ECE 8823 (Convex Optimization), Spring 2017

Non-smooth Optimization Quiz

Friday April 21

1. Consider the following non-smooth function on the real line:

$$f(x) = \max\left((x+1)^2, (x-3)^2\right).$$

Describe the subdifferential $\partial f(x)$ at every point $x \in \mathbb{R}$.

2. Prove or disprove: the subdifferential $\partial f(\boldsymbol{x})$ of a convex function is a convex set at every $\boldsymbol{x} \in \mathbb{R}^N$.

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3. Recall the subdifferential for the nuclear norm for an $N_1 \times N_2$ matrix $\boldsymbol{X}$.

$$\partial \|\boldsymbol{X}\|_* = \left\{ \boldsymbol{U}\boldsymbol{V}^{\mathrm{T}} + \boldsymbol{W} \ : \ \boldsymbol{U}^{\mathrm{T}}\boldsymbol{W} = \boldsymbol{0}, \ \boldsymbol{W}\boldsymbol{V} = \boldsymbol{0}, \ \|\boldsymbol{W}\| \leq 1 \right\}.$$

In the expression above, $\boldsymbol{X}$ has rank R and its SVD is $\boldsymbol{X} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathrm{T}}$, where $\boldsymbol{U}$ is $N_1 \times R$, $\boldsymbol{\Sigma}$ is $R \times R$ and $\boldsymbol{V}$ is $N_2 \times R$. Recall that

$$\begin{aligned} \boldsymbol{X}^+ &= \operatorname{prox}_{t\|\cdot\|_*}(\boldsymbol{X}) \\ &= \arg\min_{\boldsymbol{Z}} \left(\|\boldsymbol{Z}\|_* + \frac{1}{2t} \|\boldsymbol{Z} - \boldsymbol{X}\|_F^2 \right), \end{aligned}$$

if and only if

$$\boldsymbol{X} - \boldsymbol{X}^+ \in t\partial \|\boldsymbol{X}^+\|_*.$$

Show that we can compute the prox operator above by singular value thresholding:

$$\boldsymbol{X}^+ = \boldsymbol{U}\boldsymbol{\Sigma}^+\boldsymbol{V}^{\mathrm{T}}, \quad \text{where} \quad \Sigma^+[i,i] = \max(\sigma_i - t, 0).$$

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