MURI presentation

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I am working on:

- Fréchet processes and associated heavy tailed sup measures;
- tail inference for upper and lower tails;
- multivariate subexponential heavy tails.
A new project in cooperation with the UMass team:

- Generating mechanisms for multivariate heavy tails.
Fréchet processes and sup measures

A common heavy tailed model arising in the scheme of taking partial maxima is $\alpha$-Fréche distribution for which

$$F(x) = F_{\alpha, \sigma}(x) = \exp\{-\sigma^{\alpha} x^{-\alpha}\}, \quad x > 0.$$ 

A stochastic process $(Y(t), t \geq 0)$ is called a Fréchet process if:

- choosing any $n \geq 1$, $a_1, \ldots, a_n > 0$ and $t_1 \geq 0, \ldots, t_n \geq 0$,
- the weighted maximum $\max_{1 \leq j \leq n} a_j Y(t_j)$ has a Fréchet law.
Fréchet processes arise when taking pointwise partial maxima of heavy tailed processes;

$Y(t)$ can be viewed as the largest observations in $(0, t)$;

it is of interest to measure the largest observation in sets of a different shape;

sup measures are the appropriate model.
A set function $m$ is a sup measure if $m(\emptyset) = 0$ and

$$m \left( \bigcup_{r \in R} G_r \right) = \sup_{r \in R} m(G_r)$$

for an arbitrary collection $(G_r, r \in R)$ of open sets.
In the space of sup measures one can

- introduce a topology,
- discuss weak convergence of random sup measures.

We have an application to the maxima of long memory heavy tailed processes.
The tails are regularly varying with exponent $\alpha > 0$.

The memory is governed by a parameter $0 < \beta < 1$.

The process that measures the largest values in intervals of the type $(0, t)$ is

$$Z_{\alpha, \beta}(t) = Z_\alpha(t^\beta), \; t \geq 0,$$

where $(Z_\alpha(t), \; t \geq 0)$ is the extremal $\alpha$-Fréchet process.
Theorem (Lacaux and Samorodnitsky, 2014)

The corresponding sup measure can be described in the form

\[ W_{\alpha, \beta}(A) = \bigvee_{i=1}^{\infty} U_i 1 \left( (R^{(i)}_{\beta} + V_i) \cap A \neq \emptyset \right), \ A \subseteq [0, \infty). \]

- \( (R^{(i)}_{\beta}) \) are i.i.d. copies of the closed ranges of 1 − \( \beta \)-stable subordinators.
- \( ((U_i, V_i)) \) form an independent Poisson random measure on \((0, \infty)^2\) with the mean measure

\[ \alpha x^{-(\alpha+1)} \, dx \, \beta y^{\beta-1} \, dy, \ x, y > 0. \]
Estimation of upper and lower tails

One of the major issues in the tail estimation is estimating the shape parameter in an extreme value distribution.

How many order statistics does one use in the estimation?

When the shape parameter is positive, a major progress was achieved in Nguyen and Samorodnitsly (2013) under this MURI support.

I am working on developing efficient procedures when the shape parameter is negative.

This is equivalent to estimation of lower tails.
Multivariate subexponential heavy tails

- There is a well developed theory of multivariate regularly varying tails.
- There is also a well developed theory of one-dimensional subexponential tails.
- There is a definition of multivariate subexponential tails (Cline and Resnick, 1992)
- Not much progress since then.
- Recent progress is achieved with a PhD student, Julian Chen.