Homework# 4: Short course on sparse recovery and compressed sensing

October 2013

Reading:

Lec. 9: A very clean argument of this nature was given in Baraniuk, Davenport, DeVore, and Wakin, "A simple proof of the restricted isometry property," in J. Fourier Analysis and Applications

Lec. 10: See Recht's "A simpler approach to matrix completion," http://arxiv.org/abs/0910.0651, and Recht, Fazel, Parrilo's *Guaranteed minimum rank solutions of linear matrix equations via nuclear norm minimization*, http://bit.ly/X48ELP. For the second part of the lecture, see Ahmed, Recht, and Romberg, *Blind deconvolution using convex programming*, http://arxiv.org/abs/1211.5608

Problems:

1. Write a one paragraph summary of what we talked about today. Use complete sentences, and avoid equations as much as possible.

2. The *Hoeffding inequality* is a classic (and often used) tail bound for the sum of a set of independent random variables. Here is a statement:

   Proposition 1 (Hoeffding). *Let $X_1, X_2, \ldots, X_N$ be independent (but not necessarily identically distributed) with $\mathrm{E}[X_i] = 0$ and $|X_i| \leq a_i$ with probability one for some fixed sequence of real numbers $a_1, a_2, \ldots, a_N$. Then for any $\lambda > 0$,*

   $$\mathrm{P}\left(\sum_{i=1}^{N} X_i > \lambda\right) \leq 2\exp\left(-\frac{\lambda^2}{2\sum_{i=1}^{N} a_i^2}\right).$$

   Use the Hoeffding inequality to prove that binary random matrices where

   $$\Phi(m,n) = \begin{cases} \frac{1}{\sqrt{M}} & \text{with probability } 1/2 \\ -\frac{1}{\sqrt{M}} & \text{with probability } 1/2 \end{cases}$$

   obey the restricted isometry property (with positive probability) when

   $$M \gtrsim \mathrm{Const} \cdot S \log(N/S).$$

   It is enough to describe how the proof for the Gaussian case given in Lecture 9 needs to be modified.

3. Let A be a matrix with 50 columns and M rows whose entries are iid Gaussian random variables with zero mean and variance $1/M$. Estimate the expected maximum singular value and expected minimum singular values of A for $M = 50, 75, 100, 200, 300, 1000$. Do this simply by doing a large number of random trials in MATLAB. Do the same for a binary $\pm 1/\sqrt{M}$ valued random matrix A.