SIMULTANEOUS TOLERANCE SYNTHESIS THROUGH VARIATION PROPAGATION MODELING OF MULTISTAGE MANUFACTURING PROCESSES

Qiang Huang and Jianjun Shi
Department of Industrial and Operations Engineering
University of Michigan
Ann Arbor, MI 48109

ABSTRACT

We developed procedures of modeling tolerance stackup for multi-stage machining process. As demonstrated by a block part machining process, product tolerance and process selection are simultaneously determined by a general two-step optimization procedure. Further, design improvement is performed by analyzing the sensitivity of process parameters.

INTRODUCTION

Tolerance synthesis is a challenging issue in mechanical design. Successful synthesis is mainly affected by two conflicting goals, that is, functional requirements of a mechanical assembly and cost effectiveness of a candidate manufacturing process to fabricate the product. To meet the two goals, traditionally two types of tolerancing activities are conducted sequentially. First, given specifications of resultant or assembly dimensions, design tolerances are allocated to specify the permissible amount of variation for component dimensions. Second, manufacturing tolerances are devised for in-process dimensions in order to select a capable and economical manufacturing process. A huge body of literature can be found for design tolerancing (Taguchi, 1986; Greenwood and Chase, 1987; Lee and Woo, 1990; among others). Relatively fewer studies are seen for manufacturing tolerancing. The major approach is tolerancing charting (Wade, 1967). A good review of the two types of research can be found in (Bjørke, 1989; Chase and Greenwood, 1988; Voelcker, 1998).

The least studied area is simultaneous tolerancing for both design and manufacturing. The problem justification is that early consideration of manufacturing constraints will reduce the number of design changes. Bjørke (1989) used process parameters to describe the process capabilities and derived manufacturing tolerances from design tolerances. Zhang et al (1992) was initiated to select the process among alternatives. The Worst Case (WC) model was used to describe the stackup of design tolerances, while the boundaries of manufacturing tolerances were associated with alternative processes. Zhang (1997) modeled each component tolerance as variation stackup of a set of machining operations and treated the assembly tolerance as stackup of component tolerances. In this line of research, there are mainly two approaches to obtain tolerance stackup models. The first approach is to use WC model, Root Sum Square (RSS) model, or interpolations of these two models. One weakness of this approach is that process variation is oversimplified. Tolerances and process parameters are linked by means of certain distributions. Therefore design burden is eased at the cost of increased difficulties in selecting an appropriate process. The second approach is Monte Carlo simulation which needs to specify component distributions in advance. A third approach is to model the impact of process parameters on tolerance stackup based on the first principle, such as the kinematic analysis of part imperfection caused by fixture errors (Rong and Bai, 1996; Cai et al, 1997; Choudhuri and De Meter, 1999) and modeling of machine tool errors (Chen et al, 1993). For multi-stage processes, tolerance or variation stackup models have also been developed for assembly processes (Mantripragada. and Whitney, 1999; Jin and Shi, 1999) and machining processes (Huang et al 2001; Djurdjanovic and Ni, 2001; Zhou et al, 2001). However, very limited work has been done to apply this approach for simultaneous tolerance synthesis, especially for a multi-stage machining process. Among them, Ding *et al* (2002) used the developed state space model to concurrently allocate component tolerances and select fixtures for assembly processes.

This paper is motivated to extend the third approach to tolerance synthesis for multistage machining processes. All the concepts and procedures are demonstrated through a block part example, though the methodologies are general in nature. The tolerance stackup model is developed in Section 2. In Section 3, simultaneous tolerance synthesis is formulated as a Linear Programming (LP) problem. In Section 4, design improvement is performed by analyzing the sensitivity of process parameters. It is further generalized as a general two-step optimization procedure. The work is summarized in Section 5.

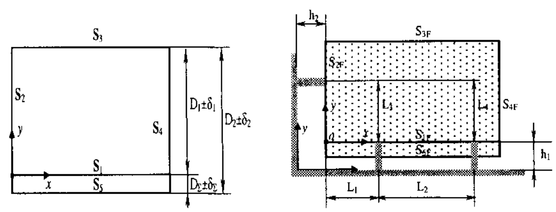
DEVELOP TOLERANCE STACKUP MODEL FOR MULTISTAGE MACHINING PROCESSES

Two fundamental issues need to be addressed in deriving the model: the way of modeling tolerance and the way of linking tolerance with process parameters. As to the former aspect, Vectorial Dimensioning and Tolerancing (VD&T) scheme is chosen because it provides a clear distinction among size, form, and orientation for each part surface (Martinsen,1993). However, the formulation of tolerance synthesis is also given for Geometric Dimensioning and Tolerancing (GD&T), scheme by considering its wide applications. As to the linkage, homogenous transformation approach is extensively applied to model setup and cutting operations and their influence on part accuracy.

Example Description and Tolerance Modeling

The chosen block part is composed of five surfaces: $\mathbf{S}_1$-$\mathbf{S}_5$ (Fig. 1a). The given design specification is $D_\Sigma \pm \delta_\Sigma$ for the clearance between $\mathbf{S}_1$ and $\mathbf{S}_5$. Under GD&T scheme, the typical tolerance synthesis problem is to determine the tolerance δ_1 and δ_2 for component dimensions D_1 and D_2. The problem for VD&T is formulated later in Section 3.

A two-operation process is given to achieve $D_\Sigma \pm \delta_\Sigma$. The first operation is to use datum surfaces $\mathbf{S}_1$ and $\mathbf{S}_2$ to mill $\mathbf{S}_3$, where $\mathbf{S}_1$ and $\mathbf{S}_2$ are assumed to be perfect. Along with $\mathbf{S}_3$, the in-process dimension and tolerance $D_1 \pm \delta_1$ is affected by face milling 1 and fixture 1. In operation 2, $\mathbf{S}_2$ and $\mathbf{S}_3$ are chosen as datums to mill $\mathbf{S}_5$. Meanwhile, the in-process $D_2 \pm \delta_2$ and resultant $D_\Sigma \pm \delta_\Sigma$ are generated. In terms of dimensional tolerance, the total transmitted variation to the resultant tolerance δ_Σ is the so-called tolerance stackup. Per the weakness of current tolerance stackup models for multi-stage machining processes, a casual model is developed to describe (1) how face milling 1 and fixture 1 affect $\mathbf{S}_3$, and (2) how face milling 2, fixture 2, and $\mathbf{S}_3$ affect $\mathbf{S}_5$.



1a FINAL BLOCK PART 1b NOMINAL SETUP 1.

Under VD&T scheme, each part surface $\mathbf{S}_i$ is specified by its orientation (n_{xi}, n_{yi}, n_{zi}), location (p_{xi}, p_{yi}, p_{zi}), and size. (Size component is ignored for planes.) Then the block part is denoted as $\mathbf{X} = (\mathbf{S}_1\ \mathbf{S}_2 \ldots \mathbf{S}_5)^T$ with $\mathbf{S}_i = (n_{xi}, n_{yi}, n_{zi}, p_{xi}, p_{yi}, p_{zi})$. For instance, $\mathbf{S}_1$ is represented as (0,1,0,0,0,0) in the part coordinate system (PCS: *xoy*) (Fig. 1a). Tolerances are specified for surface orientation, location, and size.

Modeling of the First Operation (No Datum Error)

Next we build up the linkage between the machined surfaces $\mathbf{S}_i$ and process parameters. The starting point is to describe setup and cutting operation. Figure 1b shows the nominal setup scheme in operation 1. Since positioning the raw workpiece $\mathbf{X}_0$ into fixture 1 can be viewed as transforming $\mathbf{X}_0$ from the PCS (*xoy*) to the fixture coordinate system (FCS: xo_Fy), homogenous transformation matrix (HTM) can be used to model setup. For surface S_i in $\mathbf{X}_0$, the HTM for nominal setup is expressed as

$$\mathbf{H}_S = \left(\begin{array}{cc|c} \mathbf{R} & 0 & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & \mathbf{R} & \begin{matrix}x\\y\\z\end{matrix} \\ \hline 0\;0\;0 & 0\;0\;0 & 1 \end{array}\right)_{7\times 7}, \quad (1)$$

where the block diagonal matrix $\mathbf{R}$ is

$$\mathbf{R}=\begin{pmatrix} \cos(\beta)\cos(\gamma) & -\cos(\gamma)\sin(\alpha)\sin(\beta)-\cos(\alpha)\sin(\gamma) & \sin(\alpha)\sin(\gamma)-\cos(\alpha)\cos(\gamma)\sin(\beta) \\ \cos(\beta)\sin(\gamma) & \cos(\alpha)\cos(\gamma)-\sin(\alpha)\sin(\beta)\sin(\gamma) & -\cos(\gamma)\sin(\alpha)-\cos(\alpha)\sin(\beta)\sin(\gamma) \\ \sin(\beta) & \cos(\beta)\sin(\alpha) & \cos(\alpha)\cos(\beta) \end{pmatrix} \quad (2)$$

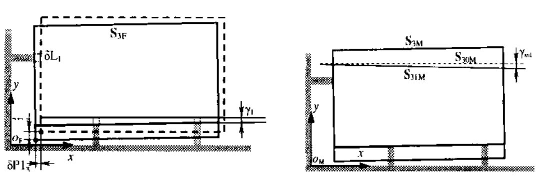
with (α,β,γ) and (x,y,z) being the amount of rotation and translation from the PCS to the FCS. Let $\mathbf{H}_{S1}$ denote HTM for nominal setup 1.

The parameters in $\mathbf{H}_S$, i.e., $(\alpha,\beta,\gamma,x,y,z)$, are nominal values determined by process design. Due to process variation, these values may deviate from their nominals and the part will be positioned differently. The setup deviation can be viewed as an additional transformation after the nominal setup. HTM can still be applied to catch the setup deviation. Let $\mathbf{H}_F$ denote HTM induced by deviations of a fixture. $\mathbf{H}_F$ has the same form as $\mathbf{H}_S$. However, under small deviation assumption, $\mathbf{R}$ is often simplified as

$$\mathbf{R} = \begin{pmatrix} 1 & -\gamma & -\beta \\ \gamma & 1 & -\alpha \\ \beta & \alpha & 1 \end{pmatrix}. \quad (3)$$

Let $(\alpha_f,\beta_f,\gamma_f)$ and (x_f,y_f,z_f) be the amount of rotation and translation caused by fixture deviations. Then $\mathbf{H}_F$ is written as

$$\mathbf{H}_F = \begin{pmatrix} 1 & -\gamma_f & -\beta_f & 0 & 0 & 0 & 0 \\ \gamma_f & 1 & -\alpha_f & 0 & 0 & 0 & 0 \\ \beta_f & \alpha_f & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_f & -\beta_f & x_f \\ 0 & 0 & 0 & \gamma_f & 1 & -\alpha_f & y_f \\ 0 & 0 & 0 & \beta_f & \alpha_d & 1 & z_f \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (4)$$



2a SETUP DEVIATION CAUSED BY FIXTURE 1, 2b (RIGHT) FACE MILLING OF S_3 IN OPERATION 1.

Suppose pin deviations of fixture 1 are (δL_1 δL_3 δL_4) (Fig. 2a), where negative value of δL_i represents the decrease in pin height. Deviations lead to the nominal part position (dash line) transformed to the final position (solid line). $\mathbf{H}_{F1}$ is derived as

$$\mathbf{H}_{F1} = \begin{pmatrix} 1 & -\gamma_1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_1 & 0 & x_{f1} \\ 0 & 0 & 0 & \gamma_1 & 1 & 0 & y_{f1} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (5)$$

where $\gamma_1 = \dfrac{-\delta L_3 + \delta L_4}{L_2}$, $x_{f1} = p_{y1}\gamma_1 + h_1\gamma_1 + \delta P1_x$, $y_{f1} = -p_{x1}\gamma_1 - h_2\gamma_1 + \delta P1_y$, $\delta P1_x = \delta L_1 + L_3\gamma_1$, and $\delta P1_y = \delta L_3 - L_1\gamma_1$ (Appendix I).

As a result, surface $\mathbf{S}_i$ in $\mathbf{X}_0$ is transformed as $\mathbf{H}_{F1}\mathbf{H}_{S1}[\mathbf{S}_i\ 0]^T$ after setup 1.

Suppose the machine tool coordinate system (MCS: xo_My) coincides with the FCS. Denote $\mathbf{S}_{iM}$ as $\mathbf{S}_i$ in the MCS. At face milling 1, $\mathbf{S}_3$ is to be removed and ideally becomes $\mathbf{S}_{30}$. The nominal values of $\mathbf{S}_{30}$ is given by design and $\mathbf{S}_{30M}$ is computed as $[\mathbf{S}_{30M}\ 0]^T = \mathbf{H}_{S1}[\mathbf{S}_{30}\ 0]^T$ (Fig. 2b). Note it is not $\mathbf{H}_{F1}\mathbf{H}_{S1}[\mathbf{S}_{30}\ 0]^T$, because the ideal tool path movement is independent of fixture errors. The machine tool errors transform $\mathbf{S}_{30M}$ to $\mathbf{S}_{31M}$ (Fig. 2b). Similar to fixture error model (Eq.(4)), the machine tool error is modeled as

$$\mathbf{H}_M = \begin{pmatrix} 1 & -\gamma_m & -\beta_m & 0 & 0 & 0 & 0 \\ \gamma_m & 1 & -\alpha_m & 0 & 0 & 0 & 0 \\ \beta_m & \alpha_m & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_m & -\beta_m & x_m \\ 0 & 0 & 0 & \gamma_m & 1 & -\alpha_m & y_m \\ 0 & 0 & 0 & \beta_m & \alpha_m & 1 & z_m \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (6)$$

where $(\alpha_m,\beta_m,\gamma_m)$ and (x_m,y_m,z_m) are the amount of rotation and translation caused by machine tool errors. $\mathbf{H}_{M1}$, the deviation of face milling 1, is expressed by Eq.(7)

$$\mathbf{H}_{M1} = \begin{pmatrix} 1 & -\gamma_{m1} & 0 & 0 & 0 & 0 & 0 \\ \gamma_{m1} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_{m1} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{m1} & 1 & 0 & y_{m1} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (7)$$

because in face millings 1 (and 2), (α_m, β_m) and (x_m, z_m) do not affect surface orientation and location.

With $\mathbf{H}_{M1}$, $\mathbf{S}_{31M}$ is derived as $[\mathbf{S}_{31M}\ 0]^T = \mathbf{H}_{M1}\mathbf{H}_{S1}[\mathbf{S}_{30}\ 0]^T$. Transforming $\mathbf{S}_{31M}$ to the PCS, the surface $\mathbf{S}_{31}$ is represented as

$$[\mathbf{S}_{31}\ 0]^T = (\mathbf{H}_{S1})^{-1}(\mathbf{H}_{F1})^{-1}\mathbf{H}_{M1}\mathbf{H}_{S1}[\mathbf{S}_{30}\ 0]^T. \quad (8)$$

Since $\mathbf{S}_{30}=(n_{x3}, n_{y3}, n_{z3}, p_{x3}, p_{y3}, p_{z3})=(0,1,0,0,D_1,0)$, $\mathbf{S}_{31}$ is computed as

$$\begin{pmatrix} n_{x3} + n_{y3}\gamma_1 - n_{y3}\gamma_{m1} \\ n_{y3} - n_{x3}\gamma_1 + n_{x3}\gamma_{m1} \\ n_{z3} \\ p_{x3} - x_{f1} + h_1\gamma_1 + p_{y3}\gamma_1 - y_{f1}\gamma_1 - h_1\gamma_{m1} - p_{y3}\gamma_{m1} \\ p_{y3} - y_{f1} + y_{m1} - h_2\gamma_1 - p_{x3}\gamma_1 + x_{f1}\gamma_1 + h_2\gamma_{m1} + p_{x3}\gamma_{m1} \\ p_{z3} \end{pmatrix}. \quad (9)$$

Modeling of the Second Operation (with Datum Error)

Setup 2 is more complicated than setup 1 because of deviation in datum $\mathbf{S}_{31}$. Let $\mathbf{H}_{S2}$ be the HTM representing nominal setup 2. To model the real setup 2, first we assume datum errors cause a transformation $\mathbf{H}_{D2}$ in addition to $\mathbf{H}_{S2}$. Note $\mathbf{H}_{D1}$ is an identity matrix **I** because of no datum error in setup 1. Second, the deviation of fixture 2 generates another transformation $\mathbf{H}_{F2}$. Hence $\mathbf{S}_i$ in $\mathbf{X}_1$ is transformed as $\mathbf{H}_{F2}\mathbf{H}_{D2}\mathbf{H}_{S2}[\mathbf{S}_i\ 0]^T$. Then we need to identify $\mathbf{H}_{F2}$ and $\mathbf{H}_{D2}$.

$\mathbf{H}_D$, transformation caused by datum errors, takes the same form of $\mathbf{H}_F$ and $\mathbf{H}_M$. Let $(\alpha_d,\beta_d,\gamma_d)$ and (x_d,y_d,z_d) are the amount of rotation and translation caused by datum errors. The general form of $\mathbf{H}_D$ is given as

$$\mathbf{H}_D = \begin{pmatrix} 1 & -\gamma_d & -\beta_d & 0 & 0 & 0 & 0 \\ \gamma_d & 1 & -\alpha_d & 0 & 0 & 0 & 0 \\ \beta_d & \alpha_d & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_d & -\beta_d & x_d \\ 0 & 0 & 0 & \gamma_d & 1 & -\alpha_d & y_d \\ 0 & 0 & 0 & \beta_d & \alpha_d & 1 & z_d \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

Figure 3a shows the nominal fixture 2. Due to deviations in $\mathbf{S}_{31}$, there exists an angle γ_2 between the *x* axis and $\mathbf{H}_{S2}[\mathbf{S}_{31}\ 0]^T$ (in dash line in Fig.3a). The part should additionally rotate by γ_2 and translate by (x_{d2}, y_{d2}) so as to contact with the three pins. $\mathbf{H}_{D2}$, which is caused by datum $\mathbf{S}_{31}$, is derived as

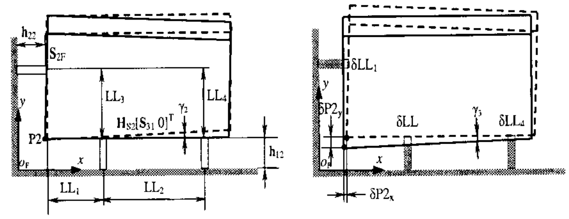
$$\mathbf{H}_{D2} = \begin{pmatrix} 1 & -\gamma_2 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_2 & 0 & x_{d2} \\ 0 & 0 & 0 & \gamma_2 & 1 & 0 & y_{d2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

where $x_{d2} = x_{f1} - h_1\gamma_1 - p_{y3}\gamma_1 + y_{f1}\gamma_1 + D_1\gamma_2 + h_{12}\gamma_2 + LL_3\gamma_2 - p_{y3}\gamma_2 + y_{f1}\gamma_2 + h_1\gamma_{m1} + p_{y3}\gamma_{m1}$, $y_{d2} = -y_{f1} + y_{m1} - h_2\gamma_1 - p_{x3}\gamma_1 + x_{f1}\gamma_1 - h_{22}\gamma_2 - p_{x3}\gamma_2 + x_{f1}\gamma_2 + h_2\gamma_{m1} + p_{x3}\gamma_{m1}$, and $\gamma_2 = \gamma_{m1} - \gamma_1$ (Appendix II).

For fixture 2 errors (Fig. 3b), $\mathbf{H}_{F2}$ is derived in the same way as $\mathbf{H}_{F1}$. Note that the high order terms of the errors are ignored.

$$\mathbf{H}_{F2} = \begin{pmatrix} 1 & -\gamma_3 & 0 & 0 & 0 & 0 & 0 \\ \gamma_3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\gamma_3 & 0 & x_{f2} \\ 0 & 0 & 0 & \gamma_3 & 1 & 0 & y_{f2} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (12)$$

where $\gamma_3 = \frac{-\delta LL_3 + \delta LL_4}{LL_2}$, $x_{f2} = -h_{12}\gamma_3 + \delta P2_x$, $y_{f2} = -(h_{22} + p_{x3})\gamma_3 + \delta P2_y$, $\delta P2_x = \delta LL_1 + LL_3\gamma_3$, and $\delta P2_y = \delta LL_3 - LL_1\gamma_3$.



3a NOMINAL FIXTURE 2 AND DATUM ERRORS, 3b (RIGHT) FIXTURE 2 ERRORS.

For the face milling of $\mathbf{S}_5$, $\mathbf{H}_{M2}$ is expressed in the same way as $\mathbf{H}_{M1}$. $\mathbf{S}_{51M}$ is computed as $[\mathbf{S}_{51M}\ 0]^T = \mathbf{H}_{M2}\mathbf{H}_{F2}\mathbf{H}_{D2}\mathbf{H}_{S2}[\mathbf{S}_{50}\ 0]^T$. Transforming $\mathbf{S}_{51M}$ to the PCS, $\mathbf{S}_{51}$ is derived as

$$[\mathbf{S}_{51}\ 0]^T = (\mathbf{H}_{S2})^{-1}(\mathbf{H}_{D2})^{-1}(\mathbf{H}_{F2})^{-1}\mathbf{H}_{M2}\mathbf{H}_{F2}\mathbf{H}_{D2}\mathbf{H}_{S2}[\mathbf{S}_{50}\ 0]^T. \quad (13)$$

Since $\mathbf{S}_{50}=(n_{x5}, n_{y5}, n_{z5}, p_{x5}, p_{y5}, p_{z5})=(0,1,0,0,-D_\Sigma,0)$, $\mathbf{S}_{51}$ is solved as

$$\begin{pmatrix} n_{x5} - n_{y5}\gamma_2 - n_{y5}\gamma_3 + n_{y5}\gamma_{m2} \\ n_{y5} + n_{x5}\gamma_2 + n_{x5}\gamma_3 - n_{x5}\gamma_{m2} \\ n_{z5} \\ p_{x5} - x_{d2} - x_{f2} + (D_2 + h_{12})\gamma_2 + (D_2 + h_{12})\gamma_3 - (D_2 + h_{12})\gamma_{m2} \\ -D_\Sigma + y_{d2} + y_{f2} - y_{m2} + (h_{22} + p_{x5})\gamma_2 + (h_{22} + p_{x5})\gamma_3 - (h_{22} + p_{x5})\gamma_{m2} \\ p_{z5} \end{pmatrix}. \quad (14)$$

Final Part Variation

The final product $\mathbf{X}_2$ becomes $(\mathbf{S}_1\ \mathbf{S}_2\ \mathbf{S}_{31}\ \mathbf{S}_4\ \mathbf{S}_{51})^T$. Dimensions D_1', D_2', and D_Σ' can be derived from $\mathbf{X}_2$. Without measurement errors, D_1', D_2', and D_Σ' are

$$\begin{aligned} D_1' &= D_1 + y_{m1} + h_2\gamma_{m1} - \left(1 + \frac{L_1}{L_2}\right)\delta L_3 + \frac{L_1}{L_2}\delta L_4 \\ D_2' &= D_1 + D_\Sigma + y_{m2} + h_{22}\gamma_{m2} - \left(1 + \frac{LL_1}{LL_2}\right)\delta LL_3 + \frac{LL_1}{LL_2}\delta LL_4 \quad (15) \\ D_\Sigma' &= D_\Sigma - y_{m1} + y_{m2} - h_2\gamma_{m1} + h_{22}\gamma_{m2} + \left(1 + \frac{L_1}{L_2}\right)\delta L_3 \\ &\quad - \frac{L_1}{L_2}\delta L_4 - \left(1 + \frac{LL_1}{LL_2}\right)\delta LL_3 + \frac{LL_1}{LL_2}\delta LL_4 \end{aligned}$$

The process parameters in the example are denoted as $\mathbf{\Theta} = (\Delta L_1, \Delta L_3, \Delta L_4, \Delta LL_1, \Delta LL_3, \Delta LL_4, y_{m1}, y_{m1}, y_{m2}, y_{m2})^T$. Not losing generality, those ten parameters in $\mathbf{\Theta}$ are assumed to be independent random variables and their standard deviations are denoted as $\boldsymbol{\sigma}_\Theta = (\sigma_{L1}, \sigma_{L3}, \sigma_{L4}, \sigma_{ym1}, \sigma_{ym1}, \sigma_{LL1}, \sigma_{LL3}, \sigma_{LL4}, \sigma_{ym2}, \sigma_{ym2})^T$. $\boldsymbol{\sigma}_\Theta$ represents process variability and determines variations of $\mathbf{S}_{31}$ and $\mathbf{S}_{51}$. Under above settings, the covariance matrices of $\mathbf{S}_{31}$ and $\mathbf{S}_{51}$, denoted as $\mathbf{K}_{S3}$ and $\mathbf{K}_{S5}$, can be calculated. Their diagonal terms diag($\mathbf{K}_{S3}$) and diag($\mathbf{K}_{S5}$) are

$$\begin{pmatrix} \sigma^2_{Ym1} + \frac{1}{L_2^2}(\sigma^2_{L3} + \sigma^2_{L4}) \\ 0 \\ 0 \\ (D_1 + h_1)^2 \sigma^2_{Ym1} + \sigma^2_{L1} + \frac{(D_1 - L_3)^2}{L_2^2}(\sigma^2_{L3} + \sigma^2_{L4}) \\ \sigma^2_{ym1} + h_2^2 \sigma^2_{Ym1} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} \\ 0 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \sigma^2_{Ym1} + \sigma^2_{Ym2} + \frac{1}{L_2^2}(\sigma^2_{L3} + \sigma^2_{L4}) + \frac{1}{LL_2^2}(\sigma^2_{LL3} + \sigma^2_{LL4}) \\ 0 \\ 0 \\ (D_\Sigma - h_1 - LL_3)^2 \sigma^2_{Ym1} + (D_2 + h_{12})^2 \sigma^2_{Ym2} + \sigma^2_{L1} + \\ \frac{(D_\Sigma + L_3 - LL_3)^2}{L_2^2}(\sigma^2_{L3} + \sigma^2_{L4}) + \sigma^2_{LL1} + \frac{(D_2 - LL_3)^2}{LL_2^2}(\sigma^2_{LL3} + \sigma^2_{LL4}) \\ \sigma^2_{ym1} + \sigma^2_{ym2} + h_2^2 \sigma^2_{Ym1} + h_{22}^2 \sigma^2_{Ym2} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \\ \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} + \left(1 + \frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL3} + \left(\frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL4} \\ 0 \\ 0 \end{pmatrix}$$

Similarly, variances of D'_1, D'_2, and D'_Σ can be calculated as

$$\sigma^2_{D1} = \sigma^2_{ym1} + h_2^2 \sigma^2_{Ym1} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4}$$

$$\sigma^2_{D2} = \sigma^2_{ym2} + h_{22}^2 \sigma^2_{Ym2} + \left(1 + \frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL3} + \left(\frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL4} \tag{17}$$

$$\sigma^2_\Sigma = \sigma^2_{D1} + \sigma^2_{D2} = \sigma^2_{ym1} + \sigma^2_{ym2} + h_2^2 \sigma^2_{Ym1} + h_{22}^2 \sigma^2_{Ym2} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} + \left(1 + \frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL3} + \left(\frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL4}$$

Since diag($\mathbf{K}_{S3}$) and diag($\mathbf{K}_{S5}$), together with σ_{D1}, σ_{D2}, and $\sigma_{D\Sigma}$, can be directly related with tolerance (e.g., $\delta_\Sigma = 6\sigma_{D\Sigma}$), the process modeling in this paper presents a tolerance stackup model which can describe how the process parameters affect part accuracy and tolerance stackup. The physics of tolerance stackup can thus be explained. Although the model is based on the block part example, the approach is generally applicable to general multistage manufacturing processes (Huang et al, 2001).

SIMULTANEOUS TOLERANCE SYNTHESIS

The simultaneous tolerance synthesis problems are formulated in Table 1. The optimal process is defined as a process satisfying the design specification at minimum tooling cost. Usually larger $\boldsymbol{\sigma}_\Theta$ is desirable, because tooling cost, i.e., fixture and machine tool cost, will be less expensive. The objective function is either to minimize total tooling cost or maximize $\boldsymbol{\sigma}_\Theta$. Maximizing $\boldsymbol{\sigma}_\Theta$ is preferred in this paper based on the following considerations: (1) At design stage, tooling cost is hard to be precisely estimated. On the other hand, maximizing $\boldsymbol{\sigma}_\Theta$ usually means minimizing tooling cost; (2) More important, it is clear to reflect the sensitivity issues of $\boldsymbol{\sigma}_\Theta$. This point will be elaborated in next section.

Table 1 Problem Formulation

Under VD&T scheme	Under GD&T scheme
Given: • Nominal final part geometry (i.e., $\mathbf{X}_{20}$) and dimensions (D_1, D_2, D_Σ) • Nominal setup geometry (L_1, L_2, L_3, L_4, h_1, h_2, LL_1, LL_2, LL_3, LL_4, h_{12}, h_{22}) • Process sequence and operations • Specifications for orientation and location components(u_{nx3}, u_{py3}, u_{nx5}, u_{py5})	Given: • Nominal final part geometry (i.e., $\mathbf{X}_{20}$) and dimensions (D_1, D_2, D_Σ) • Nominal setup geometry (L_1, L_2, L_3, L_4, h_1, h_2, LL_1, LL_2, LL_3, LL_4, h_{12}, h_{22}) • Process sequence and operations • Resultant tolerance δ_Σ
Find: • The optimal process $\boldsymbol{\sigma}_\Theta$ to fabricate the part.	Find: • The optimal process $\boldsymbol{\sigma}_\Theta$ • Component tolerance δ_1 and δ_2

To find an explicit objective function, one option is to maximize a variation component of surfaces or dimensions with the maximum number of $\boldsymbol{\sigma}_\Theta$, e.g. σ_{py5} or $\sigma_{D\Sigma}$. However, the problem with this choice is that some of the process variables in $\boldsymbol{\sigma}_\Theta$ might not be close to their optima. Fortunately this drawback can be compensated by design improvement. A two-step optimization procedure will be presented later in Section 4.

For the block part, the optimization model under VD&T scheme is to maximize

$$\sigma^2_{ym1} + \sigma^2_{ym2} + h_2^2 \sigma^2_{Ym1} + h_{22}^2 \sigma^2_{Ym2} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} + \left(1 + \frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL3} + \left(\frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL4}$$

subject to

$$\sigma^2_{Ym1} + \frac{1}{L_2^2}(\sigma^2_{L3} + \sigma^2_{L4}) \le u^2_{nx3}$$

$$\sigma^2_{ym1} + h_2^2 \sigma^2_{Ym1} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} \le u^2_{py3} \tag{18}$$

$$\sigma^2_{Ym1} + \sigma^2_{Ym2} + \frac{1}{L_2^2}(\sigma^2_{L3} + \sigma^2_{L4}) + \frac{1}{LL_2^2}(\sigma^2_{LL3} + \sigma^2_{LL4}) \le u^2_{nx5}$$

$$\sigma^2_{ym1} + \sigma^2_{ym2} + h_2^2 \sigma^2_{Ym1} + h_{22}^2 \sigma^2_{Ym2} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} + \left(1 + \frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL3} + \left(\frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL4} \le u^2_{py5}$$

$$\boldsymbol{\sigma}_\Theta > 0$$

u_{px3} and u_{px5} need not to be considered, because the location component of a plane is free in *x* direction. Under GD&T scheme, the optimization model is to maximize σ^2_Σ, subject to

$$\sigma^2_\Sigma = \sigma^2_{ym1} + \sigma^2_{ym2} + h_2^2 \sigma^2_{Ym1} + h_{22}^2 \sigma^2_{Ym2} + \left(1 + \frac{L_1}{L_2}\right)^2 \sigma^2_{L3} + \left(\frac{L_1}{L_2}\right)^2 \sigma^2_{L4} + \left(1 + \frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL3} + \left(\frac{LL_1}{LL_2}\right)^2 \sigma^2_{LL4} \le \delta_\Sigma / 6$$

$$\boldsymbol{\sigma}_\Theta > 0 \tag{19}$$

Both of the models are in LP form, because their constraints and objective functions are linear in variance σ_i^2's. Additional constraints might be considered from subject matter expertise. For example, process variation $\boldsymbol{\sigma}_\Theta$ is often bounded with $\boldsymbol{\sigma}_\Theta \le \mathbf{u}_\Theta$. Or equal precision is assumed for three pins in a fixture, i.e., $\sigma_{L1} = \sigma_{L3} = \sigma_{L4}$ and $\sigma_{LL1} = \sigma_{LL3} = \sigma_{LL4}$.

For illustration, assign L_1=100mm, L_2=300mm, L_3= $_4$=200mm, h_1=h_2=10mm, LL_1=100mm, LL_2= 300mm, LL_3=LL_4= 200mm, h_{12}=h_{22}=10mm, D_1= 295mm, D_2=300mm, D_Σ=5mm, u_{nx3}= 0.0001radiant, u_{py3}=0.02mm, u_{nx5}=0.0003radiant, u_{py5}= 0.06mm, and δ_Σ=0.36mm. To compare the two operations, the geometrical information of two fixtures is purposely chosen to be the same. δ_Σ is selected as $6u_{py5}$ so as to compare the results under two schemes. Additional constraint is $\mathbf{u}_\Theta$= (0.5mm, 0.5mm, 0.5mm, 0.5mm, 0.5mm, 0.5mm, 0.01radiant, 0.01radiant)^T.

Table 2 Optimal Process $\boldsymbol{\sigma}_\Theta^*$

$\boldsymbol{\sigma}_\Theta^*$	Fixture #1			Machine #1	
	σ_{L1}	σ_{L3}	σ_{L4}	σ_{ym1}	σ_{ym1}
VD&T	*0.006*	0.006	0.005	0.00003	0.012
GD&T	*0.102*	0.013	0.102	0.00154	0.009
$\boldsymbol{\sigma}_\Theta^*$	Fixture #2			Machine #2	
	σ_{LL1}	σ_{LL3}	σ_{LL1}	σ_{LL3}	σ_{LL1}
VD&T	*0.077*	0.004	*0.077*	0.004	*0.077*
GD&T	*0.102*	0.013	*0.102*	0.013	*0.102*

The optimal solution $\boldsymbol{\sigma}_\Theta^*$ is given in Table 2. Tooling tolerance can be obtained as $6\boldsymbol{\sigma}_\Theta^*$. Since σ_{L1} and σ_{LL1} do not appear in objective functions and constraints, there are no solutions for these two process variables. Although insensitive to tolerance stackup, they are chosen as σ_{L1}=Max(σ_{L3},σ_{L4}) and σ_{LL1}=Max(σ_{LL3},σ_{LL4}) to avoid violating the small deviation assumption. Component tolerance δ_1 and δ_2 are solved from Eq.(17) as $\delta_1=6\sigma_{D1}$=0.25mm and $\delta_2= 6\sigma_{D2}$= 0.25mm, i.e., the resultant tolerance is equally distributed to component dimensions. Difference is expected if extra tolerances are considered, e.g., the parallelism between $\mathbf{S}_3$ and $\mathbf{S}_5$.

SENSITIVITY ANALYSIS AND DESIGN IMPROVEMENT

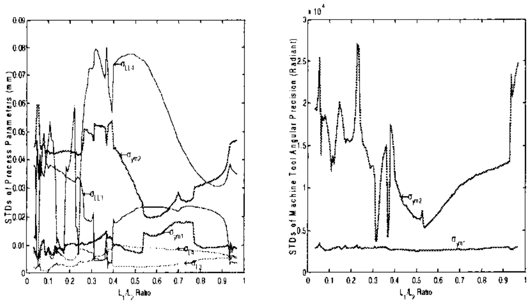
Sensitivity analysis is to study how the tolerance stackup reacts differently to the changes of *n* process variables $\mathbf{\Theta}$. A variable with high sensitivity has big impact on tolerance stackup. Design improvement is to study how the process design can be optimized in terms of minimizing tolerance stackup. These two topics are closely related.

The changes of process variables are described by $\boldsymbol{\sigma}_\Theta$ or $\boldsymbol{\sigma}_\Theta^2$, and the tolerance stackup is modeled by diag($\mathbf{K}_{Si}$) in Section 2. Suppose that tolerances are specified for *m* elements, e.g., n_{x3}, n_{x5}, p_{y3}, p_{y5}, D_1, D_2, and D_Σ. The variation of element i (i ≤ *m*) has been modeled as a linear function of $\boldsymbol{\sigma}_\Theta^2$, i.e., $\mathbf{a}_i^T \boldsymbol{\sigma}_\Theta^2$, where the jth entry of $\mathbf{a}_i$, denoted as a_{ji}, represents the sensitivity of the jth process variable in $\boldsymbol{\sigma}_\Theta^2$ (j ≤ *n*). Generally speaking, the larger the coefficient a_{ji}, the bigger the impact has the jth process variable on element i. The sensitivity matrix $\mathbf{A}$ is defined as $\mathbf{A}=[\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_m]$ and an example of $\mathbf{A}$ is shown in Table 3.

Table 3 Sensitivity Matrix for Block Part

σ_Θ^2 \ Element		Operation 1: σ^2_{nx3} $\mathbf{a}_1$	Operation 1: $\sigma^2_{py3} = \sigma^2_{D1}$ $\mathbf{a}_2$	Operation 2: σ^2_{nx5} $\mathbf{a}_3$	Operation 2: $\sigma^2_{py5} = \sigma^2_\Sigma$ $\mathbf{a}_4$	Operation 2: σ^2_{D2} $\mathbf{a}_5$
Fixture 1	σ^2_{L1}	0	0	0	0	0
	σ^2_{L3}	$\frac{1}{L_2^2}$	$\left(1+\frac{L_1}{L_2}\right)^2$	$\frac{1}{L_2^2}$	$\left(1+\frac{L_1}{L_2}\right)^2$	0
	σ^2_{L4}	$\frac{1}{L_2^2}$	$\left(\frac{L_1}{L_2}\right)^2$	$\frac{1}{L_2^2}$	$\left(\frac{L_1}{L_2}\right)^2$	0
Machine 1	σ^2_{Ym1}	1	h_2^2	1	h_2^2	0
	σ^2_{ym1}	0	1	0	1	0
Fixture 2	σ^2_{LL1}	0	0	0	0	0
	σ^2_{LL3}	0	0	$\frac{1}{LL_2^2}$	$\left(1+\frac{LL_1}{LL_2}\right)^2$	$\left(1+\frac{LL_1}{LL_2}\right)^2$
	σ^2_{LL4}	0	0	$\frac{1}{LL_2^2}$	$\left(\frac{LL_1}{LL_2}\right)^2$	$\left(\frac{LL_1}{LL_2}\right)^2$
Machine 2	σ^2_{Ym2}	0	0	1	h_{22}^2	h_{22}^2
	σ^2_{ym2}	0	0	0	1	1

Process variables which have variable coefficients are related to setup geometry information, e.g., L_1 L_2, and h_2. Their sensitivities vary with setup geometry. For instance, small L_1/L_2 ratio will reduce the transmission of fixture variation σ^2_{L3} and σ^2_{L4} to $\sigma^2_{P_{y5}}$. However, a process variable with relatively smaller sensitivity coefficient does not necessary take larger tolerance in a specific tolerance synthesis problem. The reason is that a set of constraints need to be satisfied in the optimization. The example in Section 3 is reused to illustrate this point. Suppose L_1= LL_1 and only these two dimensions are varied. $\boldsymbol{\sigma}_\Theta^*$ is obtained by solving Eq.(18) under different L_1/L_2 ratios. As shown in Fig. 4, though the sensitivity of σ^2_{L3} to $\sigma^2_{P_{y5}}$ is not less than that of σ^2_{L4}, σ^2_{L3} could have relatively larger values when L_1/L_2<0.25. (STD in Figs. 4 and 5 denotes standard deviation.) Under the problem setting, Fig. 4 also indicates that the precision of machine tool 2 can always be lower than that of machine tool 1. Compared with fixture 1, precision requirement for fixture 2 is generally lower. Besides, another interesting finding is that $\sigma^2_{Y_{m1}}$ is very robust to the changes of L_1/L_2 ratio. These properties are determined by the design specifications or the constraints in the optimization model.



4. σ^*_Θ UNDER DIFFERENT L_1/L_2 RATIO.

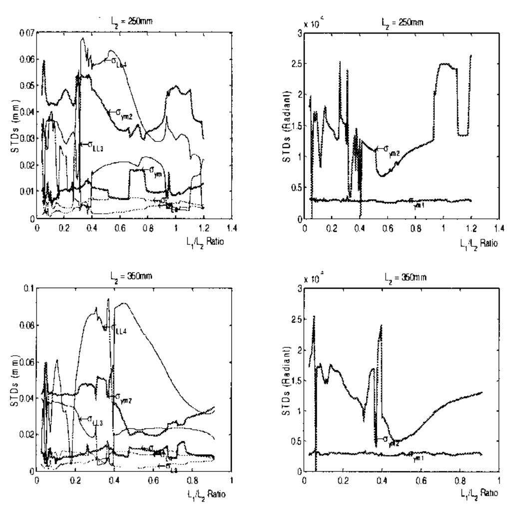
Existence of variable coefficients suggests possible opportunities to change the sensitivities and to reduce tolerance stackup by adjusting setup geometry. In another sense, process design could be improved. This spurs the idea of two-step optimization procedure to maximize $\boldsymbol{\sigma}_\Theta$.

(i) Given certain setup geometry, let $\mathbf{a}_{(m)}$ be the $\mathbf{a}_i$ with maximum dimension in sensitivity matrix $\mathbf{A}$, i.e., $\mathbf{a}_{(m)} = \text{Max}[\text{Dim}(\mathbf{a}_i)]$ for i=1,…,m. Solve $\boldsymbol{\sigma}^*_\Theta$ under the LP formulation

$$\text{Max } \mathbf{a}^T_{(m)}\, \boldsymbol{\sigma}^2_\Theta \text{, s.t.}$$
$$\mathbf{A}^T \boldsymbol{\sigma}^2_\Theta \le \mathbf{b} \qquad (20)$$
$$0 < \boldsymbol{\sigma}_\Theta \le \mathbf{u}_\Theta.$$

where $\mathbf{b}$ is a constraint vector.

(ii) Change process design or setup geometry within permissible ranges and repeat step 1. Among a set of $\{\boldsymbol{\sigma}^*_\Theta\}$, choose the one meeting other practical considerations, such as cost.



5 $\boldsymbol{\sigma}^*_\Theta$ UNDER L_2=250MM AND 350MM.

Due to practical constraints, the number of iterations in step 2 will not be large. For instance, L_2 will be limited by the length of workpiece. Further, setup geometry, e.g. L_2, can be divided into several levels to reduce the number of iterations and the complicatedness of comparing $\boldsymbol{\sigma}^*_\Theta$'s.

For the example, besides varying $L_1(=LL_1)$, we choose three levels for $L_2(=LL_2)$, i.e., L_2= 250mm, 300mm, and 350mm. Figures 4 and 5 show the computation results. Suppose the practical consideration is to lower the precision of machine tool 1 so as to reduce tooling cost. As $\sigma^2_{\gamma_{m1}}$ is very robust to the changes of L_1/L_2 ratio, the magnitude of $\sigma^2_{\gamma_{m1}}$ determines the selection. The maximum $\sigma_{\gamma m1}$ is achieved at level L_2= 250mm with $\sigma_{\gamma m1}$= 0.0184mm and L_1/L_2=0.672. Both tolerance synthesis problem and design improvement are achieved.

CONCLUSION

This paper developed the procedures of modeling tolerance stackup for multistage machining processes. A two-step optimize procedure is proposed to simultaneously determine component tolerances and process selection. Further, design improvement is performed by analyzing the sensitivity of process parameters.

APPENDIX I

In the FCS (xo_Fy) of Fig. 2a, γ_1 is the angle that the part in dash line rotates around z axis (pointing out of the paper) to have the same orientation as the part in solid line. By fixture 1 layout, γ_1 is approximated as $\gamma_1 = \frac{\delta L_4 - \delta L_3}{L_2}$.

Denote the low left corner of the part as P1 and ($\delta P1_x$, $\delta P1_y$) as P1's displacement caused by fixture error (δL_1 δL_3 δL_4). By geometric relationship, $\delta P1_x$ and $\delta P1_y$ are expressed as $\delta P1_x = \delta L_1 + L_3\gamma_1$, and $\delta P1_y = \delta L_3 - L_1\gamma_1$.

In the PCS (xoy), let $\mathbf{S}_1$ = (n_{x1}, n_{y1}, n_{z1}, p_{x1}, p_{y1}, p_{z1}) with location (p_{x1}, p_{y1}, p_{z1}) representing the coordinates of P1. $\mathbf{H}_{S1}[\mathbf{S}_1\ 0]^T$ is the ideal position of $\mathbf{S}_1$ in fixture 1 and $\mathbf{H}_{F1}\mathbf{H}_{S1}[\mathbf{S}_1\ 0]^T$ is the real position due to fixture errors. Their difference in location equals to ($\delta P1_x$, $\delta P1_y$, 0). By solving the equalities, we can solve the amount of translation (x_{f1},y_{f1}) in $\mathbf{H}_{F1}$.

APPENDIX II

Since $\mathbf{S}_{31}$ tilts by an angle of (γ_{m1}-γ_1), the angle between the x axis and $\mathbf{H}_{S2}[\mathbf{S}_{31}\ 0]^T$ equals to $|\gamma_{m1}-\gamma_1|$, i.e., $\gamma_2 = \gamma_{m1}-\gamma_1$ (Fig. 3a). Denote the low left corner of the part as P2. Ideally, P2's coordinate is (h_{22}, h_{12}) in the FCS. With deviation in $\mathbf{S}_{31}$, P2's coordinate is changed to ($h_{22}+\gamma_2 LL_3$,

h_{12}). The coordinate in *y* direction remains same because fixture 2 is assumed to be perfect in this step. In the PCS, $\mathbf{S}_{31}$ is given by Eq.(9) with location (p_{x3}, p_{y3}, p_{z3}) representing the ideal coordinate of P2. The location component of $\mathbf{H}_{D2}\mathbf{H}_{S2}[\mathbf{S}_{31}\ 0]^T$ should be equal to ($h_{22}+\gamma_2 LL_3$, h_{12}, 0). By solving the equalities, we can solve the amount of translation (x_{d2},y_{d2}) in $\mathbf{H}_{D2}$.

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