Q. Rong
General Electric Corp.,
Cleveland, OH 44112

J. Shi
Dept. of Industrial and Operations Engineering,
The University of Michigan,
Ann Arbor, MI 48109
e-mail: shihang@engin.umich.edu

D. Ceglarek
Dept. of Industrial Engineering,
The University of Wisconsin-Madison,
Madison, WI 53706-1572
e-mail: darek@engr.wisc.edu

Adjusted Least Squares Approach for Diagnosis of Ill-Conditioned Compliant Assemblies

Least squares (LS) estimation has been extensively used for parameter identification and model-based diagnosis. However, if ill-conditioning is present, the LS estimation approach tends to generate imprecise results and thus impacts the diagnostic performance. In this paper, an adjusted least squares approach is proposed to deal with the ill-conditioning problem in the diagnosis of compliant sheet metal assembly process. The adjusted LS approach is able to overcome the ill-conditioning and give precise results for certain linear combinations of the faults. Simulations and industrial case study are conducted to compare the diagnostic performance of the adjusted and regular LS approach. In addition, a two-stage assembly model is developed for further fault isolation with inclusion of additional measurement information. [DOI: 10.1115/1.1365116]

1 Introduction

The dimensional integrity of an automotive body has tremendous impact on the quality of the final vehicle. A typical body-in-white (BIW), which is the automotive body without closure panels such as the doors, hood, and deck lid, and without paint applied, consists of approximately 150–250 stamped sheet metal parts. Those parts are assembled into the BIW through 55–75 assembly stations throughout the whole assembly process.

Several key characteristics of the assembly processes have great impact on the dimensional quality of the automotive body assembly. These characteristics include product characteristics such as the part geometry and part-to-part joint functions as well as process characteristics such as part locating elements and fixture locating layout. The dimensions of the BIW are measured by the in-line optical coordinate measuring machine (OCMM). An automotive body usually has 100 to 150 measurement locating points (MLPs) measuring the ''*X*, *Y*, and *Z*'' coordinates on major subassemblies throughout the assembly process. Figure 1 shows the MLP layout of an automotive BIW.

Various research efforts have previously been made into the development of diagnostic methodologies for BIW assembly processes. Diagnostic approaches for single fault of fixture failures in assembly processes were proposed by Ceglarek and Shi [1]. This work was extended to multi-stations diagnosis by Jin and Shi [2] and Ding et al. [3], based on state space modeling technique. Wang and Nagarkar [4], Khan and Ceglarek [5] and Khan et al. [6] studied the locator and sensor placement for the automated coordinate checking fixtures and assembly systems, respectively, for rigid parts. The modeling and diagnosis of sheet metal assembly considering the compliant characteristics such as the part-to-part interferences and the part fabrication errors has been studied. Shiu et al. [7] proposed a beam-based model for dimensional control of compliant assemblies. Chang and Gossard [8] studied the impacts of compliant nonideal parts and locators on the CAD modeling. A diagnostic approach was developed based on the beam model and the principal component analysis (PCA) to isolate single fault in compliant assemblies [9]. Recently, Apley and Shi [10] have expended the diagnosis of fixture failures in BIW assembly processes to detect multiple faults using the LS approach.

The least squares (LS) approach is a common technique for parameter estimation and multiple fault diagnosis. The LS-based parameter estimation method has been applied in many different technical fields [11–13]. However, if the system/process being diagnosed is ill-conditioned, the LS approach may not work properly. Researchers [14–16] have indicated that when ill-conditioning is present, the parameter estimate based on the LS approach tends to be inflated, and there is possibility that some of the estimations may be imprecise. As a result, the LS solutions will lose the optimal properties of minimal 2-norm. For multiple faults diagnosis based on parameter estimation, the ill-conditioning problem will significantly lower the diagnostic performance.

This paper proposes a new approach for the diagnosis of multiple faults in ill-conditioned systems. An adjusted LS approach is developed based on the singular value decomposition (SVD), which is able to precisely estimate certain linear combinations of faults that generate similar fault signatures in compliant assemblies. In addition, a new method is developed to isolate these faults based on additional process information in the assembly procedures.

The paper is organized as follows. After the introduction, the fault diagnosis approach using LS estimation is reviewed in Section 2. The ill-conditioning problem in the diagnosis of compliant assembly is also discussed. In Section 3, an adjusted LS approach is developed based on SVD. The diagnostic performances of the regular LS approach and the adjusted LS approach are quantitatively analyzed in Section 4. In Section 5, a fault isolation approach is proposed based on a two-step assembly model and multivariate statistical techniques such as principal component analysis (PCA). The conclusions are summarized in Section 6.

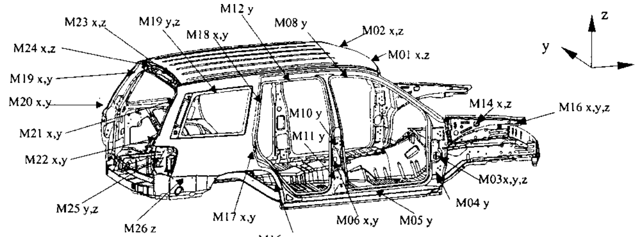


Fig. 1 The MLP layout of an automotive BIW

Contributed by the Manufacturing Engineering Division for publication in the JOURNAL OF MANUFACTURING SCIENCE AND ENGINEERING. Manuscript received August 1999; revised March 2000. Technical Editor: E. C. DeMeter.

2 Multiple Fault Diagnosis of Ill-Conditioned Systems

2.1 Fault Diagnosis Using Least Squares Estimation. A diagnostic approach has been developed for panel assembly processes based on the least squares estimation technique, which aims at detecting and isolating multiple fixture faults simultaneously [10]. Diagnosis for compliant assemblies based on the LS approach has also been studied by Rong et al. [17]. In this approach, a linearized diagnostic model was formulated as:

$$\mathbf{x}(j)=\mathbf{D}\mathbf{p}(j)+\mathbf{w}(j) \tag{1}$$

where $\mathbf{x}\in R^{n\times 1}$ is the measurement vector at the corresponding MLPs on the assembly structure, n is the total number of MLPs; $\mathbf{p}\in R^{m\times 1}$ is the vector representing the total structure faults, which are the results of part-to-part interferences at the parts joint surfaces, and will lead to assembly deformations, m is the number of faults in the assembly structure; $\mathbf{w}\in R^{n\times 1}$ is the noise vector which includes the measurement noise and any unmodeled factors in the process; j counts the product units, which is the sequential number of product units measured during real manufacturing process; $\mathbf{D}\in R^{n\times m}$ is the diagnostic matrix describing the relationship between the inputs (fault vector) and responses (measurement vector) in the diagnostic model. It was shown that for the compliant assembly model [7,9] $\mathbf{D}$ corresponds to the inversed stiffness matrix of the assembly structure, and the columns of $\mathbf{D}$ represent the impacts of unit faults on the structure deformations.

The least squares estimates of $\mathbf{p}(j)$ can be obtained as

$$\hat{\mathbf{p}}(j)=[\mathbf{D}^T\mathbf{D}]^{-1}\mathbf{d}^T\mathbf{x}(j) \tag{2}$$

Apley and Shi [10] developed the diagnostic statistics, and the diagnostic statistic for the ith fault is

$$F_i=\frac{\hat{\sigma}_i^2}{[\mathbf{D}^T\mathbf{D}]_{i,j}^{-1}\hat{\sigma}_w^2} \quad i=1,2,\ldots,m \tag{3}$$

where $\hat{\sigma}_i^2$, $\hat{\sigma}_w^2$ are the estimated variances of the ith fault and the noise respectively, $[\mathbf{D}^T\mathbf{D}]_{i,j}^{-1}$ is the ith diagonal element of matrix $[\mathbf{D}^T\mathbf{D}]^{-1}$. The diagnostic threshold is set as $F_0=F(1-\alpha,N,N(n-m))$, where $(1-\alpha)$ is the confident level of the test, and N is the sample size (total number of measured units). The diagnostic criteria is to compare the diagnostic statistic with the threshold, if $F_i>F_0$, the ith fault is detected.

2.2 The Ill-Conditioning Problem

2.2.1 An Example in Autobody Diagnosis. This LS-based diagnostic approach requires that the columns of the diagnostic matrix $\mathbf{D}$ are independent. Unfortunately, this requirement may not be satisfied for some compliant assemblies. As an example, a diagnosis based on the LS approach was performed for the rear doorframe of an automotive body as shown in Fig. 2. In this rear doorframe structure, there are a total of six nodes to represent part geometry and part joints. Eight measurement dimensions are at nodes 3, 4, 5, 6 in X and Z directions. There are four potential faults at nodes 4 and 5, which are the part jointing locations in X and Z directions.

By using the regular LS approach, the diagnostic results are obtained. Table 1 summarizes the diagnostic results for selected faults. In this simulation, $n=8$, $m=4$, the sample size was set as $N=50$, and $(1-\alpha)=0.999$. So, $F_0=1.9011$.

It can be seen from Table 1 that the regular LS approach can detect fault 2 and fault 4 effectively, but cannot detect faults 1 and fault 3, which are the faults in X direction at nodes 4 and 5. This is due to the ill-conditioned diagnostic matrix of the modeled assembly structure. The columns in $\mathbf{D}$ that correspond to fault 1 and fault 3 are nearly collinear. In the physical structure, this indicates that these two faults generate similar fault signatures.

When multicollinearity is presented among columns of $\mathbf{D}$, the matrix $\mathbf{D}$ will be singular or ill-conditioned. In the extreme case when the columns are perfectly collinear, the diagnostic matrix $\mathbf{D}$ is singular. In this situation, $\mathbf{D}$ is not full rank, i.e. *rank* ($\mathbf{D}$) $<m$. In practice, the columns of $\mathbf{D}$ may not be perfectly collinear, thus, $\mathbf{D}$ will be near singular or ill-conditioned.

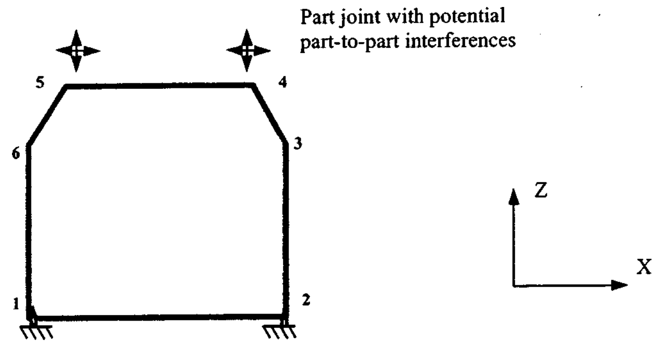


Fig. 2 The beam structure of the rear doorframe

2.2.2 The General Ill-Conditioning Problems. Considering the diagnosis of a physical system such as an assembly process, the fault-symptom relation can be formulated as functions of response variables and explanatory variables

$$\mathbf{x}=\mathbf{g}(\mathbf{p})+\mathbf{e} \tag{4}$$

where $\mathbf{x}\in R^{n\times 1}$ is the multivariate response vector, $\mathbf{p}\in R^{m\times 1}$ and $\mathbf{e}\in R^{n\times 1}$ are fault vector and errors vector respectively. Fault diagnosis based on the measured responses of the system or process can be considered as an inverse problem or a parameter estimation issue. Equation (4) can also be expressed as

$$\begin{aligned} x_1&=g_1(p_1,\cdots,p_m)+e_1\\ x_2&=g_2(p_1,\cdots,p_m)+e_2\\ &\cdots\cdots\cdots\cdots\cdots\cdots\\ x_n&=g_n(p_1,\cdots,p_m)+e_n \end{aligned} \tag{5}$$

where $g_i(p_l,\ldots,p_m)$ denotes the differentiable functions of the unknown parameters $p_1,\ldots,p_m$; x_i denotes the observations at the ith measurement point; and e_i represents the corresponding error. From Taylor expansion, and by neglecting the higher terms, the linearized relationship between responses and unknown parameters can be reduced to

$$\mathbf{x}=\mathbf{D}\mathbf{p}+\mathbf{w} \tag{6}$$

where

$$\mathbf{D}=\begin{bmatrix} \partial g_1/\partial p_1|\mathbf{p_0} & \partial g_1/\partial p_2|\mathbf{p_0} & \cdots & \partial g_1/\partial p_m|\mathbf{p_0}\\ \partial g_2/\partial p_1|\mathbf{p_0} & \partial g_2/\partial p_2|\mathbf{p_0} & \cdots & \partial g_2/\partial p_m|\mathbf{p_0}\\ \vdots & \vdots & \cdots & \vdots\\ \partial g_n/\partial p_1|\mathbf{p_0} & \partial g_n/\partial p_2|\mathbf{p_0} & \cdots & \partial g_n/\partial p_m|\mathbf{p_0} \end{bmatrix} \tag{7}$$

$\mathbf{D}\in R^{n\times m}$ is linearized coefficient matrix, $\mathbf{w}\in R^{n\times 1}$ is defined in the same way as in Eq. (1), and $\mathbf{p_0}$ is the initial condition of vector $\mathbf{p}$, which can be thought as the nominal position of $\mathbf{p}$. Notice that Eq. (1) has exactly the same format if including the sampling sequence $j=1,2,\ldots,N$.

In general, the parameter $\mathbf{p}$ can be estimated by using the LS approach only when $\mathbf{D}$ has rank m. When $\mathbf{D}$ is rank deficient, ill-conditioning problems occur, and the estimated parameters may not be uniquely determined.

Ill-conditioning of matrix $\mathbf{D}$ is often encountered in inverse problems or parameter identification [18]. Ill-conditioning can be caused by many factors. For example, in modal analysis and identification problems, ill-conditioning can result from the type and location of external excitation or the selection of parameters to be identified, as well as from the sampling interval of the responses.

Table 1 The simulation results for the regular LS approach

Fault situation	Fault 1 ($p1$)		Fault 2 ($p2$)		Fault 3 ($p3$)		Fault 4 ($p4$)	
	F_1	Test result	F_2	Test result	F_3	Test result	F_4	Test result
Fault 1 (X4)	***0.7968***	***not detected***	1.5747	no false alarm	0.8	no false alarm	1.2781	no false alarm
Fault 2 (Z4)	0.8814	no false alarm	***7.7856***	***detected***	0.8846	no false alarm	0.7354	no false alarm
Fault 1&2 (X4Z4)	***0.713***	***not detected***	***8.024***	***detected***	0.7178	no false alarm	1.1128	no false alarm
Fault 3&4 (X5Z5)	0.8142	no false alarm	0.8518	no false alarm	***0.8260***	***not detected***	***5.3303***	***detected***
Fault 1&3 (X4X5)	***1.0537***	***not detected***	1.5997	no false alarm	***1.0365***	***not detected***	1.076	no false alarm
Fault 1&3&4 (X4X5Z5)	***1.423***	***not detected***	0.9068	no false alarm	***1.4093***	***not detected***	***5.0392***	***detected***
Fault 1&2&3 (X4Z4X5)	***0.8288***	***not detected***	***8.7063***	***detected***	***0.8039***	***not detected***	0.6485	no false alarm
Fault 2 & 4 (Z4Z5)	1.1574	no false alarm	***6.2666***	***detected***	1.161	no false alarm	***5.5549***	***detected***
Fault 1,2,3,4 (X4Z4X5Z5)	***1.2035***	***not detected***	***5.9795***	***detected***	***1.23***	***not detected***	***4.9274***	***detected***

In structural dimensional analysis such as assembly processes, ill-conditioning may be induced by the design and properties of the assembly structure as well as by the fixturing scheme that locates the structure.

2.2.3 Evaluation of Ill-Conditioning Problems. One of the explanations of ill-conditioning was given by Klema and Laub [19]. The problem was stated to compute $f(z)$ for given $z \in \mathbf{z}$. Frequently, only an approximation z^* to z is known. If $f(z^*)$ is ''near'' $f(z)$, the problem is said to be well-conditioned. If $f(z^*)$ may potentially differ greatly from $f(z)$, when z^* is near z, the problem is said to be ill-conditioned.

The ill-conditioning can be evaluated quantitatively by the condition number. For a linear system represented by

$$\mathbf{A}\mathbf{y} = \mathbf{b} \tag{8}$$

where $\mathbf{A} \in R^{n\times n}$, $\mathbf{b} \in R^{n\times 1}$ are known, we seek to solve the unknown $\mathbf{y}$. When the system is perturbed or noisy, the problem becomes

$$(\mathbf{A} + \Delta\mathbf{A})\mathbf{y} = \mathbf{b} + \delta\mathbf{b} \tag{9}$$

where $\Delta\mathbf{A}$ and $\delta\mathbf{b}$ represent the perturbations in $\mathbf{A}$ and $\mathbf{b}$. The condition number is defined as

$$C_n = \|\mathbf{A}\| \cdot \|\mathbf{A}\|^{-1} \tag{10}$$

where $\|\mathbf{A}\|$ is the norm of $\mathbf{A}$ and $\|\mathbf{A}\|$ can be the 2-norm or F-norm. The condition number, C_n, gives a measure of how much errors in $\mathbf{A}$ and/or $\mathbf{b}$ may be magnified in the solutions. The condition number can also be expressed as

$$C_n = \frac{\xi_1}{\xi_r} \tag{11}$$

where ξ_1 and ξ_r are the largest and the smallest singular values of the matrix $\mathbf{A}$. If C_n is small, a small relative change in $\mathbf{b}$ cannot produce a very large relative change in $\mathbf{y}$. On the other hand, if C_n has a large value then a large relative change in y may result from a small perturbation in $\mathbf{b}$.

3 An Adjusted LS Approach

Singular value decomposition (SVD) can be used to detect ill-conditioning of the parameter estimation problems, which examines the singular condition of the matrix. The advantage of SVD is that it can quantitatively analyze the singular condition in the relationship between parameters and the responses.

Using singular value decomposition (SVD) algorithm, matrix $\mathbf{D}$ can be expressed as

$$\mathbf{D} = \mathbf{U}\Lambda\mathbf{V}^T \tag{12}$$

where both $\mathbf{U} \in \mathbf{R}^{n\times n}$ and $\mathbf{V} \in \mathbf{R}^{m\times m}$ are orthogonal matrices, and

$$\Lambda = \begin{bmatrix} \mathbf{S} & 0 \\ 0 & 0 \end{bmatrix} \in \mathbf{R}^{n\times m},$$

with $\mathbf{S} = \mathrm{diag}(\xi_1, \xi_2, \cdots, \xi_r)$, $r \leq \min(n,m)$, ξ_i's are the non-zero singular values of $\mathbf{D}$. The columns of $\mathbf{U}$ are the eigenvectors of $\mathbf{D}\mathbf{D}^T$, and the columns of $\mathbf{V}$ are the eigenvectors of $\mathbf{D}^T\mathbf{D}$.

In the past, SVD has been used to solve various ill-conditioned problems. Penny et al. [20] suggested the use of SVD to select the optimal measurement locations for dynamic tests. Hasan and Viola [18] applied SVD to identify physical parameters in time domain modal identification. Mottershead and Foster [21] discussed the application of SVD in dealing with ill-conditioning in spatial parameter estimation from measured vibration data. The ill-conditioning problems in these applications can be dealt with by commonly used approaches such as variable selection or experimental design. However, in compliant assemblies such as the autobody assembly, the inputs-responses are constrained by the physical properties of the assembly structure. The parameters that need to be estimated are the potential faults, which are determined by product/process design and cannot be simply modified by experimental design. Moreover, any potential faults may occur during the manufacturing process. Thus, it is not appropriate for the diagnostic approach to reduce the parameter domain based on variable selection. In this paper, an adjusted LS approach is developed based on the SVD and matrix partition technique for the diagnosis of compliant assemblies. Applying the adjusted LS approach, certain linear combinations of the faults with collineared fault signatures can be precisely estimated and detected under the physical constraints of the modeled assembly structure.

If $\mathbf{D}$ is ill-conditioned, one can partition $\mathbf{D}$ into $[\mathbf{D_1}|\mathbf{D_2}]$, where $\mathbf{D_1} \in \mathbf{R}^{n\times k}$ includes all independent columns of $\mathbf{D}$, and $\mathbf{D_2} \in \mathbf{R}^{n\times(m-k)}$ includes all collineared columns of $\mathbf{D}$. Thus, $\mathbf{D_2}$ is ill-conditioned and rank deficient. Partition $\mathbf{p}$ into $[\mathbf{p_1}|\mathbf{p_2}]^T$ accordingly, where $\mathbf{p_1} \in \mathbf{R}^k$, and $\mathbf{p_2} \in \mathbf{R}^{(m-k)}$. For the convenience of discussion, we define $p_i \in \mathbf{p_1}$ as a *type one* fault, $p_i \in \mathbf{p_2}$ as a *type two* fault. The criteria and procedure of the partition can be summarized as follows:

The linear dependency among *type two* faults can be statistically analyzed by using linear correlation between the columns of matrix $\mathbf{D}$. If the parameters (faults) are independent, the correlation between them is low; If the faults are collinear, the correlation is high. The correlation between two faults can be computed by [22]

$$r_{kl}=\frac{s_{kl}}{\sqrt{s_{kk}s_{ll}}} \tag{13}$$

where s_{kl} and s_{kk} are the components of the parameter covariance. The collinearity of parameters can also be evaluated based on the SVD results. It can be computed that $s_{kl}=\Sigma_{i=1}^{m}\nu_{ki}\nu_{li}/\xi_i^2$, where ν_{ki},ν_{li} are the elements of the singular vectors, and ξ_i's are the singular values.

Therefore, the *type two* faults can be grouped by the correlation analysis of the SVD of matrix $\mathbf{D}$, and the columns can be partitioned accordingly. If more than one set of *type two* faults are present, the diagnostic matrix $\mathbf{D}$ will be partitioned into different submatrices by the correlation analysis procedure.

So,

$$\begin{aligned}\mathbf{x}&=[\mathbf{D_1}\ \ \mathbf{D_2}]\begin{bmatrix}\mathbf{p_1}\\ \mathbf{p_2}\end{bmatrix}\\ &=\mathbf{D_1p_1}+\mathbf{D_2p_2}\end{aligned} \tag{14}$$

Since $\mathbf{D_2}$ is rank deficient, there is no unique solution for variables in $\mathbf{p_2}$, and only certain linear combinations of these variables can be obtained. Thus, $\mathbf{x}$ is the overall results of each individual effect of $p_i\in\mathbf{p_1}$, plus a combined effect of p_i'$s\in\mathbf{p_2}$. It is clear that *type one* fault can be detected accurately by using the regular LS estimation because $\mathbf{D_1}$ is full rank. However, *type two* faults cannot be effectively detected using the LS approach due to the rank deficiency in $\mathbf{D_2}$.

By using SVD, we can decompose $\mathbf{D_2}$ as

$$\mathbf{D_2}=\mathbf{U}^{(2)}\Lambda^{(2)}\mathbf{V}^{(2)^T} \tag{15}$$

where both $\mathbf{U}^{(2)}\in\mathbf{R}^{n\times(m-k)}$ and $\mathbf{V}^{(2)}\in\mathbf{R}^{(m-k)\times(m-k)}$ are column orthogonal matrices, and $\Lambda^{(2)}\in\mathbf{R}^{(m-k)\times(m-k)}$ is a diagonal matrix.

First, let us consider the extreme case. Assume the $(m-k)$ columns of $\mathbf{D_2}$ are exactly collinear, thus *rank* $(\mathbf{D_2})=1$. So, the $(m-k-1)$ singular values of $\mathbf{D_2}$ are zeros. Then

$$\mathbf{D_2}=\xi_1\mathbf{u}_1^{(2)}\mathbf{v}_1^{(2)^T} \tag{16}$$

If the columns are not exactly collinear but nearly rank deficient, the singular values of $\mathbf{D_2}$ will not be zeros. In this situation,

$$\mathbf{D_2}=\sum_{i=1}^{\mathrm{Rank}(\mathbf{D_2})}\xi_i\mathbf{u}_i^{(2)}\mathbf{v}_i^{(2)^T} \tag{17}$$

where $\mathbf{u}_i^{(2)}$'s and $\mathbf{v}_i^{(2)}$'s are the corresponding SVD vectors of this $\mathbf{D_2}$. In this case, one of the singular values of $\mathbf{D_2}$ is large, and all others are small. Thus $\mathbf{D_2}$ can be approximated by the linear combination associated with the largest singular value. Denoting

$$\tilde{\mathbf{D}}_2=\xi_1\mathbf{u}_1^{(2)}\mathbf{v}_1^{(2)^T} \tag{18}$$

From the results of Golub and Van Loan [14], $\tilde{\mathbf{D}}_2$ is the best rank-sufficient approximation of $\mathbf{D_2}$. The detail of this property is provided in the Appendix.

From Eq. (18),

$$\tilde{\mathbf{D}}_2\cdot\mathbf{p_2}=(\xi_1\mathbf{u}_1^{(2)})(\mathbf{v}_1^{(2)^T}\mathbf{p_2}) \tag{19}$$

Let $\mathbf{D_A}=\xi_1\mathbf{u}_1^{(2)}$, and $p_A=\mathbf{v}_1^{(2)^T}\mathbf{p_2}=\Sigma_{i=1}^{m-k}v_{i1}^{(2)}p_{2i}$. Now p_A is a new variable that represents the combined effect of the undistinguishable variables in vector $\mathbf{p_2}$.

From Eq. (19), Eq. (1) can be adjusted into

$$\mathbf{x}(j)=\tilde{\mathbf{D}}\cdot\tilde{\mathbf{p}}(j)+\mathbf{w_A}(j) \tag{20}$$

where $\tilde{\mathbf{p}}=[\mathbf{p_1}|p_A]^T=[p_1,p_2,\cdots,p_k|p_A]^T$, and $\tilde{\mathbf{D}}=[\mathbf{D_1}|\mathbf{D_A}]$, and $\mathbf{w_A}$ is the overall noise.

By doing so, the linear combination of $\mathbf{p_2}$ can be precisely estimated by using the LS approach without affecting other parameters.

4 Diagnostic Performance Analysis

In this section, the diagnostic performance of the regular LS approach and the adjusted LS approach are compared with simulations. The beam structure model of the rear doorframe shown in Fig. 2 is used in these simulations.

The diagnostic results of the adjusted LS approach using the same data set as in Table 1 are summarized in Table 2.

From Table 2, it can be seen that the adjusted LS approach can effectively detect the combination of $p1$ and $p3$ in all fault situations when these faults occur, which gives more precise detection compared to the regular LS approach as listed in Table 1. The detection performance of detecting fault 2 and fault 4 is the same as in the regular LS approach.

In order to evaluate and compare the diagnostic performance of the regular and adjusted LS diagnostic approaches, type I error (false alarm or α error) and type II error (miss detection or β error) are investigated by using Monte Carlo simulations.

The signal-to-noise ratio is defined as

$$S_n=\frac{\overline{\sigma}^2}{\sigma_w^2} \tag{21}$$

where $\overline{\sigma}^2$ is the pooling variance of the n measurement points in $\mathbf{x}$, and

$$\overline{\sigma}^2=\frac{\sum_{i=1}^{n}\mathrm{Var}[x_i(\cdot)]}{n}. \tag{22}$$

From Eq. (1), we have

$$x(j)=\sum_{k=1}^{m}d_{ik}p_k(j)+w_i(j) \tag{23}$$

and

$$\mathrm{Var}[x_i(\cdot)]=\sum_{k=1}^{m}d_{ik}^2\,\mathrm{Var}[p_k(\cdot)]+\sigma_w^2 \tag{24}$$

Table 2 The simulation results for the adjusted LS approach

Fault / Fault situation	Fault 2 (*p2*)		Fault 4 (*p4*)		Fault 1&3 (*p1&P3*)	
	F_2	Test result	F_4	Test result	F_A	Test result
Fault 1 (X4)	1.7051	no false alarm	1.3802	no false alarm	***3.6824***	***detected***
Fault 2 (Z4)	***8.089***	***detected***	0.7983	no false alarm	1.3418	no false alarm
Fault 1&2 (X4Z4)	***8.6548***	***detected***	1.128	no false alarm	***3.4748***	***detected***
Fault 3&4 (X5Z5)	0.8475	no false alarm	***5.8528***	***detected***	***3.3247***	***detected***
Fault 1&3 (X4X5)	1.6275	no false alarm	0.9613	no false alarm	***7.1431***	***detected***
Fault 1&3&4 (X4X5Z5)	0.9202	no false alarm	***4.7838***	***detected***	***5.3285***	***detected***
Fault 1&2&3 (X4Z4X5)	***9.2587***	***detected***	0.6419	no false alarm	***5.9303***	***detected***
Fault 2 & 4 (Z4Z5)	***6.249***	***detected***	***5.526***	***detected***	1.1593	no false alarm
Fault 1,2,3,4 (X4Z4X5Z5)	***5.8115***	***detected***	***4.9752***	***detected***	***4.8213***	***detected***

Table 3 The Monte Carlo simulation results for the regular and adjusted LS approach

Fault situations	Regular LS approach								Adjusted LS approach					
	Fault 1		Fault 2		Fault 3		Fault 4		Fault 2		Fault 4		Fault 1&3	
	Type I error	Type II error	Type I error	Type II error	Type I error	Type II error	Type I error	Type II error	Type I error	Type II error	Type I error	Type II error	Type I error	Type II error
Fault 1 (X4)		*1.0*	0		0.001		0.001		0		.002			*.029*
Fault 2 (Z4)	0.002			*0*	.002		.002			*0*	.002		.002	
Fault 1&2 (X4Z4)		*0.999*		*0*	.001		.001			*0*	.001			*.033*
Fault 3&4 (X5Z5)	.003		0			*.997*		*0*	0			*0*		*.038*
Fault 1&3 (X4X5)		*1.0*	.001			*.999*	.003		0		.003			*.004*
Fault 1&3&4 (X4X5Z5)		*.999*	.002			*.999*		*0*	.002			*0*		*0*
Fault 1&2&3 (X4Z4X5)		*.999*		*.007*		*.999*	.002			*.004*	.002			*.004*
Fault 2 & 4 (Z4Z5)	.001			*0*	.001			*0*		*0*		*0*	.002	
Fault 1,2,3,4 (X4Z4X5Z5)		*1.0*		*0*		*1.0*		*0*		*0*		*0*		*0*

Here we assume that $p_s(j)$ and $p_t(j)$ are independent $(s,t = 1,2,\ldots,m,\ s\neq t)$. Also, $p_s(j)$ and $w_s(j)$ are independent.

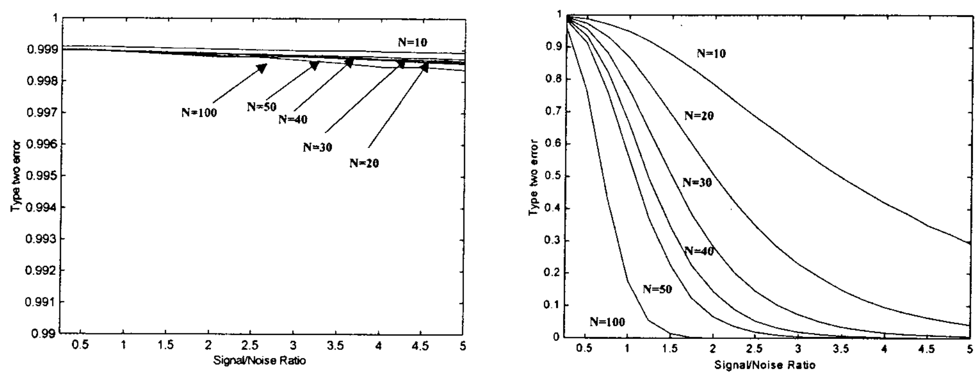
If only the kth fault occurs, then

$$\mathrm{Var}[x_i(\cdot)] = d_{ik}^2 \cdot \mathrm{Var}[p_k(\cdot)] + \sigma_w^2 \tag{25}$$

From Eq. (22),

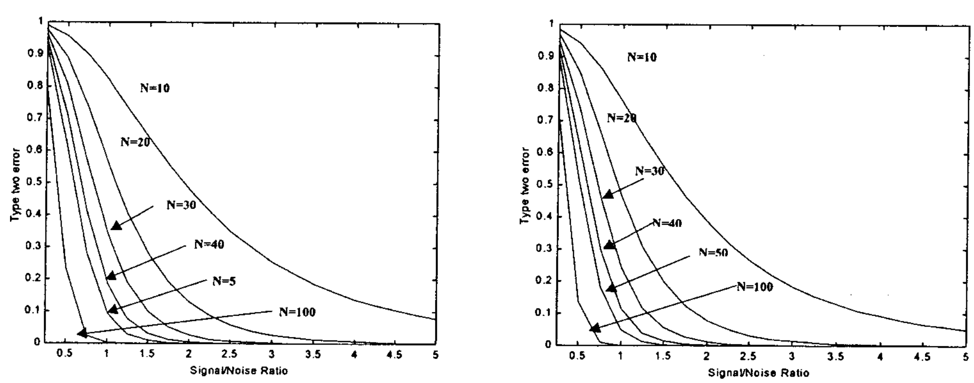
$$\overline{\sigma^2} = \frac{\sum_{i=1}^{n} (d_{ik}^2 \cdot \sigma_k^2 + \sigma_w^2)}{n} = \frac{\sigma_k^2 \sum_{i=1}^{n} d_{ik}^2 + n\sigma_w^2}{n} = \frac{\sigma_k^2}{n} + \sigma_w^2 \tag{26}$$

So, the signal-to-noise ratio for the kth fault



(a) Regular LS approach (b) Adjusted LS approach

Fig. 3 The OC curve of *type two* faults (*p*1 and *p*3)



(a) Regular LS approach (b) Adjusted LS approach

Fig. 4 The OC curves of *type one* fault (*p*2 and *p*4)

$$S_{nk}=\frac{\sigma_k^2}{n\sigma_w^2}+1 \tag{27}$$

In this simulation, we set $S_{nk}=2$, the sample size $N=50$, $(1-\alpha)=0.999$, the total number of simulations $n_k=10^4$.

The results for the regular LS and adjusted LS are summarized in Table 3. It should be noticed that when a fault occurs, the diagnostic performance can be evaluated by the type II error (miss detection); on the other hand, when a fault is not present, the diagnostic performance is evaluated by type I error (false alarm).

It can be seen from Table 3 that the adjusted LS approach has a very good performance in detecting different fault combinations containing the *type two* faults $p1$ and $p3$. Both type I error (false alarm) and type II error (miss detection) for the adjusted LS approach are very low compared to the regular LS approach. Meanwhile, both approaches have the similar performance in detecting faults $p2$ and $p4$.

The OC curve for each fault are constructed and illustrated in Figs. 3 to 4.

Comparing the diagnostic performance for the *type two* fault in Fig. 3, the adjusted LS significantly reduces the β error. For a moderate signal-to-noise ratio and sample size, say $S_n=3$, and $N=40$, by using the adjusted LS, the β error is below 0.04; while for the regular LS, the β error is about 0.999. For *type one* fault, the adjusted LS and regular LS have similar performance (see Fig. 4(*a*) and (*b*)).

5 Fault Isolation with Additional Information

5.1 Fault Isolation Based on Optimal Design. The purpose of this section is to improve the diagnostic performance of the proposed approach by studying the diagnosability and isolability of *type two* faults. The adjusted LS approach can precisely detect the combination of *type two* faults. However, each individual fault cannot be isolated based on the information provided at the measurement points. Therefore, an effective approach is needed to isolate the faults.

5.1.1 Optimal Design Technique. One common approach of improving parameter estimations in experimental design issues is to add new observations to the diagnostic model [14,15]. In the diagnosis of automotive sheet metal assembly, this concept is equivalent to adding new MLPs on the assembly structure.

In order to isolate *type two* faults, a new measurement point is added to the diagnostic model. Thus, the corresponding model becomes

$$\mathbf{x}^*=\mathbf{D}_2^*\cdot\mathbf{p}_2+\mathbf{w} \tag{28}$$

where $\mathbf{x}^*=(x_1,x_2,\ldots,x_n,x_{n+1})^T-\mathbf{x}^{(p1)}$, $\mathbf{x}^{(p1)}$ is the effect of *type one* fault $\mathbf{p}_1$ on the dimensional displacement of the assembly structure, and $\mathbf{D}_2^*=(\mathbf{D}_2\mathbf{d}_{n+1}^{(2)})^T$, $\mathbf{d}_{n+1}^{(2)}$ is the new row in the diagnostic matrix $\mathbf{D}_2^*$ corresponding to the new measurement point $\mathbf{x}_{n+1}$.

Based on the Fisher information matrix [23], the optimal criterion is to choose a new MLP that maximizes the determinant of the Fisher information matrix. It can be expressed as follow

$$\underset{x_{n+1}}{\mathrm{Max}}\|(\mathbf{D}_2^*)^T\mathbf{D}_2^*\| \tag{29}$$

Silvey [14] showed that an equivalent criterion is to choose x_{n+1} in order to maximize the minimum eigenvalue of the new information matrix

$$(\mathbf{D}_2^*)^T\mathbf{D}_2^*=\mathbf{D}_2^T\mathbf{D}_2+\mathbf{d}_{n+1}^{(2)}\mathbf{d}_{n+1}^{(2)^T} \tag{30}$$

5.1.2 Limitations of the Optimal Design in Compliant Assemblies. It should be noticed that in optimal experimental design problems, i.e. in a linear system $\mathbf{Ay}=\mathbf{b}$, $\mathbf{A}$ and $\mathbf{b}$ are all experimental data. By choosing the experimental setup, one can get new observations (new row in $\mathbf{A}$) that is able to meet the optimal criteria. Unlike the optimal experimental design problems, all responses of MLPs (the contents of the diagnostic matrix $\mathbf{D}$) are determined by the properties of the modeled assembly structure in the diagnostic issue in this paper. For some cases, an optimal solution may not exist.

For compliant assembly system with potential multiple faults, if two or more faults generate the same fault signature on the assembly structure, adding a new MLP will not provide an optimal solution. In another word, the faults cannot be isolated no matter where the additional MLP is added to the structure.

To show this, assume two faults in the *type two* fault domain occurring, let us say faults p_{I} and p_{II}. Thus $\mathbf{D}_2=[\mathbf{d}_{\mathrm{I}}^{(2)}\mathbf{d}_{\mathrm{II}}^{(2)}]$, $\mathbf{d}_{\mathrm{I}}^{(2)}$ and $\mathbf{d}_{\mathrm{II}}^{(2)}$ are the column vectors corresponding to faults p_{I} and p_{II}. Notice that the *type two* faults generate the same fault signature, thus $\mathbf{d}_{\mathrm{I}}^{(2)}$ and $\mathbf{d}_{\mathrm{II}}^{(2)}$ are correlated with relation of $\mathbf{d}_{\mathrm{II}}^{(2)}=c\mathbf{d}_{\mathrm{I}}^{(2)}$. Constrained by the assembly structure properties, the elements in the new row of the diagnostic matrix corresponding to the new MLP are proportional, which is $d_{n+1,\mathrm{II}}=cd_{n+1,\mathrm{I}}$. The relationship can be represented by

$$\begin{bmatrix}\mathbf{x}\\ x_{n+1}\end{bmatrix}=\begin{bmatrix}\mathbf{D}_2\\ \mathbf{d}_{n+1}^{(2)}\end{bmatrix}\cdot\begin{bmatrix}p_{\mathrm{I}}\\ p_{\mathrm{II}}\end{bmatrix} \tag{31}$$

From Silvey's criteria in Eq. (30),

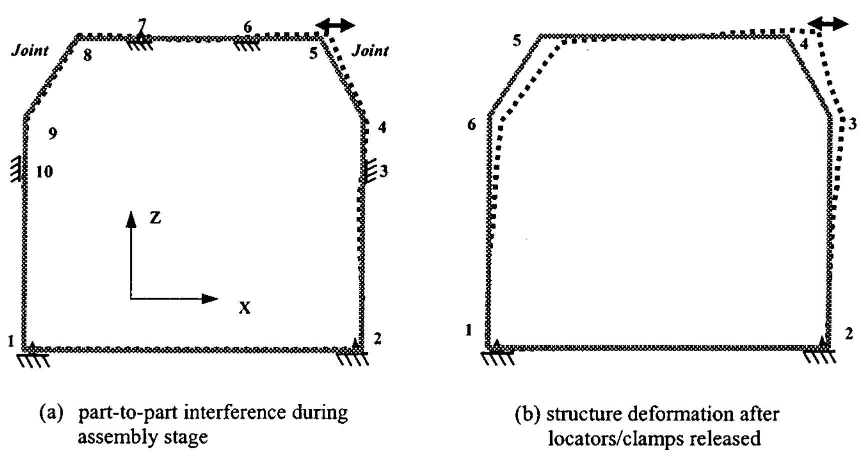


Fig. 5 The two-stage assembly model

$$\begin{bmatrix} \mathbf{D}_2 \\ \mathbf{d}_{n+1}^{(2)} \end{bmatrix}^T \cdot \begin{bmatrix} \mathbf{D}_2 \\ \mathbf{d}_{n+1}^{(2)} \end{bmatrix} = (\mathbf{D}_2)^T \mathbf{D}_2 + (\mathbf{d}_{n+1}^{(2)})^T \mathbf{d}_{n+1}^{(2)}$$

$$= [\mathbf{d}_{\mathrm{I}}^{(2)} \ \ \mathbf{d}_{\mathrm{II}}^{(2)}]^T [\mathbf{d}_{\mathrm{I}}^{(2)} \ \ \mathbf{d}_{\mathrm{II}}^{(2)}]$$

$$+ [d_{n+1,\mathrm{I}}^{(2)} \ \ d_{n+1,\mathrm{II}}^{(2)}]^T [d_{n+1,\mathrm{I}}^{(2)} \ \ d_{n+1,\mathrm{II}}^{(2)}] \tag{32}$$

$$= \left(\sum_{i=1}^{n} (d_{i,\mathrm{II}}^{(2)})^2 + (d_{n+1,\mathrm{II}}^{(2)})^2 \right) \begin{bmatrix} 1 & c \\ c & c^2 \end{bmatrix}$$

It can be computed that one of the eigenvalues in Eq. (32) is zero. Thus the optimal criteria cannot be satisfied.

5.2 Fault Isolation Based on the Two-Step Assembly Model. Since adding a new MLP at the final measurement station will not help to isolate the *type two* faults, other information is needed.

Now let us consider the assembly procedure of sheet metal parts. The assembly procedure can be described by two stages: (1) fixturing stage, which refers to the stage where parts are positioned and located by pins and clamps. In this stage, the parts deformation are caused by part-to-part interferences during assembly when the locators/clamps are applied, and (2) released stage, which refers to parts which are assembled and all locators (pins and clamps) have been released. Thus, the measurements are taken when the part is in a free state (without fixtures holding the part). The deformation/spring back results from self-constraint forces of the assembled structure after the locators/clamps have been released. The two stages are illustrated in Fig. 5.

Now consider the force-deformation relations in the two stages. For the released stage:

$$\mathbf{x}^{(2)} = \mathbf{f}^{(2)} \cdot \mathbf{p}^{(2)} \tag{33}$$

The model in the released stage is the same as the diagnostic model discussed in Eq. (1), where $\mathbf{x}^{(2)}$, $\mathbf{f}^{(2)}$ and $\mathbf{p}^{(2)}$ are corresponding to $\mathbf{x}$, $\mathbf{D}$ and $\mathbf{p}$.

For the fixturing stage:

$$\mathbf{x}^{(1)} = \mathbf{f}^{(1)} \cdot \mathbf{p}^{(1)} \tag{34}$$

$\mathbf{x}^{(1)} \in R^{k \times 1}$ and $\mathbf{p}^{(1)} \in R^{m \times 1}$ correspond to the deformation of the structure and the force vector at the fixturing stage. $\mathbf{f}^{(1)} \in R^{k \times m}$ is the diagnostic matrix when the clamps/locators are applied.

In this modeling procedure, the friction and friction forces between any two parts are not considered. Modeling the effects of friction is beyond the scope of this research. Under this assumption, we have the relationship of $\mathbf{p}^{(1)} = \mathbf{p}^{(2)}$. Notice that the diagnostic matrices of the two assembly stages are quite different because of the effects of locator/clamps. Inferring from the design principle of fixtures requires that the locating elements are orthogonal (or as orthogonal as possible). In this sense, the diagnostic matrix of the fixturing stage will not be ill-conditioned. So, additional measurement information from the assembly of the fixturing stage will be very helpful for isolating the *type two* faults.

A decision tree for the rear doorframe can be formulated as shown in Fig. 6. It can be seen that the fault isolation problem at the released stage can be reduced to single fault cases in the fixturing stage, which means that if the signature matches any particular fault, that fault is the corresponding source of the dimensional problem at the released stage. Thus, the fault diagnostic approach based on PCA for single fault [9] can be applied here to isolate the *type two* faults by using the additional measurement information at the fixturing stage. From Rong et al. [9] a diagnostic vector $\mathbf{d}_i^{(1)}$ can be defined which is the ith column of matrix $\mathbf{f}^{(1)}$. The diagnostic vector $\mathbf{d}_i^{(1)}$ $(i = 1, 2, \ldots, k)$ is a constant vector, which is determined by the properties of the modeled assembly structure, and measures the effect of the ith fault on $\mathbf{x}^{(1)}$. The fault isolation is based on multiple hypothesis tests. The hypotheses are defined as follows

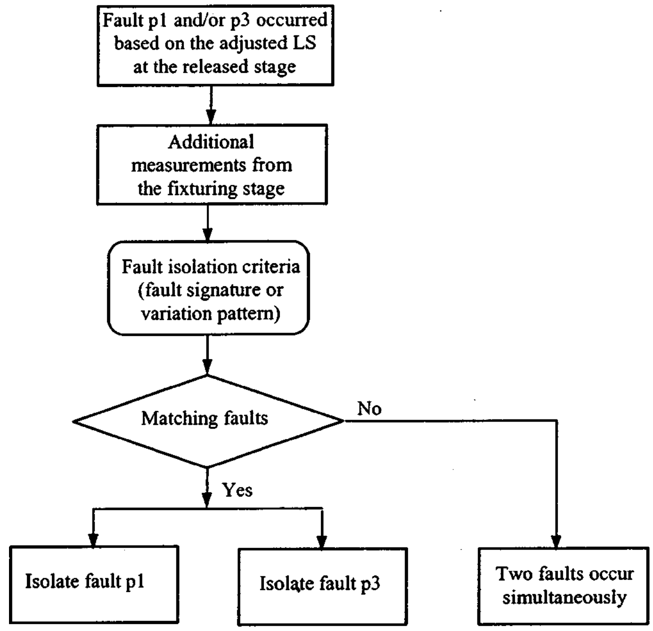


Fig. 6 The decision flowchart for the isolation of *type two* faults

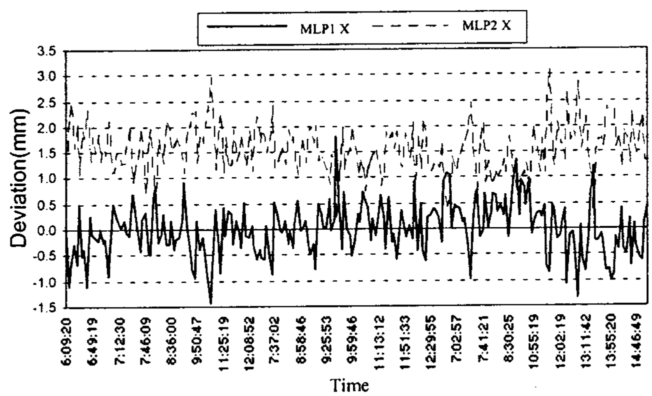


Fig. 7 The in-line measurement data of MLP 1 and MLP 2 on the rear door

Table 4 The normalized diagnostic vectors for different hypothetical faults

Fault	at node 5 in X	at node 5 in Z	at node 8 in X	at node 8 in Z
	$\mathbf{d}_1^{(1)}$	$\mathbf{d}_2^{(1)}$	$\mathbf{d}_3^{(1)}$	$\mathbf{d}_4^{(1)}$
Diagnostic vector	$\begin{bmatrix} 0.7671 \\ 0.6416 \end{bmatrix}$	$\begin{bmatrix} 0.4200 \\ -0.9086 \end{bmatrix}$	$\begin{bmatrix} 0.6416 \\ 0.7671 \end{bmatrix}$	$\begin{bmatrix} 0.9086 \\ -0.4200 \end{bmatrix}$

$$H_0: \ \mathbf{a}_1 = \mathbf{d}_i^{(1)}$$

$$H_1: \ \mathbf{a}_1 \neq \mathbf{d}_i^{(1)}$$

where $\mathbf{a}_1$ is the eigenvector that is corresponding to the largest eigenvalue of the covariance matrix of $\mathbf{x}^{(1)}$. For single fault, $\mathbf{a}_1$ explains the variation pattern of the measured data of $\mathbf{x}^{(1)}$. The particular diagnostic vector $\mathbf{d}_i^{(1)}$ that satisfies the null hypothesis will isolate the corresponding fault in $\mathbf{p}^{(1)}$. The test statistic can be formulated as

Table 5 The hypothesis test results for the diagnostic vectors

Diagnostic vector	$\mathbf{d}_1^{(1)}$	$\mathbf{d}_2^{(1)}$	$\mathbf{d}_3^{(1)}$	$\mathbf{d}_4^{(1)}$
Fault	**at node 5 in x**	at node 5 in z	at node 8 in x	at node 8 in z
Ω_i	**3.11**	1629.40	32.92	1519.90
$\chi^2_{(0.01,1)} = 6.63$	$\Omega_2 < \chi^2_{(0.01,1)}$	$\Omega_3 > \chi^2_{(0.01,1)}$	$\Omega_4 > \chi^2_{(0.01,1)}$	$\Omega_5 > \chi^2_{(0.01,1)}$

$$\Omega_i=(N-1)\left(\lambda_1\mathbf{d}_i^{(1)^T}\mathbf{s}^{-1}\mathbf{d}_i^{(1)}+\frac{1}{\lambda_1}\mathbf{d}_i^{(1)^T}\mathbf{s}\mathbf{d}_i^{(1)}-2\right) \qquad (35)$$

and Ω_i follows χ^2_{n-1} [8]. Here **S** is the covariance matrix of $\mathbf{x}^{(1)}$, λ_1 is the largest eigenvalue of **S**, N is the sample size of the measurements. The decision criteria is that if $\Omega_i>\chi^2_{(\alpha,n-1)}$, reject H_0. Here α represents the confidence level of the tests.

5.3 Industrial Case Study. The presented case study was implemented in the assembly process of sport-utility vehicles in one of the domestic assembly plants. The schematic diagram of the automotive body for this vehicle is presented in Fig. 1, and model of the doorframe in Fig. 5. The described process has installed an in-line measurement gage, which allows for measurement of 100 percent-produced vehicles. The sample of 200 measured vehicles was used for data evaluation. The data are shown in Fig. 7.

Following the procedure from Rong et al. [9], the normalized diagnostic vectors of the assembly structure are derived and shown in Table 4.

PCA analysis of the measurement data shows the first eigenvalue is $\lambda_1=0.3569$, and the eigenvector corresponding to the 1st eigenvalue is $\mathbf{a}_1=[0.7398\ 0.6729]^T$. Hypothesis tests for each diagnostic vector $\mathbf{d}_i^{(1)}$ in Table 4 against the fault vector $\mathbf{a}_1$ are conducted. Set $\alpha=0.01$, thus the decision criteria $\chi^2_{(\alpha,n-1)}=\chi^2_{(0.01,1)}=6.63$. The test results are shown in Table 5.

Based on the decision criteria of the multiple hypothesis tests and the results of test statistics Ω_i's in Table 5, the null hypothesis for diagnostic vectors $\mathbf{d}_2^{(1)}$, $\mathbf{d}_3^{(1)}$ and $\mathbf{d}_4^{(1)}$ are rejected, and $\mathbf{d}_1^{(1)}$ is concluded as the matching diagnostic vector of fault variation pattern $\mathbf{a}_1$. From Table 4, diagnostic vector $\mathbf{d}_1^{(1)}$ corresponds to node 5 in the X direction. Therefore, fault $p1$ (at node 5 in the X direction) was identified as the root cause of the dimensional problem described by the analyzed data. Following the same approach a few more case studies were identified and root causes were isolated.

6 Conclusion

The least squares (LS) approach can be used to estimate faults in compliant assembly processes based on the measured assembly responses. The diagnostic matrix for the assembly structure may be ill-conditioned because of the compliant characteristics of the structure itself. If it occurs, the LS solutions are imprecise and the diagnostic performance is poor.

In this paper, an adjusted LS approach is proposed. In this approach, the diagnostic matrix is partitioned by using singular value decomposition. The adjusted LS approach is able to precisely estimate certain linear combinations of the faults that generate similar fault signatures. The comparison results between the regular LS and the adjusted LS indicate that the performance of the adjusted LS overperforms the regular LS for *type two* faults. Meanwhile, they both have the same performance for *type one* faults. The Monte Carlo simulation results show that both type I error (false alarm) and type II error (miss detection) are very satisfactory for the adjusted LS approach. In order to isolate the *type two* faults, a two-stage assembly model is further developed. By adding additional measurements following this two-stage model, the faults can be isolated by using a predeveloped PCA based diagnostic algorithm.

Appendix: Prove $\tilde{\mathbf{D}}_2=\xi_1\mathbf{u}_1^{(\mathbf{D}_2)}\mathbf{v}_1^{(\mathbf{D}_2)T}$ is the Best Rank-Sufficient Approximation of D

Let $\mathbf{A}\in\mathbf{R}^{n\times m}$, from SVD, we have

$$\mathbf{U}^T\mathbf{A}\mathbf{V}=\mathrm{diag}(\xi_1,\cdots,\xi_k), \qquad (A\text{-}1)$$

where $k=\min\{n,m\}=\mathrm{rank}(\mathbf{A})$, and $\xi_1\geqslant\xi_2\geqslant\ldots\geqslant\xi_k\geqslant 0$.

If exist matrix $\mathbf{A_r}$, $(\mathbf{A_r})=r<k$, and

$$\mathbf{A_r}=\sum_{i=1}^{r}\xi_i u_i \nu_i^T \qquad (A\text{-}2)$$

From Eq. (A-2), we have

$$\mathbf{U}^T\mathbf{A_r}\mathbf{V}=\mathrm{diag}(\xi_1,\cdots,\xi_r,0,\cdots,0) \qquad (A\text{-}3)$$

From Golub and Van Loan [14],

$$\begin{aligned}\|\mathbf{A}-\mathbf{A_r}\|_2&=\|\mathbf{U}^T(\mathbf{A}-\mathbf{A_r})\mathbf{V}\|_2\\&=\|\mathrm{diag}(0,\cdots,0,\xi_{r+1},\cdots,\xi_k)\|_2\\&=\xi_{r+1}\end{aligned} \qquad (A\text{-}4)$$

Notice that, the singular values are arranged in a decreased sequence, so all $\xi_i\in\mathbf{A_r}$ are larger than ξ_{r+1}. Thus, $\mathbf{A_r}$ is the best rank-sufficient approximation of **A** in the senses of minimum 2-norm. The proving of $\tilde{\mathbf{D}}_2$ follows.

Acknowledgment

This research is partially supported by the National Science Foundation (NSF) Grant: NSF-DMI 9713654; by General Motors Satellite Research Laboratory at the University of Michigan, Advanced Body Division, and by the Bascom Graduate School at the University of Wisconsin-Madison.

References

[1] Ceglarek, D., and Shi, J., 1996, ''Fixture Failure Diagnosis for Autobody Assembly Using Pattern Recognition,'' ASME J. Eng. Industry, **118**, No. 1, pp. 55–66.
[2] Jin, J., and Shi, J., 1999, ''State Space Modeling of Sheet Metal Assembly for Dimensional Control,'' ASME J. Manuf. Sci. Eng., **121**, No. 4, pp. 756–762.
[3] Ding, Y., Ceglarek, D., and Shi, J., 2000, ''Modeling and Diagnostics of Multi-Station Assembly Processes: Part II—Fault Diagnostics,'' *2000 Japan-USA Symposium on Flexible Automation*, Ann Arbor, MI.
[4] Wang, Y., and Nagarkar, S. R., 1999, ''Locator and Sensor Placement for Automated Coordinate Checking Fixtures,'' ASME J. Manuf. Sci. Eng., **121**, No. 4, pp. 709–719.
[5] Khan, A., and Ceglarek, D., 2000, ''Optimization for Fault Diagnosis in Muiti-Fixture Assembly Systems with Distributed Sensind,'' ASME J. Manuf. Sci. Eng., **122**, No. 1, pp. 215–226.
[6] Khan, A., Ceglarek, D., Shi, J., Ni, J., and Woo, T. C., 1999, ''Sensor Optimization for Fault Diagnosis in Single Fixture Systems: A Methodology,'' ASME J. Manuf. Sci. Eng., **121**, No. 1, pp. 109–117.
[7] Shiu, B., Ceglarek, D., and Shi, J., 1997, ''Flexible Beam-Based Modeling of Sheet Metal Assembly for Dimensional Control,'' Trans. of NAMRI, **XXV**, pp. 49–54.
[8] Chang, M. H., and Gossard, D. C., 1997, ''Modeling the Assembly of Compliant, Non-ideal Parts,'' Comput.-Aided Des., **29**, No. 10, pp. 701–708.
[9] Rong, Q., Ceglarek, D., and Shi, J., 2000, ''Dimensional Fault Diagnosis for Compliant Beam Structure Assemblies,'' ASME J. Manuf. Sci. Eng., **122**, No. 4, 773–780.
[10] Apley, D., and Shi, J., 1998, ''Diagnosis of Multiple Fixture Faults in Panel Assembly,'' ASME J. Manuf. Sci. Eng., **120**, No. 4, pp. 793–801.
[11] Beck, J. V., and Arnold, K. J., 1977, *Parameter Estimation in Engineering and Science*, John Wiley & Sons.
[12] Koh, C. G., and See, L. M., 1994, ''Identification and Uncertainty Estimation of Structural Parameters,'' J. Eng. Mech., **120**, No. 6, pp. 1219–1236.

[13] Beck, J. V., and Woodbury, K. A., 1998, ''Inverse Problems and Parameter Estimation: Integration of Measurements and Analysis,'' Meas. Sci. Technol., **9**, pp. 839–847.
[14] Silvey, S. D., 1969, ''Multicollinearity and Imprecise Estimation,'' Technometrics, No. 3, pp. 539–552.
[15] Golub, G. H., and Van Loan, C. F., 1983, *Matrix Computations*, The Johns Hopkins University Press.
[16] Van Huffel, S., and Vandewalle, J., 1987, ''Subset Selection Using the Total Least Squares Approach in Collinearity Problems with Errors in the Variables,'' Linear Algebr. Appl., **88/89**, pp. 695–714.
[17] Rong, Q., Ceglarek, D., and Shi, J., 1999, ''Methodology of Multiple Faults Diagnostics in Compliant Structure Assemblies,'' University of Michipan-Ann Arbor and the University of Wisconsin-Madison.
[18] Hasan, W. M., and Viola, E., 1997, ''Use of Singular Value Decomposition Method to Detect Ill-conditioning of Structural Identification Problems,'' Comput. Struct., **63**, No. 2, pp. 267–275.
[19] Klema, V. C., and Laub, A. J., 1980, ''The Singular Value Decomposition: Its Computation and Some Applications,'' IEEE Trans. Autom. Control, **AC-25**, No. 2, pp. 164–173.
[20] Penny, J. E. T., Friswell, M. I., and Garvey, S. D., 1994, ''Automatic Choice of Measurement Locations for Dynamic Testing,'' AIAA J., **32**, pp. 407–414.
[21] Mottershead, J. E., and Foster, C. D., 1991, ''On the Treatment of Ill-conditioning in Spatial Parameter Estimation from Measured Vibration Data,'' Mech. Syst. Signal Process., **5**, pp. 139–154.
[22] Johnson, R. A., and Wichern, D. W., 1992, *Applied Multivariate Statistical Analysis*, Prentice Hall, New Jersey.
[23] Fedorov, V. V., 1972, *Theory of Optimal Experiments*, Academic Press, New York.