine vehicles with EVs in the near future, traditional car sharing systems need substantial revisions correspondingly (Luè et al. 2012). Such change mandates a comprehensive system design framework for an EV sharing system that determines both one-time infrastructure investment strategies and day-to-day operational decisions. The quantitative design of such a comprehensive EV sharing system is a very complex decision process because it needs to simultaneously determine EV sharing station locations, the EV fleet size at each station, traveler demand management and vehicle allocation operations. In particular, different from traditional ICEV sharing systems, an EV needs to be recharged when its battery is about to deplete, which may take considerable time. An EV's charging time likely cuts into the available time of this EV, which may conflict with frequent demand for this EV in the car-sharing service. Further, the recovery of the battery energy of an EV is likely a nonlinear function of the charging time, which further complicates this system's operations. As far as our knowledge, Li et al. (2017) is the only existing study that investigates how to systematically plan such an EV sharing system considering both planning and operational costs. They proposed a macroscopic-continuum model that successfully reveals spatial patterns of a near-optimum design for an EV sharing system and the high-level trade-offs among key cost components. However, the study assumes relatively homogeneous settings across the space and simplifies complicated EV sharing operations under the non-linear charging time constraints into low-resolution approximations. To complement the existing study, this paper aims to build an EV sharing design model that considers detailed individual vehicle operations along the total planning horizon. This model has a two level structure. The lower level is a high-fidelity simulation module that evaluates the expected cost from detailed day-to-day operations (i.e., traveler waiting cost, vehicle holding and rebalance cost) under stochastic demand for a given set of system design variables (i.e., EV station locations and fleet sizes). The upper level problem optimizes the design variables to minimize the total system cost including both the long-term infrastructure investment and the expected day-to-day operation cost. A numerical example in Yantai city, China is conducted to illustrate the efficiency of our proposed model and algorithm.

2. Literature Review

Car sharing services grew rapidly in the past decade worldwide and have been well received by both service providers and travelers (Millard-Ball 2005, Loose 2010, Ferrero, Francesco, et al., 2017). Early attempts in establishing car sharing systems date back to 1940s in Europe (Cohen, P., and S. Shaheen, 2006). The first large-scale car sharing programs appear in late 1980s (Millard-Ball, 2005). Early studies on quantitative design of car-sharing systems suffered from a limited number of facility members in their system models (Bonsall, Peter, 1982). Car sharing system, at the beginning is two-way or called roundtrip systems, i.e. customers obtain and return the vehicle at same locations. Later on, researchers tried to evaluate the practical car-sharing programs from different points of view including travel behaviors (Steininger et al., 1996) and cost-benefit analysis (Fellows and Pitfield, 2000; Huwer, 2004). Several car-sharing modeling methods were developed, such as fleet size problems based on queuing models (George and Xia, 2011), fleet assignment problems (Du and Hall, 1997, Chauvet et al., 1997; Fan et al., 2008), facility location designs (Correia and Antunes, 2012; Kumar and Bierlaire, 2012) and vehicle relocation problems (Kek et al., 2009). However, later on, one-way car sharing became popular. Given the demand is asymmetric, locating car sharing stations for a one-way system is different from two-way system. Different methodologies have been applied to study the location problems of one-way car sharing (Jorge et al. 2012; Jorge, Diana et al. 2014; Boyacı, Burak et al., 2015). Besides, some other aspects of car sharing systems have been analyzed such as customer behavior simulation (Bonsall, 1980), vehicle availability and energy management (Barth and Todd, 1999), costs and revenues optimization (Ciari et al., 2008) and travel demand estimation (Ciari et al., 2010).

The most important part of a successful car sharing systems refers to its infrastructure location design. Traditional studies attempted to determine the optimal serving facility locations to capture maximum customer flow as well as to minimize the number of facilities required to cover a given flow (Berman et al., 1992; Hodgson, 1990). For more information on location design problems see (Daskin, 1996; Owen and Daskin, 1998, p. 22; Snyder, 2006; An et al., 2014). Some other studies provided joint design problems that simultaneously consider facility location problem with other network elements such as telecommunication applications (Melkote and Daskin, 2001), disruption risks (Shen et al., 2003) and warehouse incorporations (Yao et al., 2010). But, emerging location design problems on a car sharing system is more complicated, because companies are replacing their internal combustion engine (ICE) vehicles with EVs, due to cheaper costs and less environmental impacts. Therefore, two types of stations, refueling and charging, have to be located optimally. Recently, some researchers investigated alternative-fuel vehicles considering the fact that these vehicles may need to stop at more than one facility along the entire path (Kuby et al., 2009; Kuby and Lim, 2005). Also, a few studies were extended by considering capacitated refueling stations (e.g. Frade et al., 2011; Upchurch et al., 2009), routing (Worley et al., 2012), fueling schedule decisions (Nourbakhsh and Ouyang, 2010), battery charging capacity (Nie and Ghamami, 2013) and battery exchange stations (Mak et al., 2013; Pan et al., 2010).

In addition to location design, determining the fleet size is another important factor in car sharing systems. A notable number of previous approaches in fleet size problems attempted to develop a mixed fleet size and vehicle routing problem (Brandão, 2009; Golden et al., 1984; Liu et al., 2009; Renaud and Boctor, 2002; Repoussis and Tarantilis, 2010). However, fewer studies have explored the fleet size problems in the context of shared vehicle systems such as vehicle rental systems (George and Xia, 2011; Li and Tao, 2010), bicycle sharing system (Nair et al., 2013; Pal et al.) and car-sharing system (Nair and Miller-Hooks, 2010). There are different issues have to be addressed considering a fleet size problem for a car sharing system. In shared vehicle frameworks, since flows between stations are not necessarily matched and fleet could be spatially imbalanced. For instance, some scholars considered the fleet size of a vehicle sharing system based on flow asymmetry (Li and Tao, 2010; Nair et al., 2013), demand uncertainty (List et al., 2003; Nair and Miller-Hooks, 2010) and user and operator cost-effectiveness analysis (Li et al., 2010).

The application of electric vehicles in car sharing systems is a novel development (Luè et al., 2012). Therefore, few researches have been conducted in the context of electric car sharing systems, and to the best of our knowledge, there are very rare studies attempted to design an electric car sharing framework (e.g. X Li, Ouyang Y et al., 2013). This paper contributes to the existing literature by developing an electric car sharing system design that considers simultaneous decisions about charging station locations and fleet sizes. Since, location design problems are generally NP-hard, typical solution approaches may not offer very computational efficient procedures. Therefore, we propose a Monte Carlo simulation to overcome this issue and find a near optimal solution (Kocsis and Szepesvári, 2006). Also, we consider a nonlinear charging function to give the design framework a more realistic view of electric car charging process.

3. Model Formulation

This paper studies the optimal design of charging stations and fleet deployment for a one-way station-based EV sharing system. The studied problem can be decomposed into two levels: the upper level problem makes the system infrastructure design decisions including siting EV charging stations and determining corresponding EV fleet sizes;

and the lower level problem evaluates system operation costs given an infrastructure design from the upper level problem. We first describe EV sharing operations in the lower level problem given the design of charging station locations and fleet sizes. We consider a set of user locations \mathbf{I} distributed in a space. We assume that the EV sharing system's operation is repeated by an operation cycle of length T (e.g., a day), and thus we can just investigate a generic operation cycle from time 0 through time T . Let $\tilde{\lambda}_{ij}(t)$ denote the stochastic rate of generated travel demand from one location $i \in \mathbf{I}$ to another location $j \in \mathbf{I}$ at any time $t \in [0, T]$. We allow $\tilde{\lambda}_{ij}(t)$ to follow a general random distribution pattern with $\lambda_{ij}(t)$ as its mathematical expectation that repeats every operation cycle (e.g., the diurnal distribution of travel demand). Each user location $i \in \mathbf{I}$ is attached with a parking lot that has \bar{V}_i parking spots and hosts a fleet of fully charged EVs of size V_i at time 0. Every EV, when fully charged, can travel at maximum a distance of C , which can be also interpreted as an EV's maximum battery level or capacity in distance units. Let c_{ij} denote the travel distance from location i to j , and thus a CV's battery level drops by c_{ij} after traveling from i to j . In this lower level problem, the EV charging stations have been already installed at a subset of locations in \mathbf{I} , denoted by $\hat{\mathbf{I}}$, to replenish batteries of EVs. We assume that every charging station has sufficient charging units, and whenever a vehicle is parked at a charging station, it will be charged immediately until reaching its battery capacity.

After the operation cycle starts, when a user from location $i \in \mathbf{I}$ arrives at its parking lot, she looks for a shared EV with a battery level no less than the travel distance to her destination $j \in \mathbf{I}$, c_{ij} . If multiple EVs have battery levels greater than c_{ij} , we assume that this user is always assigned with the one with the minimum battery level among them (so as to spare EVs with higher battery levels for future users traveling longer distances). If such an EV exists at location i , then this user immediately takes this vehicle to start the trip to her destination. Otherwise, she has to wait in a queue until one EV with the battery level greater than c_{ij} becomes available at this location. We assume that the unit-time waiting cost of a user is ν . The EV that this user will pick up could be one brought to location i by a user from another location. Or if location i is equipped with a charging station, i.e., $i \in \hat{\mathbf{I}}$, this EV could be an existing one parked at this location that had insufficient battery level but just got charged to c_{ij} . Further, we consider that the rate of EVs traveling from lot i to lot j cannot exceed μ_{ij} , the bottleneck capacity between i and j , $\forall i, j \in \mathbf{I}$. This capacity allows us to incorporate roadway traffic congestion in this study. We allow one-way EV sharing, and thus a user can just leave the EV at the destination parking lot when she arrives without returning it. By the end of the operation cycle, due to one-way vehicle movements, the ending fleet size at parking lot i may not be exactly identical to its initial fleet size V_i . In this case, we need to restore its fleet size to the initial value V_i to prepare for the next operation cycle. If the ending fleet size of parking lot i is less than V_i , a proper number of vehicles will be brought to this lot from some other lots with vehicle surpluses to make up the difference. Otherwise, the extra EVs from this lot will be sent out to other lots with deficit. Let \tilde{b}_{ij} denote the number of EVs moved from lot i to lot j , and r_{ij} denote the cost of moving an EV from i to j , $\forall i, j \in \mathbf{I}$. After this fleet balancing operation, every lot i shall just have V_i EVs and thus the next operation cycle of the EV sharing system would start from the same state.

To facilitate the formulation of the system operations, we introduce the following intermediate variables. Let $\tilde{q}_{ij}(t)$ denote the waiting queue length of users at time t who plan to travel from lot i to lot j , $\forall i \neq j \in \mathbf{I}$, $t \in [0, T]$. We assume that all queues are cleared at the end of an operation cycle (e.g., midnight), and thus every queue at the beginning of a cycle shall be zero, i.e.,

$$\tilde{q}_{ij}(0) = 0, \forall i \neq j \in \mathbf{I}. \quad (1)$$

To formulate the change of \tilde{q}_{ij} over time, we define $\tilde{d}_{ij}(c, t)$ as the departing rate of EVs with a battery level no less than C from lot i to lot j , $\forall i \neq j \in \mathbf{I}$, at time t , $c \in [0, C]$. Then we obtain

$$\frac{d\tilde{q}_{ij}(t)}{dt} = \tilde{\lambda}_{ij}(t) - \tilde{d}_{ij}(c_{ij}, t), \forall i \neq j \in \mathbf{I}, t \in [0, T]. \quad (2)$$

To formulate departing rate $\tilde{d}_{ij}(c, t)$, we further let $\tilde{I}_i(c, t)$ denote the number of the EVs at location i with a battery level no less than C at time t , $\forall c \in [0, C], t \in [0, T]$. Then $\tilde{d}_{ij}(c, t)$ can be quantified as follows. Note that no EV with a battery level less than c_{ij} can serve a trip from i to j , therefore

$$\tilde{d}_{ij}(c, t) = \tilde{d}_{ij}(c_{ij}, t), \forall i \neq j \in \mathbf{I}, t \in [0, T], c \in [0, c_{ij}]. \quad (3)$$

For a $c \geq c_{ij}$, $\tilde{d}_{ij}(c, t)$ is determined by the available EVs at or above battery level C , the arriving user demand, the roadway capacity, and the queue length, i.e.,

$$\tilde{d}_{ij}(c, t) = \begin{cases} 0, & \text{if } \tilde{I}_i(c, t) = 0; \\ \min\{\mu_{ij}, \tilde{\lambda}_{ij}(t)\}, & \text{if } \tilde{I}_i(c, t) > 0, \tilde{q}_{ij}(t) = 0; \forall i \neq j \in \mathbf{I}, t \in [0, T], c \in [c_{ij}, C]; \\ \mu_{ij}, & \text{if } \tilde{I}_i(c, t) > 0 \text{ and } \tilde{q}_{ij}(t) > 0, \end{cases} \quad (4)$$

We assume that EV movements are negligible at the end of the previous operation cycle (e.g., due to the low demand at that time), and thus all vehicles are fully charged at the beginning of this operational cycle, i.e.,

$$\tilde{I}_i(c, 0) = V_i, \forall i \in \mathbf{I}, c \in [0, C]. \quad (5)$$

For a parking lot i without a charging station, the change of $\tilde{I}_i(c, t)$ over time is determined by the difference between the arrival and the departure rates (or the net arrival rate) of EVs at or above battery level C . Any EV with a battery level above $c + c_{ji}$ departed from any lot $j \neq i$ at time $t - t_{ji}$ and destined at lot i contributes to a unit increase of $\tilde{I}_i(c, t)$, and any EV with a battery level above C departed from lot i at time t contributes to a unit decrease of $\tilde{I}_i(c, t)$. Therefore,

$$\frac{\partial \tilde{I}_i(c, t)}{\partial t} = \sum_{j \neq i \in \mathbf{I}} [\tilde{d}_{ji}(c + c_{ji}, t - t_{ji}) - \tilde{d}_{ij}(c, t)], \forall i \in \mathbf{I} \setminus \hat{\mathbf{I}}, c \in [0, C], t \in [0, T]. \quad (6)$$

Whereas if lot i has a charging station, besides the EV net arrival rate, $\frac{\partial \tilde{I}_i(c, t)}{\partial t}$ also results from charging EVs from a slightly lower battery level to C . Note that the EV's battery charging could be a nonlinear function of the charging time. To capture a general charging curve, we denote the battery level of an EV charged for time t from a zero battery level (or a completely drained battery) with a general non-decreasing function $F(t)$ that satisfies $F(0) = 0$, $\lim_{t \rightarrow \infty} F(t) = C$. Further, function $F(t)$ shall be strictly increasing before reaching capacity C , and let T^C denote the critical time such that $F(t) < C, \forall t \in [0, T^C)$ and $F(t) = C, \forall t \in [T^C, \infty)$. With slight abuse of notation, let $F^{-1}(c)$ denote the minimum t that satisfies $F(t) = c, \forall c \in [0, C]$. Due to the nonlinearity of the charging curve $F(t)$, this charging process is not quite intuitive and we illustrate it with Figure 1. As shown

in Figure 1(a), for an infinitesimal time interval dt , the battery level at time $t - dt$ equals $F(t) - dF(t)$, which means EVs with battery levels no less than $F(t) - dF(t)$ at time $t - dt$ can be charged to battery levels no less than $F(t)$ at time t . Therefore, at a charging parking lot i , aside from the net arrivals, the increase of $\tilde{I}_i(c, \cdot)$ from time $t - dt$ to time t , denoted by $d\tilde{I}_i(c, t)$, is essentially the number of EVs with battery levels between $c - dF(t)$ and c . Note that function $\tilde{I}_i(\cdot, t)$ is a non-increasing function as illustrated in Figure 1(b), and thus $d\tilde{I}_i(c, t)$ is identical to $-\frac{\partial \tilde{I}_i(c, t)}{\partial c} dF(t) = -\frac{\partial \tilde{I}_i(c, t)}{\partial c} dF(F^{-1}(c))$. Therefore, by further considering the net arrivals, we obtain

$$\begin{aligned} \frac{\partial \tilde{I}_i(c, t)}{\partial t} &= \sum_{j \neq i \in \mathcal{I}} \left[\tilde{d}_{ji}(c + c_{ji}, t - t_{ji}) - \tilde{d}_{ij}(c, t) \right] \\ &\quad - \frac{\partial \tilde{I}_i(c, t)}{\partial c} \frac{dF(F^{-1}(c))}{dt}, \forall i \in \hat{\mathcal{I}}, c \in [0, C], t \in [0, T]. \end{aligned} \quad (7)$$

To unify the previous two formulations of $\frac{\partial \tilde{I}_i(c, t)}{\partial t}$ (Equation (6) and Equation (7)), define $\mathbf{X} := \{X_i\}_{i \in \mathcal{I}}$ to denote whether or not a charging station is installed at each parking lot, i.e.,

$$X_i = \begin{cases} 1, & \text{if } i \in \hat{\mathcal{I}}; \\ 0, & \text{otherwise,} \end{cases} \forall i \in \mathcal{I}. \quad (8)$$

Then we obtain

$$\begin{aligned} \frac{\partial \tilde{I}_i(c, t)}{\partial t} &= \sum_{j \neq i \in \mathcal{I}} \left[\tilde{d}_{ji}(c + c_{ji}, t - t_{ji}) - \tilde{d}_{ij}(c, t) \right] \\ &\quad - \frac{\partial \tilde{I}_i(c, t)}{\partial c} \frac{dF(F^{-1}(c))}{dt} X_i, \forall i \in \mathcal{I}, c \in [0, C], t \in [0, T]. \end{aligned} \quad (9)$$

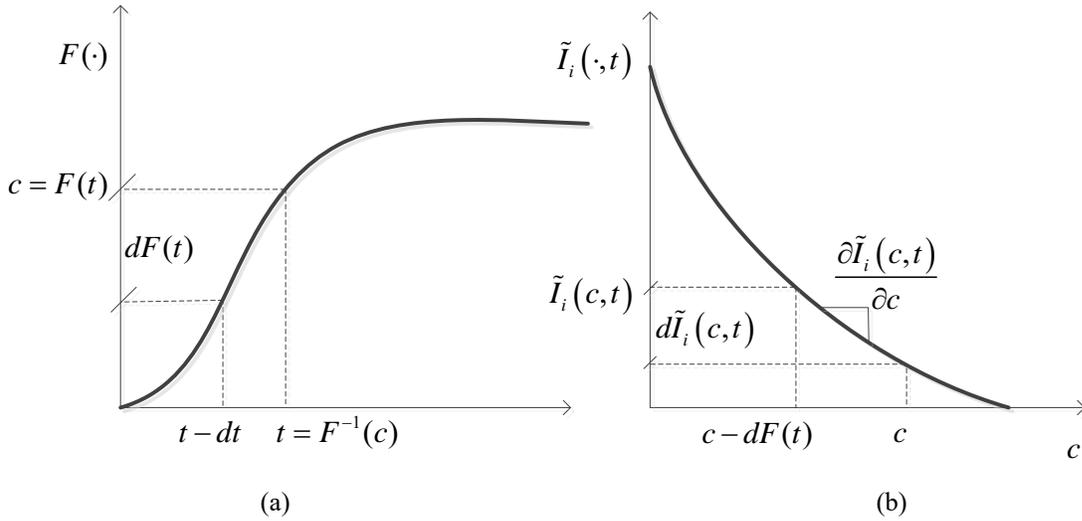


Figure 1. Illustration of how EV charging affects the change of $\tilde{I}_i(c, t)$ over time.

Finally, at the end of the operational cycle, vehicles shall be moved around to balance the fleet size for every parking lot, i.e.,

$$\tilde{I}_i(0, T) + \sum_{j \neq i \in \mathbf{I}} (\tilde{b}_{ji} - \tilde{b}_{ij}) = V_i, \forall i \in \mathbf{I}. \quad (10)$$

Equations (1)-(10) together describe the operational dynamics of a given EV system. Note that since the user arrival rate $\tilde{\lambda}_{ij}(t)$ is a random variable, intermediate variables $\tilde{q}_{ij}(t)$, $\tilde{d}_{ij}(c, t)$, $\tilde{I}_i(c, t)$ and \tilde{b}_{ij} are all random variables. During one operation cycle, we consider user waiting cost and vehicle rebalance cost. To facilitate the formulation, we stack the following variables and parameters, $\mathbf{X} := \{X_i\}_{i \in \mathbf{I}}$, $\mathbf{V} := \{V_i\}_{i \in \mathbf{I}}$, $\tilde{\boldsymbol{\lambda}} = \{\tilde{\lambda}_{ij}(t)\}_{i \neq j \in \mathbf{I}, t \in [0, T]}$ and $\tilde{\mathbf{b}} = \{\tilde{b}_{ij}\}_{i \neq j \in \mathbf{I}}$. Then the lower level problem is to evaluate the total operation cost, which includes selecting $\tilde{\mathbf{b}}$ to minimize the rebalance cost, for given charging locations \mathbf{X} and fleet sizes \mathbf{V} and a given realization of user demand $\tilde{\boldsymbol{\lambda}}$, as formulated below

$$\tilde{C}(\mathbf{X}, \mathbf{V}, \tilde{\boldsymbol{\lambda}}) := \min_{\tilde{\mathbf{b}}} \tilde{C}(\mathbf{X}, \mathbf{V}, \tilde{\boldsymbol{\lambda}}, \tilde{\mathbf{b}}) := \sum_{i \in \mathbf{I}} \sum_{j \neq i \in \mathbf{I}} \left(v \int_0^T \tilde{q}_{ij}(t) d_t + r_{ij} \tilde{b}_{ij}^* \right), \quad (11)$$

subject to (1)-(10), where \tilde{b}_{ij}^* denotes the optimal number of EVs which will be moved from i to j , $\forall i, j \in \mathbf{I}$ at the end of an operation cycle.

Now we are ready to describe the upper level design problem. Basically, the upper level problem determines where to locate charging stations (or \mathbf{X} values) and how many initial vehicles each parking lot needs to hold (or \mathbf{V} values) to minimize the system cost over the planning horizon. The system cost first includes the expectation of the operation cost across all realizations of $\tilde{\boldsymbol{\lambda}}$, formulated as follows,

$$C(\mathbf{X}, \mathbf{V}) := \mathbf{E}_{\tilde{\boldsymbol{\lambda}}} \left\{ \tilde{C}(\mathbf{X}, \mathbf{V}, \tilde{\boldsymbol{\lambda}}) \right\}. \quad (12)$$

In addition, the system cost includes the initial planning cost to open the EV sharing system. Let f_i denote the fixed cost (prorated to an operation cycle) to install a charging station at parking lot $i \in \mathbf{I}$. Let h denote the cost to hold an EV in the system (prorated to an operation cycle) due to purchasing and maintaining this EV. Then the initial planning cost can be formulated as

$$\sum_{i \in \mathbf{I}} (f_i X_i + h V_i). \quad (13)$$

Combining operation cost (Defined in Equation (12)) and planning cost (Defined in Equation (13)), the entire EV system design problem (EVSDP) can be formulated as follows

$$\text{EVSDP} : \min_{\mathbf{X}, \mathbf{V}} S(\mathbf{X}, \mathbf{V}) := \sum_{i \in \mathbf{I}} (f_i X_i + h V_i) + C(\mathbf{X}, \mathbf{V}), \quad (14)$$

$$X_i \in \{0, 1\}, \forall i \in \mathbf{I}, \quad (15)$$

$$V_i \leq \bar{V}_i \in \mathbb{N}^+, \forall i \in \mathbf{I}. \quad (16)$$

Equations (15) and (16) specify the feasible ranges for variables \mathbf{X}, \mathbf{V} .

4. Solution Algorithms

The proposed EVSDP is very challenging to solve. A simple facility location problem where the system operation can be characterized with linear, deterministic and static formulations is already NP-hard (Ref Mark Daskin book). In addition to this inherit complexity from a facility location problem, we see from Section 3 that EVSDP has highly nonlinear, stochastic and dynamic operations. It is very challenging to analytically solve the expectation of the stochastic system operation cost which is shown in Equation (12) even for a given system design of charging stations and fleet sizes. These obstacles motivate us to seek a simulation-based optimization approach to solve this problem.

In this paper, we use simulations to evaluate the expected system operation cost for the lower-level problem after discretizing the operation cycle into a finite number of small intervals. To implement the simulation, the operation cycle has to be discretized. At the end of the simulation, the EV counts at all parking lots are realized and the fleet balancing is just a balanced transportation problem that can be solved by a linear programming solver. Then Genetic Algorithm (GA) is used to solve the upper level facility planning problem.

4.1 Simulation of Lower Level Problem

We first discretize the operation cycle length T into N equal time intervals numbered sequentially from 1 to N and length of each interval is denoted by ΔT . Then, we redefine essential parameters and variables proposed in Section 3 in a discrete manner as follows.

$\bar{\lambda}_{ij}(n)$: random user demand generated from location $i \in \mathbf{I}$ to location $j \neq i \in \mathbf{I}$ during interval $n \in [1, N]$;

$\bar{q}_{ij}(n)$: demand queue length from location $i \in \mathbf{I}$ to location $j \neq i \in \mathbf{I}$ during interval $n \in [1, N]$.

Specifically, $\bar{q}_{ij}(0)$ represents the initial user queue length when each operation cycle starts.

$\bar{d}_{ij}(c, n)$: departing number of EVs with a battery level no less than $c \in [0, C]$ from lot i to lot j , $\forall i \neq j \in \mathbf{I}$, during interval $n \in [1, N]$;

$\bar{I}_i(c, n)$: number of the EVs at location i with a battery level no less than $c \in [0, C]$ at the beginning of interval $n \in [1, N]$. Specifically, $\bar{I}_i(c, N+1)$ represents the number of the EVs at location i with a battery level no less than C at the end of each operation cycle.

$\bar{A}_i(c, n)$: number of the EVs arriving at location i from other locations with a battery level no less than $c \in [0, C]$ at the beginning of interval $n \in [1, N]$. Here, we assume the travel between each location pair will cost integer time intervals.

Now we are ready to deduce the equations describing the EV sharing operation in discrete manner. For each time interval $n \in [1, N]$, the user queue length from location $i \in \mathbf{I}$ to location $j \neq i \in \mathbf{I}$, i.e.

$\bar{q}_{ij}(n) = \int_{(n-1)\Delta T}^{n\Delta T} \tilde{q}_{ij}(t) dt$ could be determined by user queue length from location $i \in \mathbf{I}$ to location $j \neq i \in \mathbf{I}$ formed in the last time interval, i.e. $\bar{q}_{ij}(n-1)$, user demand generated from location $i \in \mathbf{I}$ to location $j \neq i \in \mathbf{I}$,

i.e. $\bar{\lambda}_{ij}(n) = \int_{(n-1)\Delta T}^{n\Delta T} \tilde{\lambda}_{ij}(t) dt$ and departing number of EVs from location $i \in \mathbf{I}$ to location $j \neq i \in \mathbf{I}$, i.e.

$\bar{d}_{ij}(c_{ij}, n) = \int_{(n-1)\Delta T}^{n\Delta T} \tilde{d}_{ij}(c_{ij}, t) dt$. Intuitively, $\bar{q}_{ij}(n)$ could be computed as the follow equation.

$$\bar{q}_{ij}(n) = \bar{q}_{ij}(n-1) + \bar{\lambda}_{ij}(n) - \bar{d}_{ij}(c_{ij}, n), \forall i \neq j \in \mathbf{I}, n \in [1, N]. \quad (17)$$

We assume that at the end of each operation cycle, all the user demand queue are cleared, that is at the beginning of each operation cycle, each user demand queue equals zero, i.e.

$$\bar{q}_{ij}(0) = 0, \forall i \neq j \in \mathbf{I}. \quad (18)$$

As stated in Equation (3), no EV with a battery level less than c_{ij} can serve a trip from i to j , therefore

$$\bar{d}_{ij}(c, n) = \bar{d}_{ij}(c_{ij}, n), \forall i \neq j \in \mathbf{I}, n \in [1, N], c \in [0, c_{ij}]. \quad (19)$$

Before deducing $\bar{d}_{ij}(c, n)$ when $c \in [c_{ij}, C]$, we first describe some EV sharing operation details in discrete manner. We define each location has a unique identifier numbered sequentially from 1 to $|\mathbf{I}|$, e.g. the identifier of

location $j \in \mathbf{I}$ is $|j|$. For each location $i \in \mathbf{I}$, during each interval $n \in [1, N]$, we sequentially decide the departing number of EVs with a battery level no less than $c \in [0, C]$ destined to location $j \neq i \in \mathbf{I}$, i.e. $\bar{d}_{ij}(c, n)$ with ascending identifier, that is the smaller identifier of the destined location is, the user demand destined to it will be met with more priority. So, $\bar{d}_{ij}(c, n)$ could be determined by the number of the EVs at location i with a battery level no less than C at the beginning of interval n , i.e. $\bar{I}_i(c, n) = \int_{(n-1)\Delta T}^{n\Delta T} \tilde{I}_i(c, t) dt$, the number of the EVs arriving at location i with a battery level no less than C at the beginning of interval n , i.e. $\bar{A}_i(c, n)$, the user demand generated from location i to location j during interval n , i.e., $\bar{\lambda}_{ij}(n)$, the average user queue length from location i to location j during interval $n-1$, i.e., $\bar{q}_{ij}(n-1)$, the sum of departing number of EVs during interval n destined to any other location $k \neq j \neq i \in \mathbf{I}$ with identifier smaller than that of location j , i.e. $\sum_{k \in \mathbf{I}_{ij}} \bar{d}_{ik}(c, n)$ in which, \mathbf{I}_{ij} represents the location set containing any location with identifier smaller than that of location j except i and the bottleneck capacity between i and j , i.e., $\Delta T \mu_{ij}$. So, we could get the following equation.

$$\bar{d}_{ij}(c, n) = \min \left\{ \Delta T \mu_{ij}, \left(\bar{I}_i(c, n) + \bar{A}_i(c, n) - \sum_{k \in \mathbf{I}_{ij}} \bar{d}_{ik}(c, n) \right), (\bar{\lambda}_{ij}(n) + \bar{q}_{ij}(n-1)) \right\}, \quad (20)$$

$$\forall i \neq j \in \mathbf{I}, n \in [1, N], c \in [c_{ij}, C]$$

Further, we formulate $\bar{A}_i(c, n)$. Since at the very beginning of an operation cycle, no EVs could arrive at location i from other locations, we have

$$\bar{A}_i(c, 1) = 0, \forall c \in [0, C]. \quad (21)$$

Intuitively, for any interval $n \in [2, N]$, we have

$$\bar{A}_i(c, n) = \begin{cases} \sum_{j \neq i \in \mathbf{I}} \bar{d}_{ji}(c + c_{ij}, n - t_{ij}), c \in [0, C - c_{ij}]; \\ 0, c \in (C - c_{ij}, C]. \end{cases} \quad (22)$$

Finally, we formulate the number of the EVs at location i with a battery level no less than $c \in [0, C]$ at the beginning of interval $n \in [1, N]$, i.e., $\bar{I}_i(c, n)$. Since at the beginning of each operation cycle, each location $i \in \mathbf{I}$ are assigned with a specific number of fully charged EVs, i.e. V_i , we have

$$\bar{I}_i(c, 1) = V_i, \forall c \in [0, C]. \quad (23)$$

For any interval $n \in [2, N]$, $\bar{I}_i(c, n)$ could be decided by the left EVs at the end of the tightly previous interval and charging or not. If no charging station is installed at location $i \in \mathbf{I}$, we have

$$\bar{I}_i(c, n) = \bar{I}_i(c, n-1) + \bar{A}_i(c, n-1) - \sum_{j \neq i \in \mathbf{I}} \bar{d}_{ij}(c, n-1) \quad (24)$$

$$, \forall c \in [0, C], n \in [2, N], X_i = 0.$$

Now, let's consider the situation if a charging station is installed at location $i \in \mathbf{I}$. We assume the charging speed is constant during each interval, but it could be different across different intervals. To be reasonable, we also assume the charging speed is relative to the current battery level. We define the charging speed $f(c), \forall c \in [0, C]$ as the charged power during one interval when current battery level at the beginning of that interval is C . So, if EVs could be charged at location $i \in \mathbf{I}$, we have

$$\begin{aligned} \bar{I}_i(c, n) &= \bar{I}_i(c - f(c), n-1) + \bar{A}_i(c - f(c), n-1) - \sum_{j \neq i \in \mathbf{I}} \bar{d}_{ij}(c - f(c), n-1) \\ &, \forall c \in [0, C], n \in [2, N], X_i = 1. \end{aligned} \quad (25)$$

To facilitate the formulation, we stack the following variables and parameters $\bar{\lambda} = \{\bar{\lambda}_{ij}(n)\}_{i \neq j \in \mathbf{I}, n \in [1, N]}$ and $\bar{\mathbf{b}} = \{\bar{b}_{ij}\}_{i \neq j \in \mathbf{I}}$. So, the lower level problem is to evaluate the total operation cost is rewritten in discrete manner, which includes selecting $\bar{\mathbf{b}}$ to minimize the rebalance cost, for given charging locations \mathbf{X} and fleet sizes \mathbf{V} and a given realization of user demand $\bar{\lambda}$, as formulated below

$$\bar{C}^D(\mathbf{X}, \mathbf{V}, \bar{\lambda}) := \min_{\bar{\mathbf{b}}} \bar{C}^D(\mathbf{X}, \mathbf{V}, \bar{\lambda}, \bar{\mathbf{b}}) := \sum_{i \in \mathbf{I}} \sum_{j \neq i \in \mathbf{I}} \left(v \sum_{n=1}^N \bar{q}_{ij}(n) \Delta T + r_{ij} \bar{b}_{ij}^* \right), \quad (26)$$

subject to (17)-(25).

In order to minimize rebalance cost, we formulate rebalance problem as a balanced transportation problem. Before formulating the problem, we define some notations as follows,

$\hat{\Gamma}$: surplus set, in which any location $i \in \mathbf{I}$ has a surplus of EVs at the end of a specific operation cycle, i.e.,

$$\left(\bar{I}_i(0, N+1) + \sum_{j \neq i \in \mathbf{I}} \sum_{n=N-t_{ji}}^N \bar{d}_{ji}(c_{ji}, n) \right) - V_i > 0, \forall i \in \hat{\Gamma}, \quad (27)$$

where t_{ji} represents the travel time from location $i \in \mathbf{I}$ to location $j \in \mathbf{I}$, which is measured in integer intervals.

$\check{\Gamma}$: deficit set, in which any location $i \in \mathbf{I}$ has a deficit of EVs at the end of a specific operation cycle, i.e.,

$$\left(\bar{I}_i(0, N+1) + \sum_{j \neq i \in \mathbf{I}} \sum_{n=N-t_{ji}}^N \bar{d}_{ji}(c_{ji}, n) \right) - V_i < 0, \forall i \in \check{\Gamma}; \quad (28)$$

S_i : surplus number of EVs at any location $i \in \hat{\Gamma}$, i.e.,

$$S_i = \left(\bar{I}_i(0, N+1) + \sum_{j \neq i \in \mathbf{I}} \sum_{n=N-t_{ji}}^N \bar{d}_{ji}(c_{ji}, n) \right) - V_i, \forall i \in \hat{\Gamma}; \quad (29)$$

D_i : deficit number of EVs at any location $i \in \check{\Gamma}$, i.e.,

$$D_i = V_i - \left(\bar{I}_i(0, N+1) + \sum_{j \neq i \in \mathbf{I}} \sum_{n=N-t_{ji}}^N \bar{d}_{ji}(c_{ji}, n) \right), \forall i \in \check{\Gamma}. \quad (30)$$

Consequently, we formulate the EV rebalance problem as follows,

$$\min z = \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{I}} r_{ij} \bar{b}_{ij} \quad (31)$$

s.t.

$$\sum_{j \in \tilde{I}} r_{ij} \bar{b}_{ij} = S_i, i \in \hat{I}, \quad (32)$$

$$\sum_{i \in \hat{I}} r_{ij} \bar{b}_{ij} = D_j, j \in \tilde{I}, \quad (33)$$

$$\bar{b}_{ij} \geq 0, i \in \hat{I}, j \in \tilde{I}. \quad (34)$$

By solving this linear integer programming problem, we could get the optimal rebalance strategy at the end of any operation cycle, i.e. $\bar{b}_{ij}^*, i \in \hat{I}, j \in \tilde{I}$.

Consequently, the expectation of the operation cost in discrete manner across all realizations of $\tilde{\lambda}$ is formulated as follows,

$$C^D(\mathbf{X}, \mathbf{V}) := \mathbf{E}_{\tilde{\lambda}} \left\{ \bar{C}^D(\mathbf{X}, \mathbf{V}, \tilde{\lambda}) \right\}. \quad (35)$$

Finally, the entire discrete EV system design problem (D_EVSDP) can be formulated as follows,

$$\text{D_EVSDP} : \min_{\mathbf{X}, \mathbf{V}} S^D(\mathbf{X}, \mathbf{V}) := \sum_{i \in \hat{I}} (f_i X_i + h V_i) + C^D(\mathbf{X}, \mathbf{V}), \quad (36)$$

s.t. (15)-(16).

For a specific infrastructure design, that is locations \mathbf{X} and fleet sizes \mathbf{V} are given, we could use limited number of simulations by randomizing the generation of user demand $\tilde{\lambda}$, to approximate the true value of $C^D(\mathbf{X}, \mathbf{V})$.

4.2 GA for Upper Level Problem

In this section, we use GA to solve the upper level problem, that is to decide the optimal values of charging stations deployment ($\mathbf{X} := \{X_i\}_{i \in \hat{I}}$) and initial EV fleet sizes ($\mathbf{V} := \{V_i\}_{i \in \hat{I}}$). In our proposed GA, a chromosome is coded with binary to represent a feasible solution for both $\mathbf{X} := \{X_i\}_{i \in \hat{I}}$ and $\mathbf{V} := \{V_i\}_{i \in \hat{I}}$. One chromosome is composed of two parts, the first part represents a feasible solution to $\mathbf{X} := \{X_i\}_{i \in \hat{I}}$. Since $\mathbf{X} := \{X_i\}_{i \in \hat{I}}$ are binary, the representation of this part is very intuitive without any specific transformation and processing. The second part represents a feasible solution to $\mathbf{V} := \{V_i\}_{i \in \hat{I}}$. Since $V_i \leq \bar{V}_i \in \mathbb{N}^+, \forall i \in \hat{I}$, we use the following equation to decide how many genes (m_i) should be used to represent a feasible solution for each V_i :

$$2^{m_i-1} \leq \bar{V}_i \leq 2^{m_i} - 1, \forall i \in \hat{I}. \quad (37)$$

When decoding, for each gene fragment *substring*_{*i*} which represents the value of V_i , we have:

$$V_i = \text{round} \left(\frac{\text{Decimal}(\text{substring}_i)}{2^{m_i} - 1} \times \bar{V}_i \right), \forall i \in \hat{I} \quad (38)$$

where, $\text{Decimal}(\text{substring}_i)$ is the decimal transformation of the corresponding binary *substring*_{*i*}. $\text{round}(\cdot)$ is a function to transform the corresponding variable into the nearest integer. Roulette wheel selection containing the elite retaining model (Goldberg, 1989) is used to select best chromosomes from previous population. We use the partially matched crossover (PMX) proposed by Goldberg and Lingle (1985) to generate new chromosomes. Two crossover points are chosen from the chromosomes randomly and equably. Then the genes of the chromosomes that are in the area between the crossover points are exchanged and two new chromosomes are created. After the crossover operation, each chromosome in new population has a specific probability to be mutated. In each generation, we assume there are always M chromosomes. To determine the survival probability of a chromosome at the next generation, a fitness function is used to evaluate each chromosome, i.e. $\forall (\mathbf{X}_i, \mathbf{V}_i), i \in [1, M]$, which is defined as follows:

$$Fitness((\mathbf{X}_i, \mathbf{V}_i)) = \max_{m \in [1, M]} \{C^D(\mathbf{X}_m, \mathbf{V}_m) - C^D(\mathbf{X}_i, \mathbf{V}_i)\} \quad (39)$$

5. Numerical Examples

In this section, we aim to illustrate the proposed model and algorithm by conducting a numerical test in Yantai City, China, which is one of the earliest pilot cities that start car sharing services under the authorization and support by governments. The first car sharing project in Yantai city was launched on September 21, 2014, which totally set 30 sharing stations and input 100 new cars. We first integrate stations which are very close in space into a bigger one and finally form an electrical car sharing system with totally 14 locations for the shared vehicles parking and as candidates for building charging stations (See Figure 2).



Figure 2 The layout of car-sharing locations in Yantai City, China.

In order to derive average hourly car sharing OD data (i.e. $\int_0^1 \lambda_{ij}(t) dt, \forall i \neq j \in I$) from the test area, we first assume the serving radius of each candidate location is 3km. The population density of the area (denoted by $\kappa_i, \forall i \in I$) where each candidate station location is obtained according to the Yantai statistical yearbook 2014. Consequently, we can obtain the potential total population served at each location. The average daily trip number of a user (denoted by Avg_trips) is assumed to be 2, which is compatible with the cases in most cities in China (Zou, et al., 2008). So the daily trip generated in each location i 's service region could be derived by:

$$T_trips_i = \varphi_i \times \kappa_i \times Avg_trips, \forall i \in I, \quad (39)$$

where T_trips_i is total number of daily trips generated by the service region of location i ; φ_i is the area of service region of location i . The trip attraction of each location is estimated mainly based on the land use, employment situation etc. Trip distributions between each pair of locations are estimated by Gravity Model (reference). Figure 3 and Figure 3 **Daily Trip productions and attractions** Figure 4 present the daily trip generation/attraction and trip distributions between each pair of locations. Hourly trip percentage adopted here is presented in Figure 5, which is set according to Zhou, et al. (2007). Up to now, we can get exact hourly OD trips which served by car sharing system through multiplying number of hourly trips and a specific share ratio (here, it is set as 1%).



Figure 3 Daily Trip productions and attractions



Figure 4 Daily trip distribution

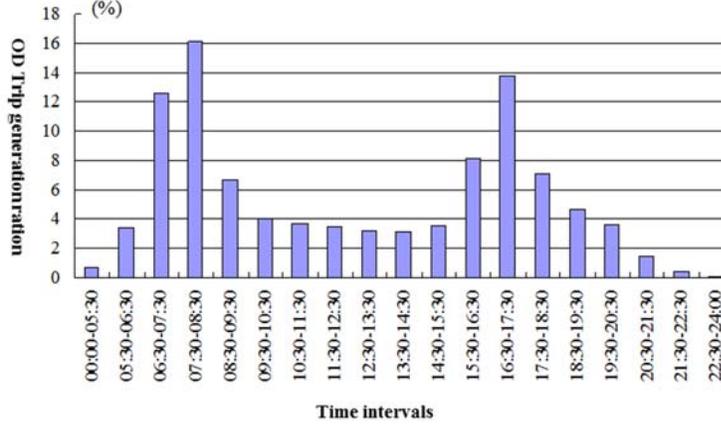


Figure 5 Hourly trip percentage

The operation cycle is set to be 24 hours which is further discretized into 96 time intervals evenly (i.e. $N = 96$ and $\Delta T = 15 \text{ min}$). The daily prorated fixed cost of each candidate station is assumed to be 3000RMB, which includes both the construction cost and daily operation cost. The value of time is set to be 22.68RMB/hour computed based on the ratio of average income of 3992RMB/person/month and 176 working hours/month in Yantai City. Distances and travel times between each OD pair are derived according to the BAIDU map (<http://map.baidu.com>). Default values of key parameters are summarized in Table 1.

Table 1 Default Values of Key Parameters

Parameter	Meaning	Default value
$f_i = f, \forall i \in I$	unit fixed cost of station	3000RMB/day
v	unit value of time	22.68RMB/hour
h	unit vehicle holding cost	200RMB/day
C	maximum mileage of an electric vehicle	150km
$\dot{F}(t)$	battery charging speed	25km/hour
$r_{ij} = r, \forall i \neq j \in I$	unit balancing cost	60RMB/veh
$\mu_{ij} = \mu, \forall i \neq j \in I$	flow capacity	1800veh/hour

5.1 Simulation Convergence Test

Theoretically, in order to get expectation cost $C^D(\mathbf{X}, \mathbf{V})$ in Equation (35) for given charging station layout \mathbf{X} and EV fleet sizes \mathbf{V} , we should conduct infinite times of simulations. Obviously, this could not be implemented in real world. So, we want to use finite times of simulations to approximate the true expectation. So, the following problem is whether the average of simulation results shows a convergent trend when increasing the number of simulations, and consequently if so, how many simulations could be relatively good for evaluating the expectation. In order to test the convergence of this simulation, we conduct many times of simulation under a specific combination of station building layout and initial vehicle fleet size. We set step size equal to 1 and 5 separately when simulation number varies from 1 to 30 and from 31 to 130. Four groups of simulation results are showed in Figure 6. From them, we can see all the simulations present an obvious convergent trend with the increasing of numbers of simulations. All the figures indicate simulations with more than 40 iterations could get a more stable result with the maximum relative errors varying from 0.327% (see Test 3) to 1.072% (see Test 4). In this research, we use the average of 40 simulations to approximate the expectation.

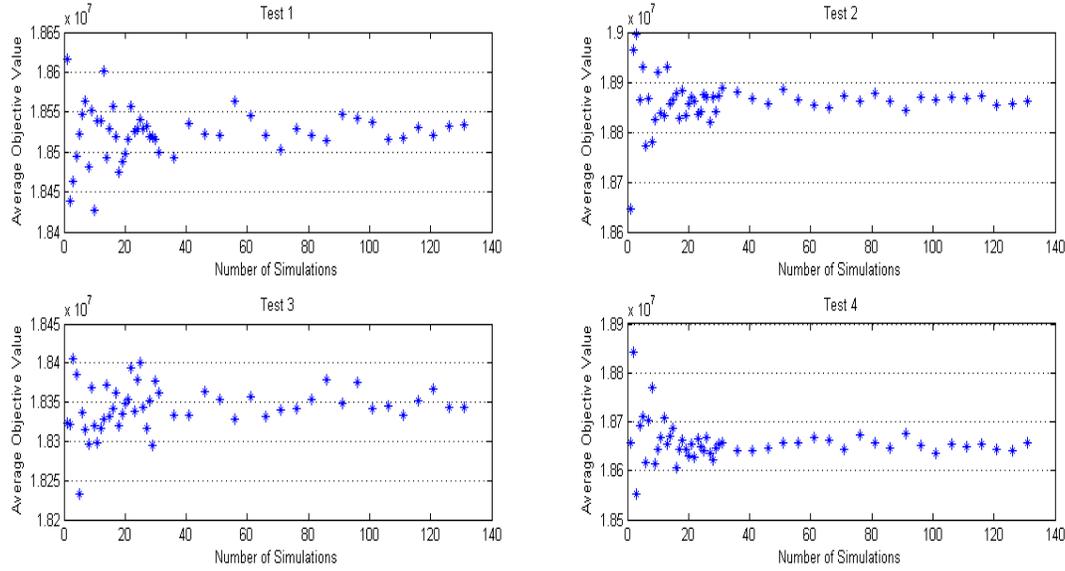


Figure 6 Four groups of convergence test of simulation

5.2 Sensitivity Analysis

In this part, we investigate how the best feasible objective cost TC and its components consisting of station related fixed cost C_f , vehicle holding cost C_h , user waiting cost C_w and vehicle rebalancing cost C_b , optimal number of total built charging stations N^* and optimal number of total deployed vehicles H^* change over some key parameters including unit fixed cost of station f , unit vehicle holding cost h , unit value of time V , unit balancing cost r , maximum mileage C and battery charging speed $\dot{F}(t)$. In order to evaluate the changing extent of a specific item $\hat{h}(\hat{\lambda})$ along with the variation of a certain parameter $\hat{\lambda}$ varying from $\hat{\lambda}_{lb}$ to $\hat{\lambda}_{ub}$, we define sensitive factor θ as follows:

$$\theta = \frac{\frac{\hat{h}(\hat{\lambda}_{ub}) - \hat{h}(\hat{\lambda}_{lb})}{\hat{h}(\hat{\lambda}_{lb})}}{\frac{(\hat{\lambda}_{ub} - \hat{\lambda}_{lb})}{\hat{\lambda}_{lb}}} \quad (40)$$

where $\hat{h}(\hat{\lambda}_{ub})$ and $\hat{h}(\hat{\lambda}_{lb})$ are the corresponding values of the investigated cost item when $\hat{\lambda}$ equals $\hat{\lambda}_{ub}$ and $\hat{\lambda}_{lb}$.

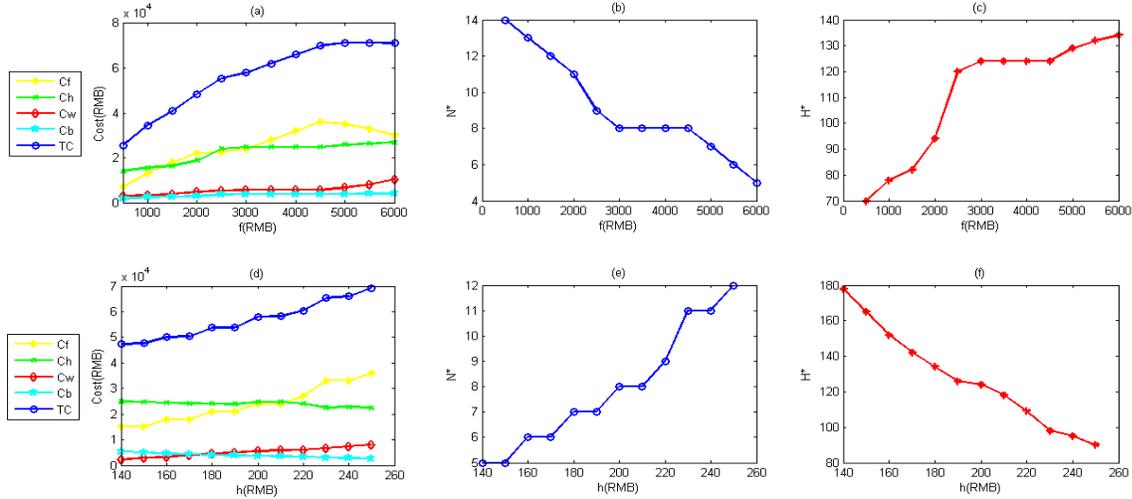


Figure 7 Sensitivity of cost components to parameters f (sub-figures (a)-(c)) and h (sub-figures (d)-(f))

Figure 7 (a)-(c) examine the sensitivity of relative cost components to unit fixed cost f . From Figure 7(a), we can see that with f increasing, the total cost TC increases with a sensitive factor of 0.231 (computed by Equation(40)) and the corresponding slope becomes flatter. This indicates that the increasing of f does not much increase the system total cost. This could be explained that when f increases, system tries to deploy more vehicles (see Figure 7 (c)) and sacrifice more user's time to decrease the number of built stations (see Figure 7(b)). We can also see that in Figure 7 (a), total fixed cost C_f experiences three different stages: first increases slower, then increases a little faster, finally decreases slowly. Since f increases, if the number of built stations does not change, the total fixed cost will increase linearly. When f increases to some extent, the system will tend to increase deployed vehicles and/or sacrifice user's benefit by increasing their waiting time to decrease the number of built stations, consequently absorbing the adverse effects resulted by f increasing. It is found that, the variation of C_f and C_h behaves certain complementarity that is the two cost components vary in different trends during the same changing interval of f . While C_w does not behave much relations with f . This indicates that, the adverse effects by increasing of f could be more effectively absorbed by changing the number of total deployed vehicles rather than increasing user's waiting time. Totally, the vehicle rebalancing cost C_b only slightly increases with a sensitive factor of 0.0082, which shows obvious that C_b is not affected by increasing f .

The sensitivity of various cost items, N^* and H^* to unit vehicle holding cost h is depicted in Figure 7(d)-(f). System total cost TC slightly increases only with a sensitive factor of 0.132, while C_f , C_h , C_w , C_b separately have a sensitive factor of 0.2, -0.123, 0.004 and -0.01. As seen in Figure 7 (a), variations of C_f and C_h also present an opposite trend, which shows at current car sharing demand, system could find a relatively good equilibrium (with slightly increasing system total cost) between vehicle number and station number when both f and h vary. As h increases, number of total deployed vehicles continuously decreases (see Figure 7 (f)) and number of built stations stabilizes and increases alternately in a very rhythmical pattern (See Figure 7(e)). User waiting cost C_w increases steadily but slightly, which illustrates increasing user's waiting time is not a primary strategy to counteract system cost increasing resulted by increasing h . C_b behaves a steady decreasing since less vehicles are used as long as h increases.

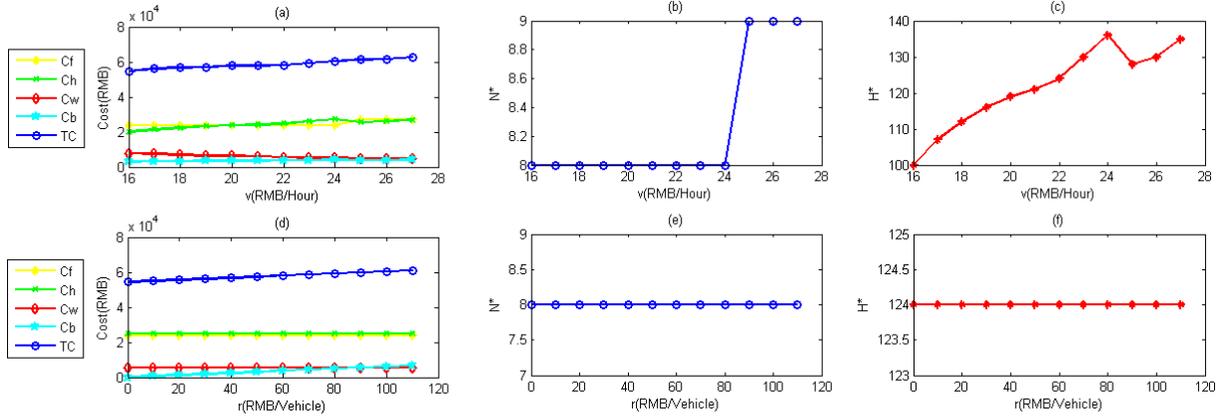


Figure 8 Sensitivity of cost components to parameters V (sub-figures (a)-(c)) and r (sub-figures (d)-(f))

Figure 8 (a)-(c) shows the sensitivity of value of time V . From Figure 8 (a), system total cost TC increases slightly along with the increasing of V with a sensitive factor of 0.2104. Overall, N^* and H^* shows an increasing trend along with the increasing of V (See Figure 8 (b) and (c)), which is consistent with what the corresponding cost items C_f and C_h (See, Figure 8(a)) behave. Due to the increasing of H^* , vehicle rebalancing cost C_b shows a slighter increasing trend. User's waiting time cost C_w totally decreases with a sensitive factor of -0.5822 due to the transportation capacity improvement resulted by increasing of N^* and H^* .

Figure 8 (d)-(f) shows how varying unit vehicle rebalancing cost r affects the system performance. Figure 8 (e) and (f) show a consistent stability across different r , which indicates that almost no influence is generated to optimal built station number and optimal total deployed vehicle number when r increases from 0 to 110 RMB. This phenomenon is also proved by Figure 8 (d) in which TC and C_b increases linearly, while C_f, C_h and C_w keep unchanged.

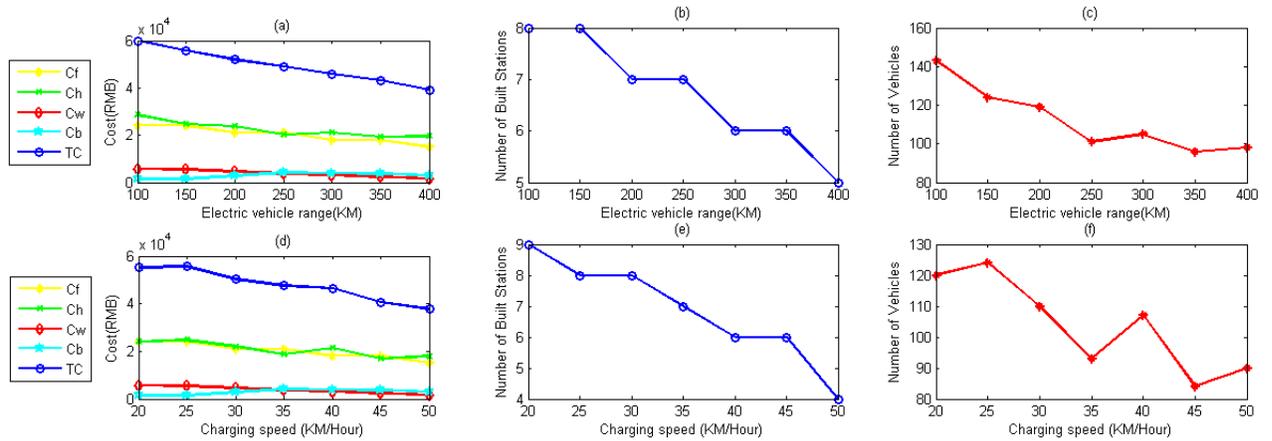


Figure 9 Sensitivity of cost components to parameters C (sub-figures (a)-(c)) and $F(t)$ (sub-figures (d)-(f))

Figure 9 (a)-(f) illustrates the changing of various cost items with the variations of maximum mileage C and charging speed $F(t)$ of electric vehicles. From Figure 9(a) and Figure 9(d), we can see both the performance improvement of maximum mileage and charging speed decrease the system total cost TC significantly, separately with a sensitive factor of -2.13 and -1.78. It is intuitive that increasing maximum mileage and charging speed both could decrease N^* (see Figure 9(b) and (e)) and H^* (see Figure 9(c) and (f)). Different with the steady decreasing behaved by N^* , H^* presents an overall decreasing trend but with intermittent rebound along with both the variations

of C and $\dot{F}(t)$. This phenomenon implies that decreasing N^* could have a better profit than decreasing H^* , when EV technology improves, system preferentially tries to decrease N^* although it will incur the relatively slighter increasing of H^* .

5.3 Optimal System Design

After investigating the sensitivity of key parameters, we aim to show how optimal system design changes according to the variations of key parameters. When investigating each key parameter, we only depict two optimal system deployments which are implemented under the boundary values of each parameter's range. All the results are presented in Figure 10 where constructed charging station locations are marked by green car icons while the user locations without charging stations are marked with grey ones. Each car icon is associated with two numbers: the black number outside of the parentheses is the location id and the red one in the parentheses shows the optimal initial EV fleet size deployed at that location.

Figure 10. (a) and (b) depict the changes of optimal system design under the unit fixed cost $f = 500$ and $f = 6000$. It is found that when unit fixed cost increases from 500 to 6000, the optimal charging station number has a great decreasing (from 14 to 5) to prevent the sudden increasing of total fixed cost. At the same time, the number of total held vehicles also has a great increasing (from 70 to 134). It is intuitive when f increases, some less necessary charging stations will not be constructed and the relative more necessary ones are reserved since they may bring more improvement of system performance by reducing total cost. Further comparing Figure 10(a) and Figure 10 (b), we can see the five locations with constructed charging stations are the locations with the highest trip attractions (see Figure 3). Also, we can find in Figure 10 (b), the numbers of initial deployed vehicles at constructed locations are generally lower than that of unconstructed locations. This could be illustrated by if one location is selected to construct charging station, vehicles visiting it could be effectively replenished, consequently reducing demand of initial vehicles. This phenomenon almost exists in all the sub figures of Figure 10.

Figure 10 (c) and (d) depict optimal designs under $h = 140$ and $h = 250$. When unit vehicle holding cost increases, system tries to reduce number of deployed vehicles and increase the number of constructed stations to slow down the increasing of total system cost.

Figure 10 (e) and (f) present optimal designs under $v = 16$ and $v = 27$. When increasing the unit value time, numbers of constructed stations and total deployed vehicles are both increased.

From Figure 10 (g) and (h), where system optimal designs under $r = 0$ and $r = 100$ are presented separately, it is found that variation of unit vehicle rebalancing cost brings no effect to optimal charging station layout and optimal number of initial deployed vehicles. This implies that under current layout of candidate locations and car sharing demand rate, total vehicle rebalancing cost only accounts for a small part of total cost no matter how unit rebalancing cost changes. Further, we can see from Figure 3 Daily Trip productions and attractions (Figure 4, car sharing trips follow a relatively uniform distribution among various OD pairs (not very much difference among line weights which represent the trip number). So, we can infer that the input and output of vehicles at each location achieves relative equilibrium, which leads to the number of vehicles need to be rebalanced is very small, consequently explain why the changing of b almost has no influence to optimal system design.

Figure 10 (i) and (j), (k) and (l) illustrate changes of the system optimal design to varying EV range C and charging speed $\dot{F}(t)$ of electric vehicles. Increasing of C and $\dot{F}(t)$ means the technology improvement of electric vehicles, which definitely will lower the fixed cost related to station construction and vehicle holding cost under specific user waiting time level.



(a) $f=500, N^*=14, H^*=70$



(b) $f=6000, N^*=5, H^*=134$



(c) $h=140, N^*=5, H^*=178$



(d) $h=250, N^*=12, H^*=90$



(e) $v=16, N^*=8, H^*=100$



(f) $v=27, N^*=9, H^*=135$



(g) $b=0, N^*=8, H^*=124$



(h) $b=100, N^*=8, H^*=124$



(i) $C=100, N^*=8, H^*=143$



(j) $C=400, N^*=5, H^*=98$



(k) $\dot{F}(t)=20, N^*=9, H^*=120$



(l) $\dot{F}(t)=50, N^*=4, H^*=90$

Figure 10 Optimal system design under the variations of different key parameters

6. Conclusion

This paper studies a one-way EV sharing system infrastructure design problem. First, as a major contribution to the literature, we formulate a set of high-fidelity simulation rules to describe the dynamic and stochastic operation process of an EV sharing system, then by integrating these simulation rules, a simulation-based optimization model for a one-way EV sharing system infrastructure design problem consisting of charging station siting and deciding fleet size for each location has been proposed. Since there is no closed-form formulations to describe the proposed optimization model, a customized heuristic algorithm integrating both discrete simulation of EV sharing operation

dynamics and GA has been developed to efficiently solve the EV system design problem. Numerical experiments are conducted with data of Yantai city, China to illustrate the efficiency of the proposed simulation rules, the optimization model and the customized solving algorithm. We found that given a specific set of installed stations and fleet sizes, average objective value of only 40 simulations could approximate the true expectation of system dynamic operation costs very well. Several groups of experiments are conducted to illustrate how the key parameters influence the best feasible cost items, total numbers of built charging stations and deployed vehicles, also the optimal EV sharing system design.

For future study, this research could be extended in several directions by relaxing a set of assumptions we made. First, in numerical experiments, we assume the EV's charging function is linear which may not be true in practice. It will be more practical and interesting to investigate optimal EV sharing system design problem under different types of charging functions in future study. Second, we assume whenever a not fully charged EV appears at an installed charging station will be charged immediately, which may not be consistent with the practice. In future studies, EV queues waiting for charging should be integrated the proposed simulation rule set. Also, since EV sharing is an emerging transportation service, in future study, integrating it into a comprehensive transportation system including multiple transportation modes (e.g., private car, transit, taxi, etc.) could be more beneficial for planning charging stations and serving individual trip

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