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State Space Modeling of Sheet Metal Assembly for Dimensional Control

In this paper, a state space modeling approach is developed for the dimensional control of sheet metal assembly processes. In this study, a 3-2-1 scheme is assumed for the sheet metal assembly. Several key concepts, such as tooling locating error, part accumulative error, and re-orientation error, are defined. The inherent relationships among these error components are developed. Those relationships finally lead to a state space model which describes the variation propagation throughout the assembly process. An observation equation is also developed to represent the relationship between the observation vector (the in-line OCMM measurement information) and the state vector (the part accumulative error). Potential usage of the developed model is discussed in the paper.

1 Introduction

Dimensional control is one of the most important challenges in automotive body assembly. Due to the complexity of the autobody assembly process, it normally requires dozens of fixtures to assemble, on average, 150–250 parts. The complexity of the assembly line places high demands on the tooling design, manufacture, and diagnosis for improving autobody quality.

In recent years, fixture modeling and design have been thoroughly studied and significant results have been achieved. Fixture designs are analyzed in terms of their ability to arrest translation and rotation, while minimizing deflection and distortion of the part during processing (Chou et al., 1989). Kinematical and mechanical methods such as screw theory (Asada and By, 1985) and force equilibrium equations (Salisbury and Roth, 1983) are most often used for functional configuration of the fixture. Menassa and De Vries (1989) presented a synthesis of this approach. Hockenberger and De Meter (1995) and De Meter (1995) conducted an experimental analysis of the causes of workpiece deflection under machining forces and optimized the layout of clamps and locators using a mini-max locating force criteria with kinematical and total restraint constraints. Rong and Zhu (1992) developed a search and retrieve (over a set of existing fixture designs) technique with kinematical requirement considerations. In the aspect of studying the locator error effect in a machining process, Rong et al. (1995) and Rong and Bai (1996) presented an effective analysis approach to study the locator error effect on the workpiece geometry accuracy by modeling of the three locating reference planes using the geometric plan constraints. Based on these models, the workpiece location and orientation can be determined and the obtained workpiece geometry error can be further used to evaluate the performance of the fixture tolerance design and layout during the tooling design stage. Although such advances in fixture design can greatly improve fixture accuracy and repeatability, fixture faults are still the major root causes of autobody dimensional variations (ABC, 1993). However, few literatures exist investigating assembly process monitoring and diagnosis based on the dimensional variations of parts.

The implementation of in-line Optical Coordinate Measurement Machines (OCMM) in the automotive industry has provided new opportunities for assembly fault diagnosis. OCMM gages are installed at the end of major assembly processes, such as framing, side frames, underbody, etc. The OCMM measures 100 to 150 points on each major assembly with a 100 percent sample rate. These inspected points are located on many of the individual parts of the autobody. As a result, the OCMM provides tremendous amounts of dimensional information, which can be used for assembly process control. However, effective utilization of this measurement information, especially for assembly fault diagnosis, is still a challenge.

Recent research exploring fault isolation issues in autobody assembly has focused on a statistical description of variation patterns (Hu and Wu, 1992) and the detection of failing assembly stations (Ceglarek et al., 1994). Hu and Wu (1992) investigated the description of dimensional faults by in-line measurement data using Principal Component Analysis (PCA). Ceglarek et al. (1994) described a systematic method of identifying failing stations and faulty parts in the assembly line. Additionally, they described a rule-based approach to identify root causes of dimensional faults in the fixture. Their rule-based approach is based on heuristic knowledge, which specifies a fixed level of detail about the position and control directions of the fixture locators. More recently, the diagnosis of a single fixture has been studied using PCA (Ceglarek and Shi, 1996) and Least Square estimation with hypothesis testing (Apley and Shi, 1998).

The aforementioned research activities have significantly advanced process monitoring and diagnosis for dimensional control of body assembly processes. However, the lack of models describing the overall assembly process has imposed a large constraint on developing advanced monitoring and diagnosis techniques for body assembly processes.

This paper attempts to resolve the above mentioned challenges by developing a state space modeling approach which defines: (1) the part accumulative dimensional error as a state vector, (2) the fixture tooling locator error as a control vector, (3) the geometric relationship and variation stack-up of assembled part as a dynamic matrix, (4) The geometric relationship between the tooling locators and part orientation as a control matrix, (5) the assembly station number which serves as the time index in the state space model, and, (6) the modeling error due to the designed tolerance and rigid part assumption as the system noise.

In this paper, assumptions and definitions will be given first in sections 2.2 and 2.3 to describe the part dimensional variations and fixture failures. Several theorems will be developed in subsections 2.3.1 and 2.3.2 to describe the inherent relationship between the tooling locator error and part orientation error in the defined body coordinates. Based on those results, a state space model is developed in sections 3.1 and 3.2 to represent the relationship between the tooling error and part dimensions for the overall assembly processes which involve all assembly stations. Some potential

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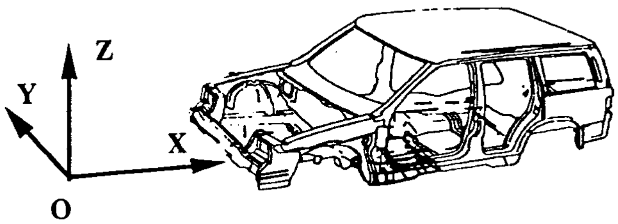

Fig. 1 Body coordinate system

usage of the developed state space model is discussed in section 4. Finally, conclusion and future work are given in section 5.

2 Description and Hypothesis of Automotive Body Assembly Process

2.1 Automotive Body Assembly Process. The automotive body without doors, hood, fenders, and trunk lid is called the "Body in White" (BIW). In a BIW assembly line, depending on the complexity of the product, there are typically 80 to 130 assembly stations which assemble 150 to 250 sheet metal parts. An assembly station normally consists of two or more assembly fixtures. Each fixture holds a single part to be assembled with other parts. In this paper, it is assumed that one assembly station contains only two fixtures. Based on their functions, the components of a BIW are usually divided into structural and non-structural parts. Structural parts, such as rails, plenum, and door hinge reinforcements are much more rigid than non-structural parts, such as the door outer panel, cowl-side, roof, etc. Past research indicates that a structural part usually has a much larger impact on the automotive body dimensional accuracy (ABC, 1993; Takezawa, 1980). Thus, only structural parts will be considered in the later modeling procedure. In the paper, a 3-2-1 fixture and rigid part assumption are made in the derivation. Those assumptions cover 68 percent of the total parts in a typical autobody (Shiu et al., 1996).

In order to conduct the state space modeling of the assembly process, a body coordinate system is shown in Fig. 1. The origin of the body coordinate system is defined in the front center of the vehicle and below the underbody. The X-Y-Z axes are shown in the figure. This definition of the body coordinate system has been widely used in auto industry in product and process design.

2.2 Fixture Layout and Fixture Error. In sheet metal assembly, locating pins and NC blocks are widely used in fixtures to determine the part location and orientation in an assembly process. For a rigid part, a 3-2-1 principle is the most common layout method. As shown in Fig. 2(*a*), a typical 3-2-1 fixture contains several key tooling locators: (1) a four-way pin P_1 to precisely locate the hole in the X and Z directions; (2) a two-way pin P_2 to locate a slot in the Z direction; these two pins constrain the part rotation and translation in the X-Z plane together; and (3) three NC blocks to locate the part in the Y direction. In this paper, a general modeling procedure is presented which focuses on the X-Z plane as shown in Fig. 2(*b*). Fixture errors (also called tooling locator errors or tooling faults) result from many different factors, such as a worn locator, missing block, broken pin, etc. In this paper, the following definitions are given to describe a fixture error.

Definition 1 *Fixture error vector:* For station i, the tooling locating error in the X-Z plane for a 3-2-1 fixture as shown in Fig. 2(*b*) is represented by:

$$\Delta\mathbf{P}(i) = (\Delta x_{P_1}(i),\quad \Delta z_{P_1}(i),\quad \Delta z_{P_2}(i))^T \tag{1}$$

where $\Delta\mathbf{P}(i)$ is the fixture error vector for locator points P_1 and P_2 of station i; $\Delta x_{P_1}(i)$ and $\Delta z_{P_1}(i)$ represent the locating errors for the 4-way pin P_1 in the X and Z directions respectively; $\Delta z_{P_2}(i)$ represents the locating error for the 2-way pin P_2 in the Z direction; and the superscript T means matrix transpose.

It should be explained that the tooling locating error is referred to as the deviation of tooling location greater than the design tolerance. The error due to the tolerance itself will be modeled as the noise, which will be discussed in detail in Section 3.1.

2.3 Part Variation. In order to study the assembly process variation, the part orientation in the body coordinates needs to be described. Based on the rigid part assumption, any single part orientation can be represented with a point on the part and an orientation angle in the body coordinates. Since variation is the focus of the research, only the deviation, or error, is included in the model. Thus, we have the following definitions on the part point and the part error vector.

Definition 2 *Part point and part error vector:* A "part point A" is defined to represent the part orientation in the body coordinates. The part error vector represented by the part point A is described as:

$$\mathbf{X}_A(i) = (\Delta x_A(i)\quad \Delta z_A(i)\quad \Delta\alpha(i))^T \tag{2}$$

where $\Delta x_A(i)$, $\Delta z_A(i)$ are the deviation errors at point A in the X and Z directions in the body coordinates at station i; $\Delta\alpha(i)$ is the part orientation angle error of this part at station i defined as positive in the counterclockwise direction.

Part point A can be any point on a part. However, the part point A should be the same throughout the assembly process for a given part. The part error vector defined in Eq. (2) can specify the part orientation error in the X-Z plane. It should be emphasized that the part error vector reflects the "accumulative error," which refers to the total assembly error occurring in previous assembly processes up to the current assembly station.

The part error vector comes from two independent root causes in the assembly process.

(i) The first root cause is due to the fixture error in the current assembly station. The fixture error could be due to worn locating pins, missing or loose tooling locators, etc. This part error is called part locating error and is defined as follows.

Definition 3 *Part locating error:* The part error, which is represented by the part point A due to the fixture error vector $\Delta\mathbf{P}(i)$ at the current assembly station i, is represented as $\mathbf{F}_A(i)$.

(ii) The other root cause is the part error due to the part reorientation around the tooling locating points at the current station i. A part error is generated after assembly in the previous stations. This part error causes a deviation on the tooling locating points on the part. Those tooling locating points are used in the current assembly station. Upon loading the part in the fixture, the deviation on those tooling locating points will be reset to zero (or

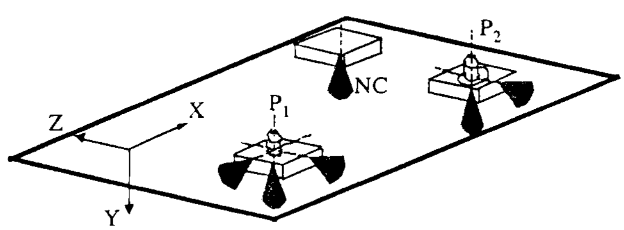

Fig. 2(*a*) 3-2-1 fixture layout

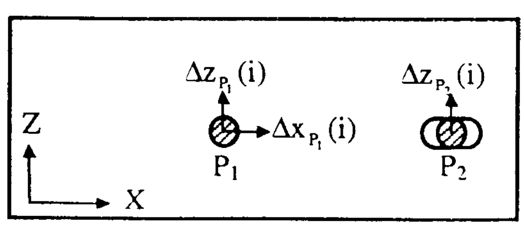

Fig. 2(*b*) *X-Z* plane fixture layout

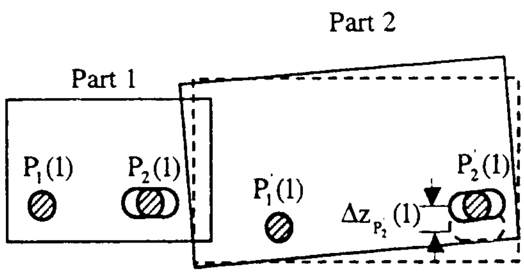

Fig. 3(*a*) Variation root causes at assembly station 1

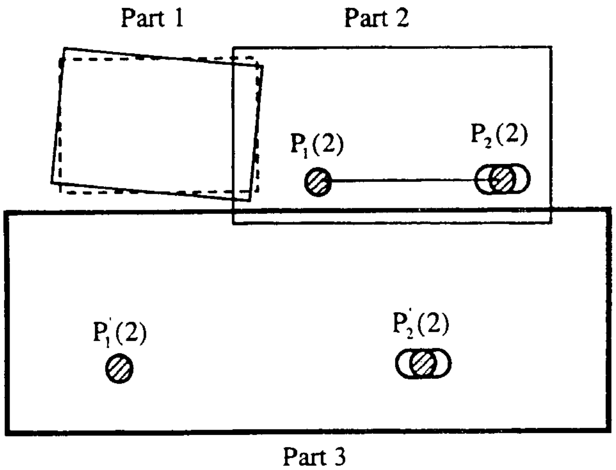

Fig. 3(*b*) Variation root causes at assembly station 2

design intent). Meanwhile, the part error represented by the part point A is changed simultaneously. This part movement, which resets the error of the tooling locating point to zero at the current station, is called the "reorientation" of the part. Due to this reorientation, the accumulative error up to the last assembly station will be "reoriented" following the tooling locating points on the part. This part reorientation error is defined in Definition 4 and explained further with an example.

Definition 4 *Part reorientation error:* Based on the design-intent, the part reorientation movement occurs when the stack-up variations (up to the previous assembly station $i - 1$) at the locator points of the current station i is reset to zero. The part error represented by the part point A due to this part reorientation movement is called the part reorientation error and represented as $\mathbf{T}_A(i - 1)$.

A simple example, as shown in Fig. 3, illustrates the variations resulting from the two error sources in an assembly process corresponding to definitions 3 and 4. In the example, three parts are assembled together by two stations. Part 1 and part 2 are assembled in station 1. The subassembly of parts 1 and 2 is assembled with part 3 in station 2.

At station 1, the locating points for part 1 are denoted as $P_1(1)$ and $P_2(1)$; and the locating points for part 2 are represented as $P'_1(1)$ and $P'_2(1)$. Assume only one locator, $P'_2(1)$, has a deviation at station 1. The assembly error due to this locator error at station 1 is the part locating error, as defined in Definition 3. The orientation of parts 1 and 2 due to this error is shown in Fig. 3(*a*). In the figure, the dashed line represents the nominal part orientation, and the solid line shows the actual orientation with the part locating error.

At station 2, the subassembly consisting of parts 1 and 2 are assembled with part 3. In this station, points $P_1(2)$ and $P_2(2)$ are designed to correspond to the same positions as $P'_1(1)$ and $P'_2(1)$. These points are used as tooling locating points to hold the first subassembly (part 1 and part 2). $P'_1(2)$ and $P'_2(2)$ are used to locate part 3. All tooling locators (pins) in station 2 are calibrated to the design-intent. Therefore, the assembly error (or deviation) accumulated in the previous assembly stations in points $P_1(2)$ and $P_2(2)$ will be reset to zero after loading the subassembly into the locating fixture. In this case, part 2 is reoriented from its deviated position in station 1 to the nominal position in station 2 as shown in Fig. 3(*b*). Due to this "reorientation," part 1, which has been assembled with part 2 in the first station, will be moved with part 2. Thus, an assembly error in part 1 is generated. This error is illustrated in Fig. 3(*b*), where the dashed line represents the nominal orientation of part 1, and the solid line shows the real position of part 1 with the reorientation error, as defined in definition 4.

Based on the definitions 3 and 4, the part variation represented by the point A can be described as follows:

$$\mathbf{X}_A(i) = \mathbf{X}_A(i - 1) + \mathbf{F}_A(i) + \mathbf{T}_A(i - 1) \tag{3}$$

where $i = 1, 2, 3, \ldots, N$ is the number of assembly stations, with N as the last assembly station in the process. The following two sections will develop the expressions for $F_A(i)$ and $T_A(i - 1)$, respectively.

2.3.1 Part Variations via Locator Errors at the Current Assembly Station. In the modeling of an assembly process, it is essential to understand the relationship between the part variation and tooling locating errors. More specifically, how does a fixture error lead to a part assembly error? The following theorem states the relationship.

Theorem 1: The part locating error represented by the point A, $F_A(i)$, can be calculated from the fixture error vector, $\Delta\mathbf{P}(i)$, using the following equation:

$$\mathbf{F}_A(i) = \mathbf{Q}_{A,P_1}(i) \quad \Delta\mathbf{P}(i) \tag{4}$$

Where $\mathbf{Q}_{A,P_1}(i)$ is the coordinate transformation matrix from the fixture error vector to the part locating error represented by the part point A at station i. $\mathbf{Q}_{A,P_1}(i)$ is given by

$$\mathbf{Q}_{A,P_1}(i) = \begin{pmatrix} 1 & \dfrac{L_Z(A, P_1)}{L_X(P_1, P_2)} & -\dfrac{L_Z(A, P_1)}{L_X(P_1, P_2)} \\ 0 & 1 - \dfrac{L_X(A, P_1)}{L_X(P_1, P_2)} & \dfrac{L_X(A, P_1)}{L_X(P_1, P_2)} \\ 0 & -\dfrac{1}{L_X(P_1, P_2)} & \dfrac{1}{L_X(P_1, P_2)} \end{pmatrix} \tag{5}$$

Proof: At station i, the part orientation angle error due to the tooling locator error is noted by $F_\alpha(i)$ as shown in Fig. 4, and can be calculated by

$$F_\alpha(i) = \frac{1}{L_X(P_1, P_2)} [\Delta z_{P_2}(i) - \Delta z_{P_1}(i)] \tag{6}$$

where $L_X(P_1, P_2) = x_{P_2} - x_{P_1}$, (x_{P_1}, z_{P_1}) and (x_{P_2}, z_{P_2}) are the coordinates of points P_1 and P_2 in the body coordinate. Based on Eq. (6), a matrix expression is obtained by:

$$\mathbf{F}_{P_1}(i) = \mathbf{Q}_{P_1,P_1}(i) \quad \Delta\mathbf{P}(i) \tag{7}$$

that is:

$$\begin{pmatrix} \Delta x_{P_1}(i) \\ \Delta z_{P_1}(i) \\ F_\alpha(i) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\dfrac{1}{L_X(P_1, P_2)} & \dfrac{1}{L_X(P_1, P_2)} \end{pmatrix} \begin{pmatrix} \Delta x_{P_1}(i) \\ \Delta z_{P_1}(i) \\ \Delta z_{P_2}(i) \end{pmatrix} \tag{8}$$

Based on the homogeneous transform, the deviation relationship between the two points of part point A and locator point P_1 can be expressed by:

$$\begin{pmatrix} \Delta x_A(i) \\ \Delta z_A(i) \\ \Delta\beta \end{pmatrix} = \begin{pmatrix} 1 & 0 & -L_Z(A, P_1) \\ 0 & 1 & L_X(A, P_1) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x_{P_1}(i) \\ \Delta z_{P_1}(i) \\ \Delta\beta \end{pmatrix} \tag{9}$$

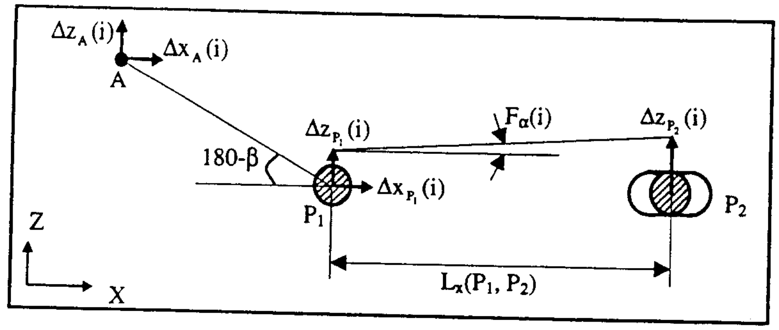

Fig. 4 Part error referenced by point A

where $L_X(A, P_1) = x_{P_1} - x_A$, $L_Z(A, P_1) = z_{P_1} - z_A$. Based on the rigid body assumption, $\Delta\beta$ is equal to $F_\alpha(i)$. For simplicity, this transform is noted as M_{A,P_1}. As a general expression, the relationship of deviations between any two points A and B on a rigid body can be represented as:

$$\mathbf{X}_B(i) = \mathbf{M}_{B,A}\mathbf{X}_A(i) \tag{10}$$

where

$$M_{B,A} = \begin{pmatrix} 1 & 0 & -L_Z(A, B) \\ 0 & 1 & L_X(A, B) \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

Based on Eqs. (7) and (10), the part locating error represented by point A can be further obtained as follows:

$$\begin{aligned} \mathbf{F}_A(i) &= \mathbf{M}_{A,P_1}(i) \quad \mathbf{F}_{P_1}(i) \\ &= \mathbf{M}_{A,P_1}(i) \quad \mathbf{Q}_{P_1,P_1}(i) \quad \Delta\mathbf{P}(i) \\ &= \mathbf{Q}_{A,P_1}(i) \quad \Delta\mathbf{P}(i) \end{aligned} \tag{12}$$

where

$$\mathbf{Q}_{A,P_1}(i) = \mathbf{M}_{A,P_1}(i) \quad \mathbf{Q}_{P_1,P_1}(i) \tag{13}$$

Thus, Eq. (4) is proved.

2.3.2 Part Variations via Part Reorientation Errors. In the last subsection, the relationship between the part locating error and the fixture error vector has been studied. The study was focused on the part locating error due to current fixture faults. In this subsection, it will focus on the effect of stack-up variations resulting from the previous station and its reorientation error due to the reorientation movements. As the first step, the accumulative locating point error is defined in definition 5.

Definition 5 *Accumulative locating point error:* The part accumulative error up to station $i - 1$ represented by the locating point P_s ($s = 1, 2$; P_s is used as the locators in station i) is called the "*accumulative locating point error*" and represented by:

$$X_{P_s}(i - 1) = (x_{P_s}(i - 1) \quad z_{P_s}(i - 1) \quad \Delta\alpha(i - 1))^T \tag{14}$$

As discussed before, the part error can be represented by any selected part point A in part error vector as defined in Definition 2. If we select the part point as the locating point P_s, the part error vector under this representation is called "accumulative locating point error" as defined in Definition 5. This definition is important when we further study the reorientation error in the following theorem.

Theorem 2: The part reorientation error represented by the part point P_1 at station i can be obtained from the part error vector represented by the part point P_1 at station $i - 1$ and the part error vector represented by the part point P_2 at station $i - 1$ using the following equations:

$$\mathbf{T}_{P_1}(i - 1) = (\mathbf{D}(i) \quad \mathbf{G}(i))\begin{pmatrix} \mathbf{X}_{P_1}(i - 1) \\ \mathbf{X}_{P_2}(i - 1) \end{pmatrix} \tag{15}$$

Where

$$\mathbf{D}(i) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & \dfrac{1}{L_x(P_1, P_2)} & 0 \end{pmatrix} \tag{16}$$

and

$$\mathbf{G}(i) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\dfrac{1}{L_x(P_1, P_2)} & 0 \end{pmatrix} \tag{17}$$

P_1 and P_2 are the tooling locators in the current station i, and can be either on the same part or different parts at station $i - 1$. P_1 and P_2 will be loaded at the design intent (or nominal) position at station i.

Proof: As shown in Fig. 5, the locator points P_1 and P_2 on station i before reorientation of the part are plotted with a solid line and their positions after reorientation are plotted with a dashed

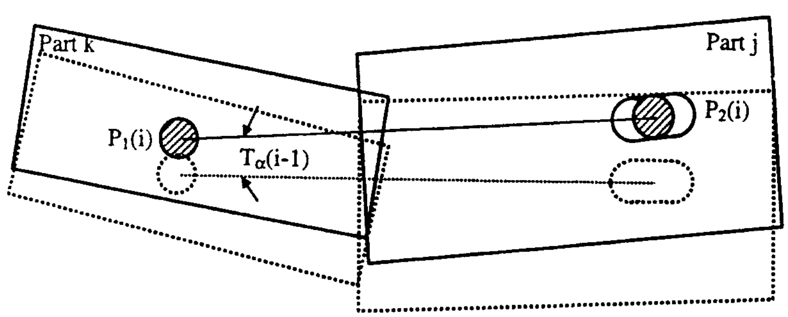

Fig. 5 Locators on the different parts k and j

line. The dimensional deviations $x_{P_1}(i-1)$, $z_{P_1}(i-1)$, and $z_{P_2}(i-1)$ of the accumulative locating point errors up to station $i-1$ are adjusted to zero at station i. The part orientation error represented by point P_1 is:

$$\mathbf{T}_{P_1}(i-1) = (T_X(i-1) \quad \mathbf{T}_Z(i-1) \quad \mathbf{T}_\alpha(i-1))^T \quad (18)$$

Which can be determined by:

$$\mathbf{T}_X(i-1) = -x_{P_1}(i-1) \quad (19)$$

$$\mathbf{T}_Z(i-1) = -z_{P_1}(i-1) \quad (20)$$

$$\mathbf{T}_\alpha(i-1) = -\frac{1}{L_X(P_1, P_2)}(z_{P_2}(i-1) - z_{P_1}(i-1)) \quad (21)$$

Equations (19) and (20) represent the dimensional deviations of P_1 and P_2 in the X and Z directions. Those deviations will be reset to zero at station i. Thus, the part orientation error represented by P_1 will be negative to the original accumulative error up to station $i-1$ in both X and Z axis.

Rewriting (19) to (21) as a matrix expression proves Eqs. (15) to (17).

Theorem 2 indicates the relationship between the part locating error represented by a locating point and the reorientation error when only two parts are involved in station i. If there are three parts involved in a station, the two locating points, P_s $(s = 1, 2)$ may either be located in two separate parts or the same part. In this situation, the reorientation error of the part, on which the locating points are located, can be obtained from theorems 3 and 4 respectively.

Theorem 3: If points A_k, A_j, and A_r are selected as the part points of part k, j, and r at station i respectively, and the locator points P_1 and P_2 are located on the different parts k and j, respectively, then the part reorientation error of part r represented by the part point A_r can be obtained by:

$$\mathbf{T}_{A_r}(i-1) = (\mathbf{H}_{rk}(i) \quad \mathbf{H}_{rj}(i))\begin{pmatrix}\mathbf{X}_{A_k}(i-1)\\ \mathbf{X}_{A_j}(i-1)\end{pmatrix} \quad (22)$$

where

$$\mathbf{H}_{rk}(i) = \mathbf{M}_{A_r,P_1}(i) \quad \mathbf{D}(i) \quad \mathbf{M}_{P_1,A_k}(i) \quad (23)$$

$$\mathbf{H}_{rj}(i) = \mathbf{M}_{A_r,P_1}(i) \quad \mathbf{G}(i) \quad \mathbf{M}_{P_2,A_j}(i) \quad (24)$$

Proof: Based on Eq. (10), it can be obtained:

$$\mathbf{T}_{A_r}(i-1) = \mathbf{M}_{A_r,P_1}(i) \quad \mathbf{T}_{P_1}(i-1) \quad (25)$$

$$\mathbf{X}_{P_1}(i-1) = \mathbf{M}_{P_1,A_k}(i) \quad \mathbf{X}_{A_k}(i-1) \quad (26)$$

$$\mathbf{X}_{P_2}(i-1) = \mathbf{M}_{P_2,A_j}(i) \quad \mathbf{X}_{A_j}(i-1) \quad (27)$$

So, substituting Eqs. (15), (26) and (27) into Eq. (25), gives Eq. (28).

$$\mathbf{T}_{A_r}(i-1) = \mathbf{M}_{A_r,P_1}(i)(\mathbf{D}(i) \quad \mathbf{G}(i)) \times \begin{pmatrix}\mathbf{M}_{P_1,A_k}(i) & \mathbf{\Theta}\\ \mathbf{\Theta} & \mathbf{M}_{P_2,A_j}(i)\end{pmatrix}\begin{pmatrix}\mathbf{X}_{A_k}(i-1)\\ \mathbf{X}_{A_j}(i-1)\end{pmatrix} \quad (28)$$

Theorem 4: If the locator points P_1 and P_2 on station i are located on the same part k, then the part reorientation error of part r represented by the part point A_r, $\mathbf{T}_{A_r}(i-1)$, can be calculated by

$$\mathbf{T}_{A_r}(i-1) = -\mathbf{M}_{A_r,A_k}(i) \quad \mathbf{X}_{A_k}(i-1) \quad (29)$$

Proof: Based on Eq. (10), the deviation vectors $\mathbf{X}_{A_j}(i-1)$ can be obtained by:

$$\mathbf{X}_{A_j}(i-1) = \mathbf{M}_{A_j,A_k}(i) \quad \mathbf{X}_{A_k}(i-1) \quad (30)$$

Substituting Eq. (30) into Eq. (22), it gives:

$$\mathbf{T}_{A_r}(i-1) = (\mathbf{H}_{rk}(i) \quad \mathbf{H}_{rj}(i)) \times \begin{pmatrix}\mathbf{I} & \mathbf{\Theta}\\ \mathbf{\Theta} & \mathbf{M}_{A_j,A_k}(i)\end{pmatrix}\begin{pmatrix}\mathbf{X}_{A_k}(i-1)\\ \mathbf{X}_{A_k}(i-1)\end{pmatrix} \quad (31)$$

Substituting Eqs. (10), (16), (17), (23) and (24) into Eq. (31), Eq. (31) is reduced to

$$\mathbf{T}_{A_r}(i-1) = -\mathbf{M}_{A_r,A_k}(i) \quad \mathbf{X}_{A_k}(i-1) \quad (32)$$

As a special case of theorem 4, where $r = k$ in Eq. (32),

$$\mathbf{T}_{A_k}(i-1) = -\mathbf{X}_{A_k}(i-1) \quad (33)$$

The interpretation of Eq. (33) is that when the locator points P_1 and P_2 are located at the same part k, the part error vector of part k at station i, $\mathbf{X}_{A_k}(i)$, is only determined by the part locating error at the current station i, i.e., substituting Eq. (33) into Eq. (3) gives

$$\mathbf{X}_{A_k}(i) = \mathbf{F}_{A_k}(i). \quad (34)$$

3 Assembly Process Modeling Based on a State Space Model

3.1 State Equation. In this section, a state space model will be developed to describe the part dimensional variations during an assembly process. A state variable vector, $\mathbf{X}(i)$, is defined by including all assembly part error vectors and represented as

$$\mathbf{X}(i) = \begin{pmatrix}\mathbf{X}_{A_1}(i)\\ \vdots\\ \mathbf{X}_{A_n}(i)\end{pmatrix} \quad (35)$$

Where, i $(i = 1, 2, \ldots, N)$ is the index of the assembly station, N is the total number of assembly stations in the assembly process; n is the total number of parts to be assembled in the whole assembly process; $\mathbf{X}_{A_j}(i)$ is the part error vector of part j represented by the part point A_j in assembly station i.

Based on the two types of part dimensional variation sources represented in Eq. (3), the state equation at station i can be expressed by:

$$\mathbf{X}(i) = [\mathbf{I} + \mathbf{T}(i-1)] \; \mathbf{X}(i-1) + \mathbf{B}(i) \; \mathbf{U}(i) + \mathbf{V}(i) \quad (36)$$

Here, $\mathbf{I}$ is the unit matrix with the dimension $3n \times 3n$. $\mathbf{X}(i)$ and $\mathbf{X}(i-1)$ include all part error vectors at station i and station $i-1$ respectively. $\mathbf{X}(0)$ is equal to a zero vector, which means the part variation caused by the stamping process is ignored. $\mathbf{V}(i)$ is the noise term in the model, which represents the imperfections in the modeling due to such factors as the rigid body assumption, the designed tolerance of the locators, etc. $\mathbf{U}(i)$ is the control vector at station i, which is defined as the fixture error vector for both subassembly parts at station i. P_s and P_s' $(s = 1, 2)$ are the locator points of subassembly parts 1 and 2 at station i respectively. $\mathbf{U}(i)$ can be expressed by:

$$\mathbf{U}(i) = \begin{pmatrix}\Delta\mathbf{P}(i)\\ \Delta\mathbf{P}'(i)\end{pmatrix} = (\Delta x_{P_1} \quad \Delta z_{P_1} \quad \Delta z_{P_2} \quad \Delta x_{P_1'} \quad \Delta z_{P_1'} \quad \Delta z_{P_2'})^T \quad (37)$$

Thus, the control matrix $\mathbf{B}(i)$ has dimension $3n \times 6$ and is given by:

$$\mathbf{B}(i) = \begin{pmatrix}\mathbf{Q}_{A_1,P_1}(i) & \mathbf{\Theta}^{3\times 3}\\ \vdots & \vdots\\ \mathbf{Q}_{A_i,P_1}(i) & \mathbf{\Theta}^{3\times 3}\\ \mathbf{\Theta}^{3\times 3} & \mathbf{Q}_{A_{i+1},P_1'}(i)\\ \mathbf{\Theta}^{3(n-i-1)\times 3} & \mathbf{\Theta}^{3(n-i-1)\times 3}\end{pmatrix} \quad (38)$$

where, $\mathbf{\Theta}^{j\times k}$ is a zero matrix with dimension $j \times k$.

The matrix $[\mathbf{I} + \mathbf{T}(i-1)]^{3n\times 3n}$ in Eq. (36) is the dynamic matrix with

$$\mathbf{T}(i-1) = \begin{pmatrix} \mathbf{H}^{3i\times 3i}(i) & \Theta^{3i\times 3(n-i)} \\ \Theta^{3(n-i)\times 3i} & \Theta^{3(n-i)\times 3(n-i)} \end{pmatrix} \tag{39}$$

If P_1 and P_2 are on the different parts k and j.

$$\mathbf{H}^{3i\times 3i}(i) = \begin{bmatrix} \Theta & \cdots & \Theta & \mathbf{H}_{1k}(i) & \Theta & \cdots & \Theta & \mathbf{H}_{1j}(i) & \Theta & \cdots & \Theta \\ \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \Theta & \cdots & \Theta & \mathbf{H}_{ik}(i) & \Theta & \cdots & \Theta & \mathbf{H}_{ij}(i) & \Theta & \cdots & \Theta \end{bmatrix} \tag{40}$$

otherwise, if P_1 and P_2 are on the same part k.

$$\mathbf{H}^{3i\times 3i}(i) = \begin{bmatrix} \Theta & \cdots & \Theta & -\mathbf{M}_{1,k}(i) & \Theta & \cdots & \Theta \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \Theta & \cdots & \Theta & -\mathbf{M}_{i,k}(i) & \Theta & \cdots & \Theta \end{bmatrix} \tag{41}$$

The dimension of all the zero matrix Θ in Eqs. (40) and (41) are 3×3.

3.2 Observation Equations. The measurement points are usually different from the part points. The observation equation can be expressed by

$$\mathbf{Y}(i) = \mathbf{C}(i) \quad \mathbf{X}(i) + \mathbf{W}(i) \tag{42}$$

(1) $\mathbf{Y}(i)$ is an observed vector related to all the measurement points at station i, which is expressed by:

$$\mathbf{Y}(i) = (\mathbf{Y}_1(i) \cdots \mathbf{Y}_r(i) \cdots \mathbf{Y}_n(i))^T \tag{43}$$

$\mathbf{Y}_r(i)$ represents the deviations at the measurement points $R_{j,r}$ ($j = 1, 2, \ldots m_r$) on part r at station i, that is:

$$\mathbf{Y}_r(i) = (x_{1,r} \quad z_{1,r} \quad \cdots \quad x_{m_r,r} \quad z_{m_r,r})^T \tag{44}$$

where m_r is the total number of measurement points on part r. Therefore, the dimension of the vector $\mathbf{Y}(i)$ is equal to $2*\sum_{r=1}^{n} m_r$.

(2) $\mathbf{C}(i)$ is an observation matrix, which can be expressed by:

$$\mathbf{C}(i) = \begin{pmatrix} \mathbf{C}_1(i) & & & & \\ & \ddots & & \Theta & \\ & & \mathbf{C}_r(i) & & \\ & & & \ddots & \\ & \Theta & & & \mathbf{C}_n(i) \end{pmatrix} \tag{45}$$

where the dimension of $\mathbf{C}_r(i)$ is $(2*m_r) \times 3$, and based on Eq. (10), the matrix $\mathbf{C}_r(i)$ related to part r can be calculated by:

$$\mathbf{C}_r(i) = \begin{pmatrix} 1 & 0 & -L_Z(R_{1,r}, A_r) \\ 0 & 1 & L_X(R_{1,r}, A_r) \\ \vdots & \vdots & \vdots \\ 1 & 0 & -L_Z(R_{m_r,r}, A_r) \\ 0 & 1 & L_X(R_{m_r,r}, A_r) \end{pmatrix} \tag{46}$$

(3) $\mathbf{W}(i)$ is white noise representing measurement noise. $\mathbf{Cov}(\mathbf{W}_{Q_{s,r}}(i), \mathbf{W}_{Q_{t,r}}(i)) = \sigma^2\delta_{Q_{s,r},Q_{t,r}}$, which means measurement error at any measurement points s and t on part r are not correlated. $\delta_{Q_{s,r},Q_{t,r}}$ is the Kronecker delta.

4 Discussion on the Usage of the Developed Model

The proposed modeling approach and results provide a new way for dimensional control in sheet metal assembly. This research lays the foundation for implementing advanced system identification and control theory in process design, monitoring, and diagnosis for body assembly. Here, a brief summary of the main problems and how these problems may be approached is provided.

(1) In-line OCMM sensor placement strategy:

In order to perform process monitoring and diagnosis, in-line dimensional measurement is essential. How to place sensors in the assembly line so as to minimize the cost and maximize the amount of information has been a challenging issue for years. Using the developed state space model and observation model, one can apply concepts similar to classical "observability" in control system theory to study the sensor placement problem. In addition, the observation matrix expresses the relationship between the observation variables (in-line sensor measurement information) and state variables (part accumulative error). Thus, a sensitivity study and optimization can be performed to select the sensor location that provides maximum diagnosability.

(2) Variation simulation for the assembly processes:

The developed state space model describes the mechanisms of variation propagation for the whole assembly processes. Thus, a variation simulation can be conducted by solving the state space equation with given initial conditions.

(3) Fixture tooling locator design and optimization:

The state space model provides a quantitative relationship between the tooling locator error and the part error vector. The concepts similar to classical "controllability" in control theory can be used to evaluate the impact and effectiveness of the tooling locators on part dimensional control. Some mechanical joint design philosophies, such as design slip plane and design gaps (Ceglarek and Shi, 1997), can be easily incorporated in the state space model as constraint conditions.

(4) Monitoring and diagnosis for assembly processes:

One of the major contributions of this research is on dimensional control of body assembly processes. Based on the state space model, many well-developed algorithms in control system theory can be directly applied to process monitoring and control of the body assembly processes. Examples include 1) Kalman filtering for the state estimation, which provides the part accumulative error and identification of large variation parts; 2) Abrupt change detection techniques for fixture tooling failure detection (e.g. broken or missing tooling pins); 3) System identification techniques to model the assembly process and compare the identified model with the design intent, which provides the evaluation of process and tooling design in production environments.

5 Conclusions and Future Work

The complexity of the assembly line due to the number of parts, stations and its high production rate, places a high demand on the tooling equipment. Tooling failure diagnosis based on in-line measurements is an important issue in the dimensional control, and various efforts have been made in the past. However, there were no models available to describe the overall body assembly processes for the purposes of process monitoring and diagnosis in manufacturing.

This paper develops a modeling technique for autobody assembly using state space models. A 3-2-1 fixturing mechanism is assumed, and emphasis on the dimension control for the X-Z plane are considered. Various variation error components (part, tooling, etc.) and their representations are defined. Furthermore, the inherent relationships among those error components are studied, and finally a state space model is developed.

The state space model is developed solely based on the assembly product design, process configuration, and tooling/fixture design. Thus, it can be obtained in the early design stage for dimensional control analysis and process monitoring and diagnosis in new product launch.

The major contribution of this research is to provide a state space model to describe the overall body assembly variation propagation. As a result, many well developed algorithms in control and system science can be used in dimensional control of body assembly.

It should be pointed out that further research on the modeling techniques is needed. In the paper, a 3-2-1 fixture and rigid part assumption are made in the derivation, which covers 68 percent of total part in a typical autobody (Shiu et al., 1996). Relaxing this assumption to nonrigid parts should be studied further. In this case, a nonlinear state-space model will be used. The concept of beam modeling (Shiu et al., 1996; 1997) has great potential in this respect. The second area of the research is to extend the modeling method for the 3-D case. A 3-D error model for single fixture has been developed in Apley and Shi (1998). The authors believe that the extension of 3-D modeling for the overall assembly process can be achieved with improved notations.

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