Imbalance Estimation for Speed-Varying Rigid Rotors Using Time-Varying Observer

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Rigid rotor dynamic model is widely used to model rotating machinery. In this paper, a speed-varying transient rigid rotor model is developed in the state space form. The states of this model are augmented to include imbalance forces and moments. A time-varying observer can then be designed for the augmented system by using canonical transformation. After obtaining an estimation of the imbalance forces and moments as the states of the augmented system, the estimated imbalance can be directly calculated. This estimation method can be used in the active vibration control or active balancing schemes for a rigid rotor. [DOI: 10.1115/1.1409935]

1 Introduction

Rotating machines, including machining tools, industrial turbomachinery, and aircraft gas turbine engines, are commonly used in industry. Vibration caused by mass imbalance is an important factor limiting the performance and fatigue life of a rotating system. There are two major categories of control methods for the suppression of excessive imbalance-induced vibration. The first category is balancing. This method tries to eliminate the rotor imbalance. Off-line balancing methods (Wowk [1]) are widely used in practice, but they are usually time-consuming and cannot be used if the distribution of imbalance changes during operation. Some researchers (Gosiewski [2,3], Van De Vegte and Lake [4], Van De Vegte [5]) tried to use some kind of mass redistribution device to actively balance the rotating systems during operation. The method directly suppresses the imbalance-induced vibration or the force transmitted to the base by using lateral force actuators such as magnetic bearings (Knospe et al. [6], Lum et al. [7], Herzog et al. [8]). All of the above research concentrated on the constant spin-speed case: the so-called "steady-state" case. Because of this constant spin-speed assumption, the influence coefficient method is used to model the rotor system. The whole rotor dynamics are lumped into constants: the influence coefficients. The imbalance is embedded in the influence coefficients. The estimation of the imbalance, which is very important in both the balancing and active vibration control schemes, is fulfilled by the estimation of influence coefficients.

An alternative way to estimate the system imbalance is provided by Reinig and Desrochers [9] and Zhu et al. [10]. In their methods, the states of the rotor dynamic system are augmented to include the imbalance forces. An observer is then used to estimate the augmented states set. Again, their methods only deal with the constant spin-speed case. Hence, the enlarged system is a time invariant linear system. The traditional Luenberger observer (Luenberger [11]) can be used to estimate the imbalance forces.

Besides the constant rotating speed case, the imbalance vibration control needs to be completed during speed-varying transient time to save time and improve performance in some other cases. For example, a machining tool needs to be engaged in cutting as soon as the spindle goes into steady state in high-speed machining. The vibration control has to be active during the acceleration period to reduce the effect on the cutting cycle time. Although several researchers (Knospe et al. [12]) have pointed out how to conduct imbalance vibration control during the startup through the critical speed, their basic method is to interpolate the influence coefficients between different speeds. This is a "quasi-steady" strategy because the requirement of a "steady-state" response is inherited in the influence coefficient method. Very little research work has been reported on the balancing and active control for the rotor system with fast acceleration and low damping ratio. Zhou and Shi [13] found an analytical expression of the imbalance-induced vibration of a rotor system during acceleration. From this analytical expression, a significant suddenly occurring free vibration component can be found in the imbalance-induced vibration if the acceleration is high and the damping is low. The "quasi-steady-state" assumption does not hold under these conditions. Zhou and Shi [14] also proposed a real-time active balancing scheme for fast acceleration case. That scheme is based on the least squares estimation of the system imbalance of the rotor system. In that scheme, a mass redistribution active balancing device is required to excite the system dynamics and hence make the estimation converge fast.

In this paper, a time-varying observer is developed to estimate the imbalance force and the imbalance itself of a rigid rotor system during acceleration. This formulation does not require steady-state response because the rotor is modeled by dynamic differential equations. Moreover, this formulation assumes that the dynamic parameters of rotor system are known and hence no extra excitation is required. This result can be used in both active balancing and direct vibration control schemes for rigid rotors.

This paper consists of five sections. In Section 2, a speed-varying transient dynamic model of a rigid rotor will be presented. It is formulated in a form suitable for imbalance force estimation. This dynamic model is a time-varying linear model due to the gyroscopic effect and the inclusion of the imbalance forces in the states. The time-varying observer design problem is addressed in Section 3. Section 4 gives some simulation results to show the effectiveness of this imbalance estimation scheme. Conclusions are given in the last section.

2 Speed Varying Dynamic Model of Rigid Rotors

The dynamics of rigid rotor are very important because the rigid rotor model has been the model used for many practical rotors. The geometric setup is shown in Fig. 1.

The basic assumptions used in the modeling procedure are:

(a) The rotor is rigid with circular cross section and the imbalance is represented as a concentrated mass on the shaft. Any static and dynamic imbalance can be represented by the mass and the position of this concentrated imbalance.

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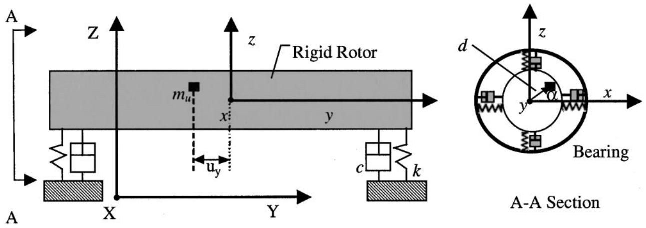

Fig. 1 The geometric setup of rigid rotor model

(b) The bearings are modeled as a set of linear springs and dampers in two orthogonal directions with the same spring rates and damping coefficients. The bearings locate at each end of the shaft and the mass center of the rotor is located at the midpoint between the two bearings. This assumption simplifies the dynamic model because the translational motion and the conical motion of the rotor are uncoupled under this assumption. The translational motion in the two orthogonal directions (X and Z) is also uncoupled under this assumption. This simplified rigid rotor model is used in the following derivation, but the extension to a more general rigid rotor model is quite straightforward.

(c) The angular acceleration of the rotational motion is assumed a known constant. The linear acceleration speed profile is common for machining spindles. We also assume that the rotating angle, speed, and acceleration are known. They can be easily measured by attaching an encoder to the spindle.

(d) The lateral vibration motions are assumed small to simplify the dynamics.

Two coordinate systems are used in the derivation: the body-fixed coordinate $oxyz$ and the inertial coordinate $OXYZ$. The body-fixed y-axis is the rotating axis of the shaft and x- and z- axes are defined by the other two principal inertia axes of the rotor. The origin of xyz is selected as the geometric center of the shaft. The XYZ coordinate system is the stationary coordinate and coincides with the xyz coordinate system when the body is at rest.

The elastic and damping forces provided by the bearings are

$$\begin{cases} \mathbf{F}_k = -2k[R_X \quad R_Z]^T, & \mathbf{M}_k = -kL^2/2[\theta \quad \psi]^T/2 \\ \mathbf{F}_c = -2c[\dot{R}_X \quad \dot{R}_Z]^T, & \mathbf{M}_c = -cL^2/2[\dot{\theta} \quad \dot{\psi}]^T/2 \end{cases}. \tag{1}$$

The equation of motion can be obtained by using Newton's law for a rigid body

$$\begin{cases} I_t\ddot{\theta} + cL^2\dot{\theta}/2 + kL^2\theta/2 - I_p\dot{\psi}\dot{\phi} - I_p\psi\ddot{\phi} = m_u d(\dot{\phi}^2 \sin(\alpha-\phi) + \ddot{\phi}\cos(\alpha-\phi))u_y \\ I_t\ddot{\psi} + cL^2\dot{\psi}/2 + kL^2\psi/2 + I_p\dot{\phi}\dot{\theta} + I_p\ddot{\phi}\theta = m_u d(-\dot{\phi}^2\cos(\alpha-\phi) + \ddot{\phi}\sin(\alpha-\phi))u_y \\ m\ddot{R}_X + 2c\dot{R}_X + 2kR_X = -m_u d(\ddot{\phi}\sin(\alpha-\phi) - \dot{\phi}^2\cos(\alpha-\phi)) \\ m\ddot{R}_Z + 2c\dot{R}_Z + 2kR_Z = m_u d(\dot{\phi}^2\sin(\alpha-\phi) + \ddot{\phi}\cos(\alpha-\phi)) \end{cases}. \tag{2}$$

Using the substitution of $u_x = d\cos\alpha$ and $u_z = d\sin\alpha$ and rewriting Eq. (2) into state space form, we can get the state space model for rigid rotor,

$$\frac{d}{dt}\begin{bmatrix} R_X \\ R_Z \\ \theta \\ \psi \\ \dot{R}_X \\ \dot{R}_Z \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{2k}{m} & 0 & 0 & 0 & -\frac{2c}{m} & 0 & 0 & 0 \\ 0 & -\frac{2k}{m} & 0 & 0 & 0 & -\frac{2c}{m} & 0 & 0 \\ 0 & 0 & -\frac{kL^2}{2I_t} & \frac{I_p\ddot{\phi}}{I_t} & 0 & 0 & -\frac{cL^2}{2I_t} & \frac{I_p\dot{\phi}}{I_t} \\ 0 & 0 & -\frac{I_p\ddot{\phi}}{I_t} & -\frac{kL^2}{2I_t} & 0 & 0 & -\frac{I_p\dot{\phi}}{I_t} & -\frac{cL^2}{2I_t} \end{bmatrix}}_{\mathbf{A}(t)} \begin{bmatrix} R_X \\ R_Z \\ \theta \\ \psi \\ \dot{R}_X \\ \dot{R}_Z \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{-m_u u_z}{m} & \frac{m_u u_x}{m} \\ \frac{m_u u_x}{m} & \frac{m_u u_z}{m} \\ \frac{m_u u_x u_y}{I_t} & \frac{m_u u_y u_z}{I_t} \\ \frac{m_u u_y u_z}{I_t} & \frac{-m_u u_x u_y}{I_t} \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \tag{3}$$

where $f_1 = \ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi$, $f_2 = \ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi$, are known functions of time.

Observing that f_2 and f_1 are the real and imaginary parts of a complex number $\mathbf{F} = (\dot{\phi}^2 + i\ddot{\phi})e^{-i\phi}$ (actually, $\mathbf{F} = -d^2(e^{-i\phi})/dt^2$) and

$$\frac{d}{dt}\begin{pmatrix} \mathbf{F} \\ \dot{\mathbf{F}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -3i\ddot{\phi} & -\dot{\phi}i \end{pmatrix}\begin{pmatrix} \mathbf{F} \\ \dot{\mathbf{F}} \end{pmatrix}, \tag{4}$$

we can augment the state space model and make the imbalance force be states. In fact,

$$\begin{bmatrix} \frac{-m_u u_z}{m} & \frac{m_u u_x}{m} \\ \frac{m_u u_x}{m} & \frac{m_u u_z}{m} \\ \frac{m_u u_x u_y}{I_t} & \frac{m_u u_y u_z}{I_t} \\ \frac{m_u u_y u_z}{I_t} & \frac{-m_u u_x u_y}{I_t} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\left\{\left(\frac{m_u u_x}{m} + \frac{m_u u_z}{m} i\right)\mathbf{F}\right\} \\ \mathrm{Im}\left\{\left(\frac{m_u u_x}{m} + \frac{m_u u_z}{m} i\right)\mathbf{F}\right\} \\ \mathrm{Re}\left\{\left(\frac{m_u u_y u_z}{I_t} + \frac{-m_u u_x u_y}{I_t} i\right)\mathbf{F}\right\} \\ \mathrm{Im}\left\{\left(\frac{m_u u_y u_z}{I_t} + \frac{-m_u u_x u_y}{I_t} i\right)\mathbf{F}\right\} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{\mathbf{F}_1'\} \\ \mathrm{Im}\{\mathbf{F}_1'\} \\ \mathrm{Re}\{\mathbf{F}_2'\} \\ \mathrm{Im}\{\mathbf{F}_2'\} \end{bmatrix}. \tag{5}$$

$\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real and imaginary part of a complex number. $\mathbf{F}_1'$ and $\mathbf{F}_2'$ are $\mathbf{F}$ multiplied by complex constants as defined in Eq. (5). It is obvious that $\mathbf{F}_1'$ and $\mathbf{F}_2'$ also satisfy Eq. (4). Equation (4) can be rewritten in real domain as

$$\frac{d}{dt}\begin{pmatrix} f_r \\ f_i \\ \dot{f}_r \\ \dot{f}_i \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3\ddot{\phi} & 0 & \dot{\phi} \\ -3\ddot{\phi} & 0 & -\dot{\phi} & 0 \end{pmatrix}}_{\mathbf{A}_1(t)} \begin{pmatrix} f_r \\ f_i \\ \dot{f}_r \\ \dot{f}_i \end{pmatrix}, \tag{6}$$

where f_r and f_i are the real and imaginary part of $\mathbf{F}$. Combining Eq. (3) through Eq. (6), the augmented state space model of the rigid rotor is

$$\frac{d}{dt}\mathbf{x} = \begin{pmatrix} \mathbf{A}(t) & \mathbf{Y}_1 & \mathbf{Y}_2 \\ 0 & \mathbf{A}_1(t) & 0 \\ 0 & 0 & \mathbf{A}_1(t) \end{pmatrix}\mathbf{x}, \qquad \mathbf{y} = [R_X \quad R_Z \quad \theta \quad \psi]^T \tag{7}$$

where $\mathbf{x} = [R_X\, R_Z\, \theta\, \psi\, \dot{R}_X\, \dot{R}_Z\, \dot{\theta}\, \dot{\psi}\, f_{1r}\, f_{1i}\, \dot{f}_{1r}\, \dot{f}_{1i}\, f_{2r}\, f_{2i}\, \dot{f}_{2r}\, \dot{f}_{2i}]^T$, f_{1r}, f_{1i} and f_{2r}, f_{2i} are the real and imaginary part of $\mathbf{F}_1'$ and $\mathbf{F}_2'$, respectively. $\mathbf{A}(t)$ and $\mathbf{A}_1(t)$ are the matrices shown in Eq. (3) and Eq. (6). $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are defined as

$$\mathbf{Y}_1 = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix} \quad \mathbf{Y}_2 = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&0&0&0 \\ 0&1&0&0 \end{bmatrix}.$$

Remarks:

- The system imbalance m_u, u_x, u_y, and u_z are not shown explicitly in Eq. (7). They are implicitly included in the model as the initial conditions of $[f_{1r}\, f_{1i}\, f_{2r}\, f_{2i}]$.
- Since this paper concentrates on the imbalance estimation problem, the control input (either lateral forces or balancer-induced forces) is not included in the model. It is straightforward to include these forces.
- The output of the model is the displacement of the mass center and the swinging angle of the rotor about X and Z direction. Since the rotor is rigid, these can be obtained by measuring the motion of any two non-coplanar points on the rotor.
- The model is a linear time variant model because the rotating speed is included in the dynamic matrices $\mathbf{A}$ and $\mathbf{A}_1$.

Using this augmented model, the imbalance estimation problem is transformed to a state estimation problem. A linear observer can give the estimation of states from incomplete states measurements.

3 Time Varying Observer Design

It is well known that the states of a time-invariant linear system can be reconstructed by a linear time-invariant observer. An enormous body of literature has been published on this topic. An excellent review can be found in O'Reilly [15]. Compared to the time-invariant observers, relatively few papers (Wolovich [16], Yuksel and Bongiorno [17], Nguyen and Lee [18], Shafai and Carroll [19]) deal with observers for time-varying systems, such as the dynamic system of Eq. (7). In this paper, the design of time varying observers follows the basic steps in Nguyen and Lee [18].

The essential results of their work are that: (1) a linear time-varying system can be transformed into an observability canonical form under certain conditions; (2) the states of the canonical system can be estimated by a full order observer; (3) the states of the original system can be calculated from the output of the full order observer. A brief review of these results is listed as follows.

The time varying system is represented by the state space model

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t), \qquad \mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) \tag{8}$$

where $\mathbf{x}(t)$, $\mathbf{u}(t)$, and $\mathbf{y}(t)$ are $(n\times 1)$, $(p\times 1)$, and $(q\times 1)$ vectors and $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are matrices with appropriate dimensions. For this system, a full-order observer can be in the form

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{F}(t)\hat{\mathbf{x}}(t) + \mathbf{G}(t)\mathbf{y}(t) + \mathbf{H}(t)\mathbf{u}(t). \tag{9}$$

This observer is an asymptotic identity state observer for system Eq. (8) if

$$\mathbf{H}(t) = \mathbf{B}(t) \quad \text{and} \quad \mathbf{F} = \mathbf{A}(t) - \mathbf{G}(t)\mathbf{C}(t) \tag{10}$$

$\mathbf{F}$ here is a constant matrix with stable eigenvalues. This result can be clearly seen by substituting Eqs. (8) and (10) into Eq. (9), which yields

$$\dot{\hat{\mathbf{x}}} - \dot{\mathbf{x}} = \mathbf{F}(\hat{\mathbf{x}} - \mathbf{x}). \tag{11}$$

The convergence rate of the observer depends on the eigenvalues of matrix $\mathbf{F}$.

Given a general $\mathbf{A}(t)$, $\mathbf{C}(t)$, and the predetermined eigenvalues of $\mathbf{F}$, it is difficult to find $\mathbf{G}(t)$. However, the procedure to find $\mathbf{G}(t)$ for an observability canonical form is simple. Under certain conditions, the system Eq. (8) can be transformed into an observability form Eq. (12) by a Lyapunov transformation.

$$\dot{\bar{\mathbf{x}}}(t) = \overline{\mathbf{A}}(t)\overline{\mathbf{x}}(t) + \overline{\mathbf{B}}(t)\mathbf{u}(t), \qquad \mathbf{y}(t) = \overline{\mathbf{C}}(t)\mathbf{x}(t) \tag{12}$$

where $\overline{\mathbf{A}}(t)$ and $\overline{\mathbf{C}}(t)$ are in the forms

$$\overline{\mathbf{A}}(t) = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1q} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2q} \\ \vdots & \vdots & \vdots & \\ \mathbf{A}_{q1} & \mathbf{A}_{q2} & \cdots & \mathbf{A}_{qq} \end{bmatrix},$$

where

$$\mathbf{A}_{ii} = \left[\, X_{n_i\times 1} \;\middle|\; \begin{matrix} I_{ni-1} \\ \cdots\cdots \\ \mathbf{0}_{1\times(n_i-1)} \end{matrix} \right],$$

$\mathbf{A}_{ij}=[X_{n_i\times 1}\mathbf{0}_{n_i\times(n_j-1)}]$ for $i,j=1,2,\ldots,q$ and $i\neq j$, n_i and n_j are the observability indices associated with the ith and jth row of $\mathbf{C}(t)$ matrix, I_j is $j\times j$ identity matrix, $\mathbf{0}_{i\times j}$ is $i\times j$ zero matrix,

$$\bar{\mathbf{C}}(t)=\left[\begin{array}{cccc|cccc|cccc} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\ X & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\ X & 0 & \cdots & 0 & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & 0\\ X & 0 & \cdots & 0 & X & 0 & \cdots & 0 & 0 & 0 & \cdots & 0\\ X & 0 & \cdots & 0 & X & 0 & \cdots & 0 & 1 & 0 & \cdots & 0\\ \multicolumn{4}{c}{\underbrace{\qquad\qquad}_{n_1}} & \multicolumn{4}{c}{\underbrace{\qquad\qquad}_{n_2}} & \multicolumn{4}{c}{\underbrace{\qquad\qquad}_{n_q}} \end{array}\right]\Bigg\} q.$$

X represents a nonzero number.

Nguyen and Lee [10] provide a simple method to find $\bar{\mathbf{G}}(t)$ that satisfies $\bar{\mathbf{F}}=\bar{\mathbf{A}}(t)-\bar{\mathbf{G}}(t)\bar{\mathbf{C}}(t)$, where $\bar{\mathbf{F}}$ is in observability canonical form with desired eigenvalues. More clearly, $\bar{\mathbf{F}}$ is in the form

$$\bar{\mathbf{F}}=\left[\begin{array}{c|c} -\beta_0 & I_{n-1}\\ -\beta_1 & \cdots\cdots\cdots\\ \vdots & \\ -\beta_{n-1} & \mathbf{0}_{1\times(n-1)} \end{array}\right]. \tag{13}$$

After obtaining the $\bar{\mathbf{G}}(t)$ matrix, we can get an asymptotic full-order observer for system (12). Since Lyapunov transformation is a nonsingular transformation, it is straightforward to estimate the states of the original system (8) if the estimations of $\bar{\mathbf{x}}(t)$ are available.

Some other observer design issues are not addressed in Nguyen and Lee [18]. The first one is the selection of the observer eigenvalues. The decay rate of the observer error is determined by the eigenvalues of the error dynamic matrix $\bar{\mathbf{F}}$. Theoretically, we can pick eigenvalues at the far left of the complex plane for $\bar{\mathbf{F}}$ to make the observer outputs converge to the true states very fast. However, fast observer poles may enlarge the effects of sensor noise. The selection of the eigenvalues is a compromise between the fast response speed and good noise smoothing capability.

Besides the decaying rate, another concern in the observer design is the transient performance. Noticing the error dynamic equation (Eq. (11)), the overall error response of the observer is determined by the $\mathbf{F}$ matrix including both the eigenvalues and the structure of $\mathbf{F}$. A large transient error will take more time to die out if other conditions are the same. Moreover, very large transient error could cause numerical problems in the observer. Hence, small transient error of the observer is desired.

The transient performance of the observer can be predicted by the fact stated in Chen [20]: For a companion form dynamic matrix of order n, if all its eigenvalues are distinct, the largest magnitude in transient. is roughly proportional to $(|\lambda|_{\max})^{n-1}$, where $|\lambda|_{\max}$ is the magnitude of the largest eigenvalue. Hence, in order to have a small transient error, the order of the largest companion-form block in the error dynamic matrix $\bar{\mathbf{F}}$ should be kept as small as possible. For a low dimensional dynamic system, the transient performance of the observer using the $\bar{\mathbf{F}}$ matrix in Eq. (13) is satisfactory. But for a high dimensional dynamic system, such as the augmented rigid rotor model with dimension 16, the transient response of the observer will be too large. For example, if the largest magnitude of the eigenvalues of $\bar{\mathbf{F}}$ is 10, then the largest transient response magnitude will be close to the order 10^{15}. To deal with this problem, we can break the matrix $\bar{\mathbf{F}}$ into small companion-form block matrix. The dimension of each block is determined by the corresponding observability index. The new matrix $\bar{\mathbf{F}}'$ has the similar structure of the $\bar{\mathbf{A}}(t)$,

$$\bar{\mathbf{F}}'=\begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} & \cdots & \mathbf{F}_{1q}\\ \mathbf{F}_{21} & \mathbf{F}_{22} & \cdots & \mathbf{F}_{2q}\\ \vdots & \vdots & & \vdots\\ \mathbf{F}_{q1} & \mathbf{F}_{q2} & \cdots & \mathbf{F}_{qq} \end{bmatrix},$$

where (14)

$$\mathbf{F}_{ii}=\left[\begin{array}{c|c} -\beta_{i,0} & I_{n_i-1}\\ -\beta_{i,1} & \cdots\cdots\\ \vdots & \\ -\beta_{i,n_i-1} & 0_{1\times(n_i-1)} \end{array}\right],$$

$$\mathbf{F}_{ij}=0_{n_i\times n_j},\quad \text{for } i,j=1,2,\ldots,q \text{ and } i\neq j.$$

The β's in Eq. (14) are determined by the desired eigenvalues. Actually, they are the coefficients of the characteristic equation of each companion form blocks.

To design a full order observer for $\bar{\mathbf{F}}'$, we need to solve the matrix equation

$$\bar{\mathbf{G}}(t)\bar{\mathbf{C}}(t)=\bar{\mathbf{A}}(t)-\bar{\mathbf{F}}'(t) \tag{15}$$

for $\bar{\mathbf{G}}(t)$. If all-zero columns are eliminated from both left and right sides of Eq. (15) by noting the special structure of $\bar{\mathbf{F}}'$, $\bar{\mathbf{A}}(t)$, and $\bar{\mathbf{C}}(t)$, Eq. (15) changes to

$$\bar{\mathbf{G}}_{n\times q}(t)\bar{\mathbf{C}}'_{q\times q}(t)=\mathbf{D}_{n\times q}, \tag{16}$$

where the columns of $\mathbf{D}$ consist of the nonzero columns of $\bar{\mathbf{A}}(t)-\bar{\mathbf{F}}'(t)$ and the columns of $\bar{\mathbf{C}}'$ consists of the nonzero columns of $\bar{\mathbf{C}}(t)$. It is clear that $\bar{\mathbf{C}}'$ is nonsingular. Equation (16) can be solved by

$$\bar{\mathbf{G}}(t)=\mathbf{D}(t)\bar{\mathbf{C}}'^{-1}(t). \tag{17}$$

The design procedures are summarized as follows:

1 Transform the original state space model (8) into observability canonical form (12) by a Lyapunov transformation $\mathbf{x}(t)=\mathbf{P}(t)\bar{\mathbf{x}}(t)$, where $\mathbf{P}(t)$ is a nonsingular matrix.
2 Select n distinct desired eigenvalues for the estimator.
3 Group these n values into q groups. Each group has n_i ($i=1\ldots q$) elements. n_i is the observability index of the ith subsystem. From these q groups of eigenvalues, get q groups of $\beta_{i,0..n_i-1}$.
4 Obtain $\bar{\mathbf{F}}'$ in Eq. (14).
5 Obtain $\bar{\mathbf{G}}(t)$ by the formula of Eq. (17).
6 Use the full-order estimator (9) to estimate the states of (12), where $\mathbf{F}(t)=\bar{\mathbf{F}}'$, $\mathbf{H}(t)=\bar{\mathbf{B}}(t)$, and $\mathbf{G}(t)=\bar{\mathbf{G}}(t)$.
7 Denoting the estimation in step 6 as $\hat{\bar{\mathbf{x}}}$, the estimation of the original system is $\hat{\mathbf{x}}=\mathbf{P}(t)\hat{\bar{\mathbf{x}}}$.

The rigid rotor system (7) can be transformed into an observability companion form. This is confirmed by the numerical example in Section 4 and also can be easily checked analytically by substituting $\mathbf{A}(t)$ and $\mathbf{C}(t)$ matrices in (7) into the checking conditions (Nguyen and Lee [18]). Following the above procedures, we can design a time-varying observer for the rigid rotor whose estimation error will go to zero asymptotically while the transient performance is satisfactory. The estimation of imbalance force can be obtained by this observer.

From the imbalance force estimation, the estimated imbalance itself can be obtained by direct algebraic calculation. If we denote $\theta_1=-m_u u_z/m$, $\theta_2=m_u u_x/m$, $\theta_3=m_u u_x u_y/I_t$, $\theta_4=m_u u_y u_z/I_t$, this method can be formulized as follows:

$$\begin{bmatrix} \theta_1 & \theta_2\\ \theta_2 & -\theta_1\\ \theta_3 & \theta_4\\ \theta_4 & -\theta_3 \end{bmatrix}\begin{bmatrix} f_1\\ f_2 \end{bmatrix}=\begin{bmatrix} \hat{f}_{1r}\\ \hat{f}_{1i}\\ \hat{f}_{2r}\\ \hat{f}_{2i} \end{bmatrix}, \tag{18}$$

where f_1 and f_2 are the same as in Eq. (3), and $[\hat{f}_{1r}\,\hat{f}_{1i}\,\hat{f}_{2r}\,\hat{f}_{2i}]^T$ are the estimated imbalance force or moment. Equation (18) can be transformed into

$$\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix}=\begin{bmatrix}f_1 & f_2 & 0 & 0\\ -f_2 & f_1 & 0 & 0\\ 0 & 0 & f_1 & f_2\\ 0 & 0 & -f_2 & f_1\end{bmatrix}^{-1}\begin{bmatrix}\hat{f}_{1r}\\ \hat{f}_{1i}\\ \hat{f}_{2r}\\ \hat{f}_{2i}\end{bmatrix}. \tag{19}$$

Since the absolute determinant of $[\begin{smallmatrix}f_1 & f_2\\ -f_2 & f_1\end{smallmatrix}]$ is $(\dot{\phi}^4+\ddot{\phi}^2)$, which is always nonzero, we do not need to worry about the singularities of the calculation during acceleration. The advantage of this method is its simplicity. The disadvantage is that the effect of noise on the estimation will be presented in the imbalance estimation directly.

The simulation result is presented in the next section to illustrate the above procedures and methods.

4 Simulation

The simulation is done with the parameters $L=0.5$ m, $r=0.1$ m, $k=1\times10^7$ N/m, $c=1000$ Ns/m, $m_u=0.5$ kg, $[u_x u_y u_z]=[0.08,0.2,0.05]$ m in the body-fixed coordinate system, and $\ddot{\phi}=100$ rad$\cdot$s^{-2}. Following the time-varying observer design procedures, we first obtain the Lyapunov transformation matrix $\mathbf{P}(t)$. Then, the equivalent observability canonical form of system Eq. (7) can be obtained. In this simulation, the four groups of eigenvalues of the $\overline{\mathbf{F}}'$ matrix are selected as $[-11\ -12\ -13\ -14]$, $[-15\ -16\ -17\ -18]$, $[-19\ -20\ -21\ -22]$, and $[-23\ -24\ -25\ -26]$. The $\overline{\mathbf{G}}(t)$ matrix can be obtained by using the formula Eq. (17). All these matrices are listed in the Appendix.

The original system Eq. (7) and the observer dynamic system in step 6 are solved by the Runge-Kutta method. The imbalance-induced translational and conical swinging motions are shown in Fig. 2. Figure 2(*a*) shows the resonant peak of the translational motion is reached at about 400 rad$\cdot$s^{-1} and Fig. 2(*b*) shows the resonant peak of the conical motion is reached at about 700 rad$\cdot$s^{-1}.

The initial states of the observer are all zeros. The output of the observer dynamic system is multiplied by $\mathbf{P}(t)$ to get the state estimation of the original system. The error of the imbalance force estimation is shown in Fig. 3. As stated in Section 3, the response of the observer is determined by the eigenvalues of $\overline{\mathbf{F}}'$ and the transient response magnitude. Since the eigenvalues of $\overline{\mathbf{F}}'$ are far from the imaginary axis, the decay rate of the estimation error is high. The transient magnitude is also satisfactory because the block canonical structure is selected for $\overline{\mathbf{F}}'$ matrix. Therefore, the observer error converges to zero well before the rotating speed hits the critical speeds. This property is desired for active balancing purpose.

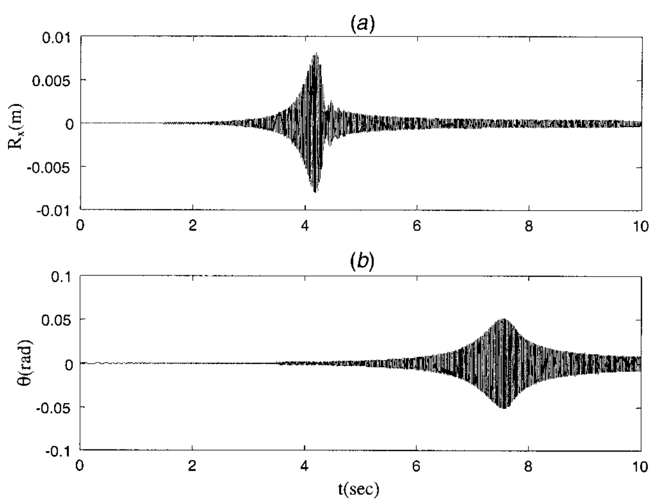

Fig. 2 The imbalance-induced vibration of rigid rotor

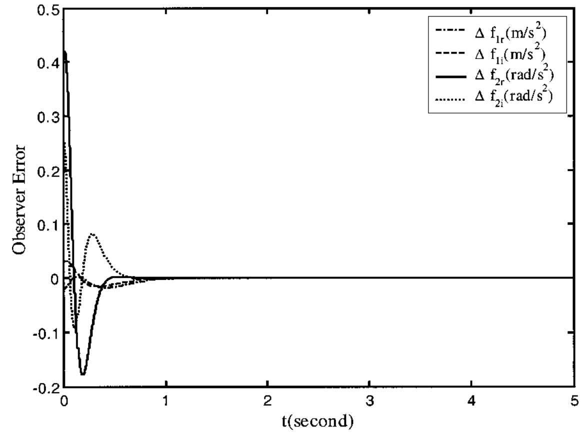

Fig. 3 The observer error

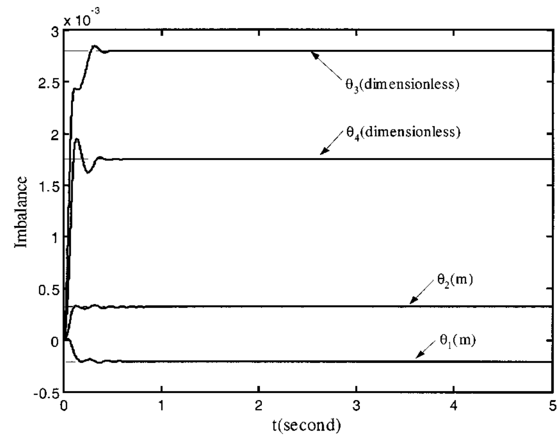

Fig. 4 The imbalance estimation by solving algebraic equation

From the imbalance force estimation, the system imbalance can be estimated using direct algebraic calculation Eq. (19). The imbalance estimation results are shown in Fig. 4. The true values of the imbalance parameters are shown by dashed lines in Fig. 4. The result shows that the imbalance estimation converges to the true value quickly.

5 Conclusions

This paper presents a new method of the imbalance estimation for the rigid rotor during acceleration. The acceleration will excite the dynamics of the rotor. Therefore, the static method such as the influence coefficient method cannot be used in this case. By viewing the imbalance forces and moments as outputs of a dynamic system, we can augment the states of the rigid rotor model to include the imbalance forces and moments. The resulting system is an autonomous time-varying linear system. Under the assumption that the displacement and the swinging angle of the rigid rotor can be measured, a time-varying observer can be designed by canonical transformation of the original system. Transient performance of the observer is improved and the upper bound of the magnitude is determined in this paper by selecting a special structure of the estimation error dynamic matrix. The simulation results show that the estimation error converges to zero quickly and the transient estimation error is kept small. This estimation method can be used in active vibration control or active balancing schemes for rigid rotor.

This paper concentrates on the imbalance estimation issues. References and Discussions on other issues of real-time active balancing of rotating machinery, such as the rotor dynamic modeling, analysis, and control, actuator and sensor layout, etc, can be found in Zhou and Shi [22].

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Nomenclature

I_p, I_t = the polar, and the diametric moments of inertia of the rotor
L = the length of the rotor
$OXYZ$ = the stationary coordinate system
R_X, R_Z = the displacements of the mass center of the rotor in X and Z directions
$\dot{R}_X$, $\dot{R}_Z$ = the velocities of the mass center of the rotor in X and Z directions
$\ddot{R}_X$, $\ddot{R}_Z$ = the accelerations of the mass center of the rotor in X and Z directions
d, α = the magnitude and angle of the imbalance vector as shown in Fig. 1.
m_u, u_x, u_y, u_z = the mass and the position of the imbalance in body-fixed coordinate xyz
m = the mass of the rotor
c, k = the viscous damping coefficient and the spring rate of the bearings
$oxyz$ = the body-fixed coordinate system
$\phi, \dot{\phi}, \ddot{\phi}$ = the rotating angle, speed, and acceleration of the rotor
ψ, θ = Euler angles to describe the orientation of the body-fixed coordinate in the stationary coordinate. (ψ, θ, ϕ) forms a body 3-1-2 Euler angle set (Kane [21])

Appendix

The explicit expressions of the matrices used in the numerical study are listed as follows.

$$\mathbf{P}(t)=\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
-16.3&1&0&0&100t&0&0&0&0&0&0&0&0&0&0&0\\
-100t&0&0&0&-16.3&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-44&1&0&0&121t&0&0&0\\
0&0&0&0&0&0&0&0&-121t&0&0&0&-44&1&0&0\\
-1\times10^4t^2&0&1&0&100&100t&0&0&0&0&0&0&0&0&0&0\\
-100&-100t&0&0&-1\times10^4t^2&0&1&0&0&0&0&0&0&0&0&0\\
-3\times10^4t&-1\times10^4t^2&0&1&-1\times10^6t^3&200&100t&0&0&0&0&0&0&0&0&0\\
1\times10^6t^3&-200&-100t&0&-3\times10^4t&-1\times10^4t^2&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1\times10^4t^2&0&1&0&100&100t&0&0\\
0&0&0&0&0&0&0&0&-100&-100t&0&0&-1\times10^4t^2&0&1&0\\
0&0&0&0&0&0&0&0&-3\times10^4t&-1\times10^4t^2&0&1&-1\times10^6t^3&200&100t&0\\
0&0&0&0&0&0&0&0&1\times10^6t^3&-200&-100t&0&-3\times10^4t&-1\times10^4t^2&0&1
\end{bmatrix}$$

$$\overline{\mathbf{C}}(t)=\begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0
\end{bmatrix}$$

$$\overline{\mathbf{A}}(t)=\begin{bmatrix}
-16.3&1&0&0&100t&0&0&0&0&0&0&0&0&0&0&0\\
-1.63\times10^5&0&1&0&1630t&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&1630+1.63\times10^7t&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&3.26\times10^7&0&0&0&0&0&0&0&0&0&0&0\\
-100t&0&0&0&-16.3&1&0&0&0&0&0&0&0&0&0&0\\
-1630t&0&0&0&-1.63\times10^5&0&1&0&0&0&0&0&0&0&0&0\\
-1.63\times10^3-1.63\times10^7t&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
-3.3\times10^7&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-44&1&0&0&120t&0&0&0\\
0&0&0&0&0&0&0&0&-4.4\times10^5+2100t^2&0&1&0&4.4\times10^3t&0&0&0\\
0&0&0&0&0&0&0&0&2.1\times10^3t&0&0&1&4400+4.4\times10^7t&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&8.7\times10^7&0&0&0\\
0&0&0&0&0&0&0&0&-1.2\times10^2t&0&0&0&-43&1&0&0\\
0&0&0&0&0&0&0&0&-4.4\times10^3t&0&0&0&-4.4\times10^5+2.1\times10^3t^2&0&1&0\\
0&0&0&0&0&0&0&0&-4400-4.4\times10^7t&0&0&0&2100t&0&0&1\\
0&0&0&0&0&0&0&0&-8.7\times10^7&0&0&0&0&0&0&0
\end{bmatrix}$$

$$
\overline{\mathbf{F}}'=\begin{bmatrix}
-50 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-935 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-7800 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-2.4\times 10^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -66 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1600 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1.8\times 10^4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -7.3\times 10^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -82 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2500 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.4\times 10^4 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.7\times 10^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -98 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3600 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5.9\times 10^4 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.6\times 10^5 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\overline{\mathbf{G}}(t)=\begin{bmatrix}
33.7 & 100t & 0 & 0\\
-1.6\times 10^5 & 1.6\times 10^3 t & 0 & 0\\
7.7\times 10^3 & 1600+1.6\times 10^7 t & 0 & 0\\
2.4\times 10^4 & 3.3\times 10^7 & 0 & 0\\
-100t & 50 & 0 & 0\\
-1.6\times 10^3 t & -1.6\times 10^5 & 0 & 0\\
-1.6\times 10^3-1.6\times 10^7 t & 1.8\times 10^4 & 0 & 0\\
-3.3\times 10^7 & 7.3\times 10^4 & 0 & 0\\
0 & 0 & 38 & 120t\\
0 & 0 & -4.3\times 10^5+2.1\times 10^3 t^2 & 4400t\\
0 & 0 & 2100t+3.4\times 10^4 & 4.4\times 10^3+4.4\times 10^7 t\\
0 & 0 & 1.8\times 10^5 & 8.7\times 10^7\\
0 & 0 & -120t & 54\\
0 & 0 & -4400t & -4.3\times 10^5+2100t^2\\
0 & 0 & -4.4\times 10^3-4.4\times 10^7 t & 2100t+5.9\times 10^4\\
0 & 0 & -8.7\times 10^7 & 3.6\times 10^5
\end{bmatrix}
$$

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