Fast Evaluation of Probabilistic Total Transfer Capability Considering Multiple Wind Farms

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***Abstract*— Before the rapid growth of wind power, most bulk power system uncertainties came from system contingencies and load fluctuation. However, when large numbers of wind farms are introduced, the stochastic nature of the wind farm output under system normal state calls for a new probabilistic framework for system operation and evaluation, which captures the uncertainty of wind and corresponding transfer constraints of the system. Total transfer capability (TTC) is a measurement of the system's maximum power transfer capacity from a set of source buses to a set of sink buses. In this paper, we propose a fast probabilistic TTC evaluation method that incorporates the less predictable nature of wind farms. The proposed method returns the probabilistic TTC as a random variable that follows a certain distribution derived from wind farm output distributions using power transfer distribution factors. Our proposed method solves the probabilistic TTC in an analytic way. Results from the IEEE 118-bus test case demonstrate the efficiency and accuracy of the proposed method.**

Keywords—total transfer capability, wind power; probabilistic model;

I. INTRODUCTION

The modern electric grid is experiencing a rapid transformation as an effort to meet new sustainability requirements while a large portion of the infrastructure is ageing at the same time. Part of this transformation is driven by the growing number of wind farms all over the world. Wind is an attractive and effective renewable energy resource, but its production is difficult to predict with high levels of accuracy. As a result, both the asset management and control strategies of the future grid must be able to accommodate the uncertainty introduced by the integration of large-scale wind power generation.

As deregulation continues, the number of entities such as power suppliers, power distributors and power aggregators involved in electricity markets will continue to grow. All of these entities must interface with each other through the grid and all transactions have the potential to impact all entities. Therefore, there is a rapidly growing demand for better tools that will allow for the calculation of power transactions from one node to another node or from a group of nodes to another group of nodes to access the state of the grid. Available transfer capability (ATC) is used to ensure the feasibility of such power transactions. The North American Electric Reliability Council (NERC) defines ATC as a measure of the remaining physical electricity network capability for further power transfers over already committed transactions. [1] Mathematically, the ATC equals to the total transfer capability (TTC) minus existing power transfer commitments and margins. This paper focuses on developing a fast and accurate probabilistic TTC evaluation method which considers the wind farm output uncertainty.

Since the introduction of the TTC concept, researchers and engineers have done significant study on how to evaluate TTC. Based on the system constraints enforced during the TTC computation, most TTC evaluation methods can be categorized into DC power flow methods [2-3] and AC power flow methods [4-6]. DC power flow methods are based on sensitivity analysis which is then used to evaluate TTC considering line thermal limits. Compared with DC power flow methods, AC power flow methods are slower but consider system voltage limits. Based on the TTC evaluation result, most TTC computation methods can also be categorized into deterministic TTC [2-6] and probabilistic TTC [7-11]. Compared with deterministic TTC evaluation, which returns a single TTC value, probabilistic TTC returns the TTC value as a random variable which follows a probability distribution.

Before large numbers of wind farms were integrated into the electric grid, most uncertainty in the power system came from system contingencies such as transmission line failures and generator failures. Using contingency screening techniques, researchers developed N-1 or robust TTC evaluation algorithms which simulate all critical system failures to cover the uncertainties brought by system contingencies. Since the deterministic TTC only considers the worst case scenario, researchers found that a deterministic TTC is too conservative and cannot evaluate the risk of carrying a specific power transaction [12]. This is especially true when large numbers of wind farms are present [8]. Given the difficulty of accurately predicting wind farm output, it is hard to determine what the worst case scenario is and the possibility for the worst case scenario to happen through deterministic TTC evaluation. Instead of returning a single TTC value, probabilistic TTC treats the TTC value as a random variable that follows a specific distribution. Probabilistic TTC can naturally integrate the uncertainty produced by wind farms into the probability distribution of the TTC value. As a result, probabilistic TTC can quantify the risk of allowing a certain power transaction when wind farms are present.

In this paper, we show that our proposed probabilistic TTC evaluation method is faster and more accurate than existing probabilistic TTC evaluation methods. Current probabilistic TTC evaluation methods derive their result from prolonged Monte-Carlo simulations and bootstrap sampling algorithms. The simulation process follows a two-step structure: select a system state and compute the TTC for the elected system state [7]. Monte-Carlo methods is a simulation based method that is widely used in power system economic and reliability analysis [13]. The system state space of the Monte-Carlo based method increases exponentially with the system complexity. Therefore, the Monte-Carlo based TTC evaluation process requires extensive length of simulation in order to capture enough system states. Instead of running time consuming simulations, we derive the results analytically by linking the wind farm output distributions with the TTC probability distribution of a power transfer through power transfer distribution factors (PTDFs). As a result, the probability distribution of TTC is solved analytically with better efficiency and accuracy.

In Section II, we introduce a novel fast probabilistic TTC evaluation method based on sensitivity analysis. In Section III, we first introduce a data-driven modeling method to generate forecasted wind farm output distributions. Then, we pass the wind farm output uncertainties to the system TTC using PTDFs. In Section IV, we compare the proposed fast probabilistic TTC evaluation method with non-sequential Monte-Carlo simulation method through the IEEE 118-bus test case. We demonstrate that the proposed method is more accurate and more efficient. Section V concludes the paper and discusses possible future research directions.

II. FAST PROBABILISTIC ATC EVALUATION

In this section, we derive the fast probabilistic TTC evaluation method from power system sensitivity analysis under linear power flow assumption, applicable to either DC power flow or linearized methods.

A. *Deterministic ATC Evaluation Based on PTDFs*

Sensitivity analysis method based on PTDFs is one of most widely used methods to calculate deterministic TTC. Under the DC approximation, the active power flow change on each transmission line l is linearly related to the scale of the power transaction $\boldsymbol{T}$, which is defined as a vector in (1).

$$\boldsymbol{T} = \boldsymbol{T}^{\text{source}} + \boldsymbol{T}^{\text{sink}}, \tag{1}$$

where $\sum \boldsymbol{T}^{\text{source}} = 1$, and $\sum \boldsymbol{T}^{\text{source}} = -1$.

PTDF reflects the influences of a specific power transaction on transmission lines, as shown in equation (2).

$$PTDF_{l,\boldsymbol{T}} = \frac{\partial p_l}{\partial p_{\boldsymbol{T}}}, \tag{2}$$

where $PTDF_{l,\boldsymbol{T}}$ is the power transfer distribution factor of transmission line l for a given power transaction $\boldsymbol{T}$; p_l stands for the real power flow on line l; and $p_{\boldsymbol{T}}$ is the scale of the power transfer $\boldsymbol{T}$.

The TTC of line l, denoted as $TTC_{l,\boldsymbol{T}}$, can be computed through (3)

$$TTC_{l,\boldsymbol{T}} = \begin{cases} \frac{\overline{p_l} - p_l}{PTDF_{l,\boldsymbol{T}}} & PTDF_{l,\boldsymbol{T}} > 0 \\ \frac{-\overline{p_l} - p_l}{PTDF_{l,\boldsymbol{T}}} & PTDF_{l,\boldsymbol{T}} < 0 \end{cases}, \tag{3}$$

where $\overline{p_l}$ stands for the thermal limit of line l. The deterministic TTC for the whole system is settled by the smallest line TTC value.

B. *Power System Uncertainties*

TTC is not only a market tool in electricity economics but also a very important reliability index that considering various system uncertainties. These uncertainties may result from line and generator failures, load forecast errors, or output fluctuation of large scale renewables. The logic behind the probabilistic TTC is to capture these power system uncertainties by reflecting them through a probability density distribution (pdf). In this paper, we model the system uncertainties as various pdf s, where the probability distribution of the TTC is derived from.

C. *Fast Probabilistic TTC Evaluation Considering Wind Farm Uncertainties*

In this paper, we assume the fluctuations of wind farm output is always balanced by the system slack bus. Without loss of generality, this may correspond to a participation factor generation dispatch modeling distributed slack generation. Under the DC assumption, the proposed fast probabilistic TTC evaluation method can be decomposed into five steps:

Step 1: Compute the base case PTDF for each transmission line l under a given power transaction $\boldsymbol{T}$ that is of interest. In the base case scenario, we remove all wind farms from the system and compute PTDFs as shown in equation (2).

Step 2: Compute the wind power PTDFs of a specific power transaction $\boldsymbol{T}_{wind}$ where the source bus is the wind farm location and the sink bus is the system slack bus. We denote the wind power PTDF for line l as $PTDF_{l,\boldsymbol{T}_{wind}}$.

Step 3: Compute the probabilistic power flow correcting term $\widehat{p_l}$ for each transmission line based on the wind farm output $\hat{p}_{wind}$ and the wind power PTDF $PTDF_{l,\boldsymbol{T}_{wind}}$, as shown in (4).

$$\hat{p}_l = PTDF_{l,\boldsymbol{T}_{wind}} \times \hat{p}_{wind}, \tag{4}$$

where $\hat{p}_{wind}$ is a random variable of wind farm output that follows a pdf denoted as $p_{\hat{p}_{wind}}(x)$, $\hat{p}_l$ is a random variable of power flow correcting term that follows a pdf denoted as $p_{\hat{p}_l}$.

However, the probabilistic correcting term $\hat{p}_l$ can be difficult to solve if multiple wind farms are presented. In Section III, we illustrate how to derive $\hat{p}_{wind}$ and $\hat{p}_l$ in those scenarios.

Step 4: Correct the deterministic line TTC by adding the probabilistic correcting term $\hat{p}_l$ computed in step 3 using (5).

$$\widehat{\text{TTC}}_{l,\boldsymbol{T}} = \begin{cases} \frac{\overline{p_l} - p_l - \hat{p}_l}{PTDF_{l,\boldsymbol{T}}} & PTDF_{l,\boldsymbol{T}} > 0 \\ \frac{-\overline{p_l} - p_l - \hat{p}_l}{PTDF_{l,\boldsymbol{T}}} & PTDF_{l,\boldsymbol{T}} < 0 \end{cases}, \tag{5}$$

where $\widehat{\text{TTC}}_{l,\boldsymbol{T}}$ is a random variable of line l TTC that follows a pdf denoted as $p_{\widehat{\text{TTC}}_{l,\boldsymbol{T}}}$.

Step 5: Determine the system TTC for transaction $\boldsymbol{T}$ using the minimum line TTC throughout the system.

III. PROBABILISTIC MODEL OF WIND FARM OUTPUT

In this section, we first develop a data-driven model for the random variable of a single wind farm output $\hat{p}_{wind}$. Then we discuss how to combine the influences of multiple wind farms and compute the pdf of the probabilistic correcting term $\hat{p}_l$.

A. *A Data-Driven Model for A Single Wind Farm Output*

Wind farm output modeling or wind farm output forecast is a conventional task that has been heavily studied by many engineers and mathematicians in the last few years. However, due to the limited accuracy of the wind forecast, the predictability of any particular wind farm is still very low for short term operation [14]. In order to dispatch the wind power, system operators require a constantly updated forecast of the wind farm output throughout time as shown in Fig. 1. Sometimes, apart from providing a constant value, the forecast provides a distribution or confidence interval for the wind farm output in the future.

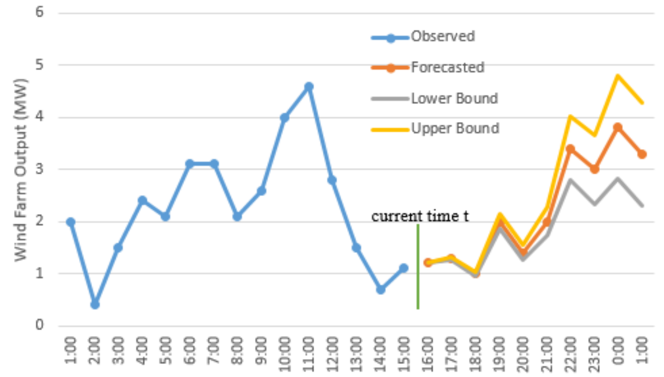

Fig. 1. Hourly wind farm output forecast.

In many cases, the pdf of the future wind farm output is not available. Therefore, we develop a data-driven method to generate a pseudo forecasted distribution of the wind farm output condition on forecasted value using historical data. In order to illustrate the modeling process, we take sample data from NREL renewable data center [15], which contains the hourly data of forecasted output and observed output of a 20 MW inland wind farm from year 2004 to 2007.

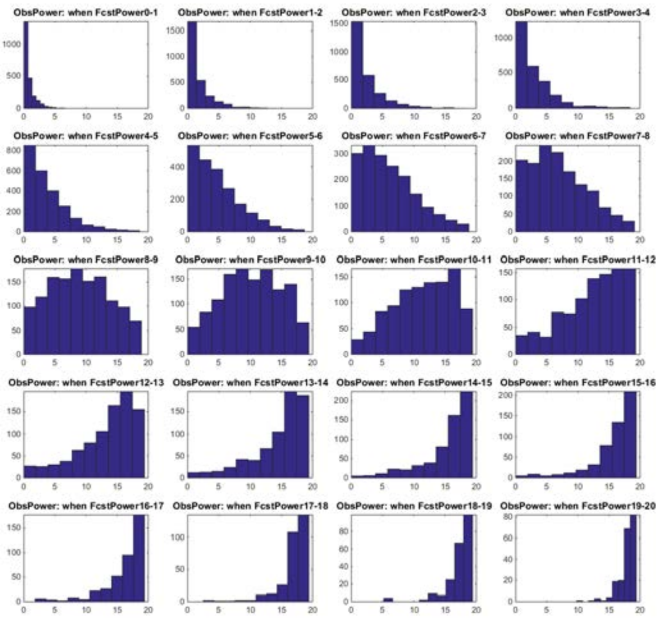

Fig. 2. Histogram of observed output condition on forecast output (MW).

We first group the data according to the forecasted output. In each group, the histogram of the observed output condition on the forecasted output is drawn as in Fig. 2. The histograms shown in Fig. 2 can be used to generate $f(p_{real}|p_{pred})$, which is the empirical probability density function of the actual wind farm output p_{real} condition on the forecasted value p_{pred}. In other words, for every wind farm output forecast $p_{pred} = p_t$ at time t, we can solve $\hat{p}_{wind}$ with equation (6).

$$\hat{p}_{wind} = f(p_{real}|p_{pred} = p_t) \tag{6}$$

B. Calculation of the Probabilistic Correcting Term $\hat{p}_l$

The key of the proposed method is the calculation of the probabilistic correcting term $\hat{p}_l$. In this section, we further explain the $\hat{p}_l$ calculation on different scenarios with a simple test system as shown in Fig. 3. The test system consists of 5 buses, 7 lines and two generators [16]. Bus 5 is the system slack bus.

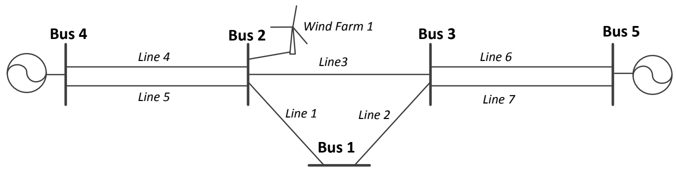

Fig. 3. 5-bus test system with one wind farm

1) Scenario 1: One Wind Farm Case

In scenario 1, we locate a 200MW wind farm on bus 2, as shown in Fig. 3. The wind farm output for the future time t is 100MW. We are interested in the power transfer capability from bus 5 to bus 2.

According to equation (4), the calculation of $\hat{p}_l$ involves $PTDF_{l,\boldsymbol{T}_{wind}}$ and $\hat{p}_{wind}$. We first compute $PTDF_{l,\boldsymbol{T}_{wind}}$, the PTDF for transaction $\boldsymbol{T}_{wind}$ from wind farm location bus 2 to the system slack bus 5. Next, we generate the distribution of the forecasted output at time t according to our empirical distribution using equation (6). In Scenario 1, we can interpret the probabilistic correcting term $\hat{p}_l$ as the wind farm output $\hat{p}_{wind}$ scaled by $PTDF_{l,\boldsymbol{T}_{wind}}$.

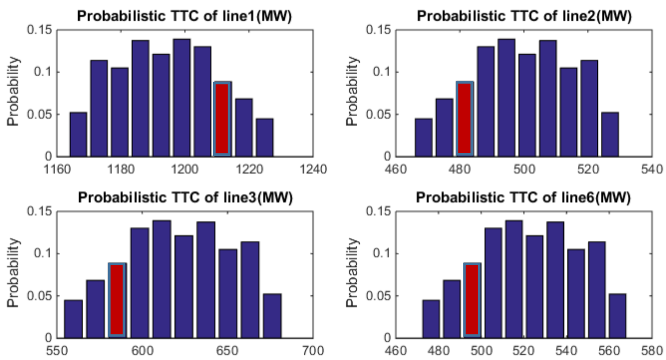

Fig. 4. pdfs of the probabilistic line TTC.

For simplicity, let's assume that line 4 and 5 have infinite capacity and line 6 and line 7 are identical. As a result, the system TTC is determined by line 1, 2, 3 and 6. The base case line TTC values for line 1, 2, 3, and 6 are 1228MW, 465MW, 553MW and 471MW respectively. Fig. 4 shows the probabilistic line TTC for these lines using the proposed method. The system TTC is derived by choosing the smallest line TTC value for each wind farm output scenario, as shown in Fig. 5. For example, when the wind farm output is between 40 MW and 60 MW (with probability 0.09), the line TTCs for line 1, 2, 3, 6 are 1180MW, 481MW, 586MW and 496MW as shown by the red bars in Fig. 4. As a result, the system TTC of this scenarios is 481MW (the minimum of the four values) with probability 0.09, shown as the red bar in Fig. 5.

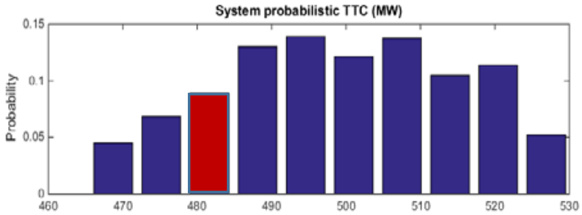

Fig. 5. pdf of the system probabilistic TTC.

2) Scenario 2: Multiple Wind Farms Case

In scenario 2, we add an additional 100MW wind farm on bus 1 as shown in Fig. 6. The wind farm output forecast for wind farm 1 and 2 are 100MW and 80 MW respectively. We are still interested in the power transfer from bus 5 to bus 2. For simplicity, it is assumed that we are only interested in the line TTC for line 6. The rest line TTCs can be derived similarly.

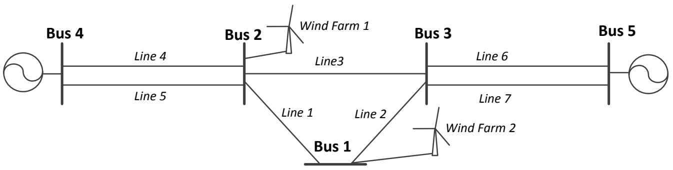

Fig. 6. 5-bus test system with multiple wind farms.

When multiple wind farms are present, we first compute the probabilistic correcting term for each wind farm independently as in scenario 1. Let $\hat{p}_{l,i}$ denote the probabilistic correcting term for wind farm i. Then, the total correcting term for line l can be computed by the sum of all wind farm correcting terms using equations (7-8). Fig. 7 shows the pdfs of random variable $\hat{p}_{wind,i}$ for wind farm 1 and 2 and the pdfs of random variable $\hat{p}_{l,i}$ for line 6.

$$\hat{p}_l = \sum_i \hat{p}_{l,i}, \tag{7}$$

$$\hat{p}_{l,i} = PTDF_{l,\boldsymbol{T}_{wind,i}} \times \hat{p}_{wind,i}, \tag{8}$$

where $PTDF_{l,\boldsymbol{T}_{wind,i}}$ is the wind farm transaction PTDF for wind farm i and $\hat{p}_{wind,i}$ is the random variable for wind farm i output.

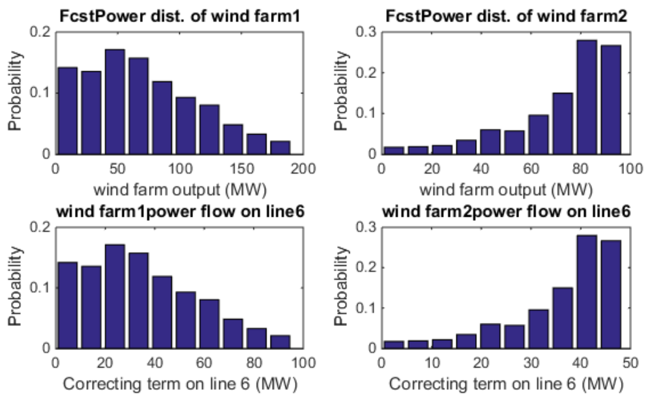

Fig. 7. pdfs of wind farm output and correcting term for line 6.

From equation (7-8), solving $\hat{p}_l$ is equivalent of solving the sum of two random variables $\hat{p}_{l,1}$ and $\hat{p}_{l,2}$. According to the dependency of different wind farm outputs, we present three ways to solve $\hat{p}_l$: numerating method, convolution method and fast Fourier method. Convolution method and fast Fourier method is much faster than the numerating method but they assume the outputs of different wind farms are independent. This may apply when the transfer of interest is large and the locations of different wind farms are relatively far away. The numerating method, on the other hand, can be applied when outputs of wind farms are strongly correlated.

a) Numerating Method

Numerating method solves the sum of two random variables by the definition as shown in equation (9), where every combination of the two random variables is numerated.

$$p_{\hat{p}_l}(\hat{p}_l) = p_{\hat{p}_{l,1},\hat{p}_{l,2}}(\hat{p}_{l,1} + \hat{p}_{l,2}), \tag{9}$$

where $p_{\hat{p}_{l,1},\hat{p}_{l,2}}$ is the joint distribution of two wind farm outputs.

b) Convolution Method

One of the most important properties of convolution is that the convolution of the distributions of two or more independent random variables equals to the distribution of the sum of these random variables [17]. As a result, when the number of wind farms is limited, compared with the numerating method, convolution is the most efficient way to solve $\hat{p}_l$.

c) Fast Fourier Transform Method

Fourier transform and its inverse are developed to transform signal between time domain and frequency domain. A fast Fourier transform (FFT) is an algorithm to compute the discrete Fourier transform (DFT) and its inverse [18]. According to the property of Fourier transform, the convolution in one domain equals to the point-wise multiplication in the other domain. As a result, we can avoid convolution operation in the convolution method by performing FFT and inverse FFT. Compared with the convolution method, the computational speed improvement of using FFT is significant when large numbers of wind farms are presented.

In the 5-bus case, if we assume the outputs of the two wind farms are independent, all three methods will generate the same $\hat{p}_l$ as shown in Fig. 8. The line TTC of line 6 can be computed using equation (5).

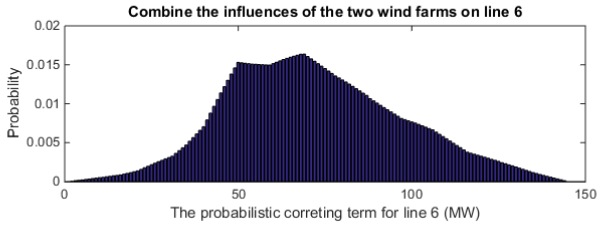

Fig. 8. pdf of $\hat{p}_l$ on line 6 using FFT.

IV. Case Study

In this section, we compare our proposed method with the non-sequential Monte-Carlo simulation method using the IEEE 118-bus test case. The IEEE 118-bus test case consists of 118 buses, 186 lines, 91 loads and 54 thermal units. Bus 69 is chosen as the system slack bus. We are interested in the power transfer from bus 100 to bus 2. In order to see the influences of multiple wind farms, two wind farms are introduced with the same capacity (both 200MW). For a specific time of interest t, the output forecast of the two wind farms are 180 MW and 50 MW respectively. The wind farm data comes from NREL renewable data center [15].

In deterministic ATC evaluation, the limiting element of the power transfer is line 60. As a result, it is critical to see how wind farms influence the power flow on line 60. Fig. 9 shows the distribution of the probabilistic correcting term $\hat{p}_l$ on line 60.

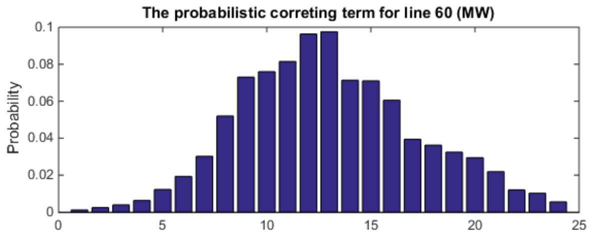

Fig. 9. pdf of $\hat{p}_l$ on line 60 using the proposed method.

A. Accuracy Comparison

Compared with the existing probabilistic TTC evaluation methods, the proposed method is more efficient and more accurate. Fig. 10 shows the probabilistic ATC result using non-sequential Monte-Carlo simulation. From Fig. 10, we can see that the accuracy of the simulation based method increases as the length of the simulation grows. However, even when the simulation length is as large as 10000, the accuracy at the tails is still not as good as the analytical solution.

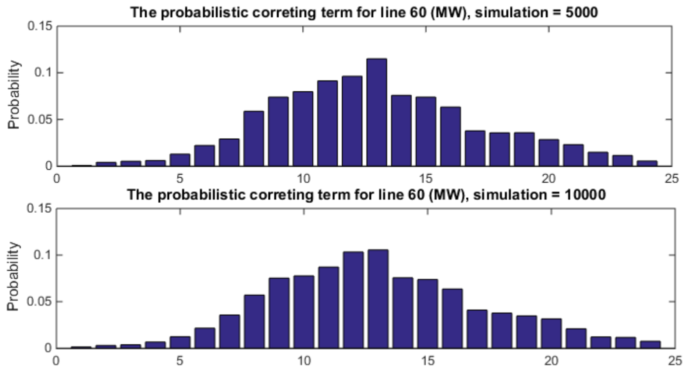

Fig. 10. pdf of $\hat{p}_l$ on line 60 using the simulation based method.

B. Computational Speed Comparison

TABLE I compares the computation time between the proposed method and the simulation based method. From TABLE I, the proposed probabilistic ATC evaluation method is very computational efficient.

TABLE I. COMPUTATIONAL TIME COMPARISON

	Proposed method	**Non-sequential Monte-Carlo Simulation**	
		Simulation = 5000	***Simulation = 10000***
Time (s)	0.0066	32.2407	63.0414

V. CONCLUSION

In this paper, we propose a fast probabilistic TTC evaluation method that considers multiple wind farms. The proposed algorithm takes the probabilistic models of different wind farm output as inputs and returns a probabilistic TTC as a random variable that follows a specific distribution. The proposed method is an analytic solver for probabilistic TTC with better computational efficiency and accuracy, compared with the common probabilistic TTC evaluation method based on prolonged simulation analysis. Future research opportunities may include how to incorporate other system uncertainties such as load fluctuations and system contingencies into the fast probabilistic TTC evaluation method.

REFERENCES

[1] *Available Transfer Capability Definition and Determination*. Princeton, NJ: North American Electric Reliability Council (NERC), June 1996.

[2] S. Grijalva, P. W. Sauer, and J. D. Weber, "Enhancement of linear ATC calculations by the incorporation of reactive power flows," *IEEE Trans. on Power Syst.*, vol. 18, no. 2, pp. 619-624, 2003.

[3] X. Zhang and S. Grijalva, "Multi-area TTC evaluation based on Kron reduction," *in Proc. 2013 IEEE Smart Energy Grid Engineering*, vol. 1, no. 6, pp. 28-30.

[4] E. D. Tuglie, M. Dicorato, M. L. Scala, and P. Scarpellini, "A static optimization approach to access dynamic available transfer capability," *IEEE Trans. on Power Syst.*, vol. 15, no. 3, pp. 1069-1076, 2000.

[5] B. Gao, G. Morison, and P. Kundur, "Towards the development of a systematic approach for voltage stability assessment of large-scale power systems," *IEEE Trans. on Power Syst.*, vol. 11, no. 3, pp. 1314-1324, 1996.

[6] C. A. Canizares, A. Berizzi, and P. Marannino, "Using FACTS controllers to maximize available transfer capability," *in Proc. 1998 Bulk Power Systems Dynamics and Control IV-Restructuring*, pp. 633-641.

[7] G. Luo, J. Chen, D. Cai, D. Shi, and X. Duan, "Probabilistic assessment of available transfer capability considering spatial correlation in wind power integrated system," *IET Generation, Transmission & Distribution*, vol. 7, no. 12, pp. 1527-1535, 2014

[8] F. Xin, F. Li, and N. Gao, " Probabilistic available transfer capability evaluation for power systems including high penetration of wind power," *in Conf. 2014 Methods Applied to Power Systems (PMAPS)*.

[9] A. B. Rodrigues, and M. G. Da Silva, "Probabilistic Assessment of Available Transfer Capability Based on Monte Carlo Method With Sequential Simulation," *IEEE Trans. on Power Syst.*, vol. 22, no. 1, pp. 484-492, 2007.

[10] M. Ramezani, and M. R. Haghifam, "Modeling and Evaluation of Wind Turbines on Total Transfer Capability," *in Proc. 2007 IEEE Power and Energy Society General Meeting*.

[11] X. Li, P. Wang, and Z. Guo, "Determination of optimal total transfer capability using a probabilistic approach." *IEEE Trans. on Power Syst.*, vol. 21, no. 2, pp. 862-868, 2006.

[12] A. M. Leite da Silva, J. Guilherme de Carvalho Costa, L. Antonio da Fonseca Manso, and G. J. Anders, "Transmission capacity: Availability, maximum transfer and reliability, " *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 843–849, Aug. 2002.

[13] X. Zhang, Z. Bie, and G. Li, "Reliability Assessment of Distribution Networks with Distributed Generations using Monte Carlo Method," *Elsevier, Energy Procedia*, vol. 12, pp. 278-286, Dec. 2011.

[14] *Wind Systems Integration Basics*. National Renewable Energy Laboratory (NREL), June 2012.

[15] The Renewable Resource Data Center (RReDC), National Renewable Energy Laboratory (NREL), available at: http://www.nrel.gov/rredc/, July 2015.

[16] X. Wang, Y. Song, and M. Irving, "Modern power system analysis," Beijing: Science Press, 2012, pp. 246-248.

[17] C. S. S. Burrus, and W. P. Thomas, "DFT/FFT and Convolution Algorithms: theory and Implementation," John Wiley & Sons, Inc., 1991.

[18] P. Duhamel, and M. Vetterli, "Fast Fourier transforms: a tutorial review and a state of the art," *Signal Processing*, vol. 19, no. 4, pp. 259-299, 1990.