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Research

Design of Regression Model-based Automatic Process Control with Reduced Adjustment Frequency

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Automatic process control based on a regression model has been adopted as one of the important techniques to improve product quality in manufacturing processes. Though more frequent adjustments generally produce a superior control performance, it may also increase control cost and impair control applicability. In this paper, the concepts of quality margin and self-compensation of noise change are introduced. Based on these concepts, a control strategy is proposed which is capable of ensuring an acceptable process performance with a reduced adjustment frequency. A case study of leaf spring forming process is conducted to compare the control performance and control adjustment frequency between the proposed approach and the existing methods. Some properties of the proposed control law are also studied. The proposed method is implemented in a hot steel rolling process to demonstrate the applicability of the proposed method. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Process control plays an important role in improving process stability, products' quality, and manufacturing efficiency. In the past decades, various control strategies have been developed based on a regression model (RM) framework. Among them, off-line optimal design techniques and online process control approaches have been commonly adopted.

Robust parameter design (RPD), which was pioneered by Taguchi[1], is considered as one of the most widely used off-line process control techniques. The prevailing work treats the values of the noise factors as unknown during the running of the process (Wu and Hamada[2]). The controllable variables are set to

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the optimal points (without further adjustments during production) so that the response is insensitive to the noise variations within the anticipated ranges.

Motivated by the fact that some noise factors may be measurable during a production run, some online process control strategies were proposed. Pledger[3] used information from observable uncontrollable factors to achieve a better performance in the robust design. Jin and Ding[4] proposed a systematical method to design an online automatic process control (APC) scheme that fully uses the information from those observable noise factors. Joseph[5] has investigated a parameter design methodology in the presence of a feed-forward control. Recently, Dasgupta and Wu[6] presented how to conduct a robust design in the process where a feed-back controller is applied. Because aforementioned control strategies are all designed using regression process models, they are viewed as a class of control scheme here named as 'RM-based APC' (RM-APC).

In the literature, all reported RM-APCs focus on achieving the optimal control performance without considering the implementation cost when different control adjustment frequencies are adopted. Generally, the off-line RPD scheme is believed to be superior from the control cost point of view because no further online adjustment is needed during production. However, this strategy may not always be acceptable from a control performance point of view because *mean-on-target* and *variation minimization* cannot always be achieved simultaneously. From another side, RM-APCs are believed superior in terms of control performance because controllable variables are adjusted in real time to compensate any changes observed from noise variables. However, RM-APCs may lead to high implementation cost due to the need of real-time adjustments. In the literature, various efforts have been reported to investigate the cost due to real-time adjustments. Process control problems under fixed adjustment costs have been studied by Box and Jenkins[7], Crowder[8], Box and Luceño[9], and Del Castillo[10]. In their study, the objective function was revised by adding a term considering the fixed adjustment costs. Through weighing the adjustment cost and quality gain, less frequent adjustments rather than continuously adjusting the process are achieved. Furthermore, their strategies are motivated by a basic assumption, that is, the quality gain and the adjustment cost should be measurable and quantified using monetary coefficients. This may bring limitations to their approach. It may not work well when the monetary coefficients are unavailable or vague. Finally, their control strategies did not consider quality specifications, hence, the control actions lead to the minimization of overall loss, but not necessarily to ensure the product quality within the tolerance specifications.

This paper aims to develop a new design approach of an RM-APC, which is able to assure the specified process performance with reduced online adjustment frequency. The motivation and objective of conducting reduced frequent adjustments in our approach differ from that of the aforementioned strategies. Our strategy is based on the viewpoint that if the quality characteristic of products is within the specified tolerance, the products are qualified or acceptable. Specifically, the concepts of quality margins and self-compensations of noise changes are proposed and explicitly discussed. Fully utilizing these two concepts leads to the development of the proposed control law design approach.

The remainder of the paper is organized as follows: Section 2 provides a brief review of RM-APC strategies and proposes the concepts of quality margins and self-compensations of noise changes. Section 3 presents the detailed procedures of the proposed control law design. Two case studies are conducted in Section 4. Finally, a summary is given in Section 5.

2. RM-APC

2.1. Review of RM-APC and RPD

In manufacturing processes, there are many process variables that interact in a complicated manner. In general, these variables are classified into controllable factors $\mathbf{x}$ and noise factors $\mathbf{n}$ (variables that vary randomly and are difficult to control online). If y denotes the system response, the relationship among $\mathbf{x}$, $\mathbf{n}$, and y can be generally expressed as

$$y=\beta_0+\boldsymbol{\beta}_1^{\mathrm{T}}\mathbf{x}+\boldsymbol{\beta}_2^{\mathrm{T}}\mathbf{n}+\mathbf{x}^{\mathrm{T}}\mathbf{B}\mathbf{n}+\varepsilon \tag{1}$$

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where $\mathbf{x}\in\Re^{m\times 1}$, $\mathbf{n}\in\Re^{p\times 1}$, and other vectors and matrices are of appropriate dimensions. $\mathbf{n}$ is assumed to be measurable online. ε is the residual error containing the remaining unobservable noise factors. It is also assumed that $\mathbf{n}$ and ε are independent of each other with a covariance of $\Sigma_{\mathbf{n}}$ and variance σ_ε^2, respectively. ε is an *i.i.d.* variable with zero mean. This type of RM, usually having no input–output dynamics, mainly focuses on the description of the dependency of the process response y on the process variables $(\mathbf{x},\mathbf{n})$ and their interactions.

In general, there are two types of goals in process control: minimizing output deviation from the target and minimizing process variation. In this paper, we focus on the on-target process control problem, namely the control law should minimize both the variance of the response y and minimize the mean deviation of the response from the target. Therefore, a quadratic loss function is selected as the control objective function, i.e.

$$L(\mathbf{x})=c(y-T)^2 \tag{2}$$

where T is a target specified by the engineering design and c is the monetary coefficient which is set to be 1 without loss of generality.

The optimal setting of controllable factor $\mathbf{x}^*$ is defined as the value that minimizes the objective function (2). When the observation of noise factor $(\hat{\mathbf{n}})$ is obtained through an online observer or a sensor, the corresponding objective function is represented as:

$$\begin{aligned}\min L(\mathbf{x}) &= \min(\beta_0-T+\boldsymbol{\beta}_1^{\mathrm{T}}\mathbf{x}+\boldsymbol{\beta}_2^{\mathrm{T}}\hat{\mathbf{n}}+\mathbf{x}^{\mathrm{T}}\mathbf{B}\hat{\mathbf{n}})^2\\ s.t.\ \|\mathbf{x}\|_\infty &\le 1\end{aligned} \tag{3}$$

Here the unit hypercube constraint of $\|\mathbf{x}\|_\infty\le 1$ comes from the fact that both $\mathbf{n}$ and $\mathbf{x}$ are coded as a value in $[-1,+1]$ during the regression analysis.

The constrained optimization as described in (3) is solved by a numerical search. More search algorithms can be found in optimization literature (Pierre[11]). There are two approaches to conduct the search: One is to conduct a search over the constrained region and find the optimal solution according to the objective function; the other is to solve an unconstrained problem first, by setting the first-order partial derivative of $L(\mathbf{x})$ to zero, i.e. $\partial L(\mathbf{x})/\partial\mathbf{x}=0$. Its solution $\mathbf{x}_p^*$ is further evaluated to see if the constraint $\|\mathbf{x}_p^*\|_\infty\le 1$ is satisfied. If so, the $\mathbf{x}_p^*$ will be adopted as the final optimal solution; if not, the optimal solution will be achieved on the boundary of $\Lambda\equiv\{\mathbf{x}:\|\mathbf{x}\|_\infty=1\}$. Denote the optimal points on the boundary:

$$\mathbf{x}_\Lambda^*=\min_{\mathbf{x}\in\Lambda}\{L(\mathbf{x})\} \tag{4}$$

Thus, the optimal solution of (3) is:

$$\mathbf{x}^*=\begin{cases}\mathbf{x}_p^* & \text{if } \|\mathbf{x}_p^*\|_\infty\le 1\\ \mathbf{x}_\Lambda^* & \text{if } \|\mathbf{x}_p^*\|_\infty> 1\end{cases} \tag{5}$$

2.2. *Quality margin and self-compensation of noise change*

In this section, the concepts of quality margin and self-compensation of noise changes are introduced. Both concepts will contribute to the reduction of online adjustment frequencies.

In general, a quality specification, typically represented as a tolerance, always comes with a target mean value. A tolerance confines the quality margin within which the deviation of response from the target mean value is acceptable, although not preferable. In order to utilize such quality specifications for the adjustment frequency reduction, the deterioration pattern of response will be studied under the conditions that noise variables deviate from their current levels while the current controllable variable settings are not changed.

Here we provide an analysis of how the control performance deteriorates as the adjustment frequency reduces. If the current noise observation $\hat{\mathbf{n}}_c$ is obtained through an online observer, the corresponding optimal

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settings for controllable factors can be calculated from (5) and denoted by $\mathbf{x}_c^*$. If a new noise observation $\hat{\mathbf{n}}_{\text{new}}$ slightly deviates from $\hat{\mathbf{n}}_c$, and the controllable factors are still set at $\mathbf{x}_c^*$ purposely without updating using (5), control performance will deteriorate. This performance deterioration can be derived as

$$\begin{aligned}\Delta\hat{y} &= \hat{y}_{\text{new}}(\mathbf{x}_c^*, \hat{\mathbf{n}}_{\text{new}}) - \hat{y}_c(\mathbf{x}_c^*, \hat{\mathbf{n}}_c) = (\boldsymbol{\beta}_2 + \mathbf{B}^{\mathrm{T}}\mathbf{x}_c^*)(\hat{\mathbf{n}}_{\text{new}} - \hat{\mathbf{n}}_c) \\ &= \underbrace{(\beta_2^1 + \mathbf{b}_1^{\mathrm{T}}\mathbf{x}_c^*)}_{k_1}\underbrace{(\hat{n}_{\text{new}}^1 - \hat{n}_c^1)}_{\Delta\hat{n}_1} + \underbrace{(\beta_2^2 + \mathbf{b}_2^{\mathrm{T}}\mathbf{x}_c^*)}_{k_2}\underbrace{(\hat{n}_{\text{new}}^2 - \hat{n}_c^2)}_{\Delta\hat{n}_2} + \cdots + \underbrace{(\beta_2^p + \mathbf{b}_p^{\mathrm{T}}\mathbf{x}_c^*)}_{k_p}\underbrace{(\hat{n}_{\text{new}}^p - \hat{n}_c^p)}_{\Delta\hat{n}_p}\end{aligned} \tag{6}$$

where

$$\begin{aligned}\boldsymbol{\beta}_2 &= [\beta_2^1\ \ \beta_2^2\ \ \cdots\ \ \beta_2^p] \\ \hat{\mathbf{n}} &= [\hat{n}_1\ \ \hat{n}_2\ \ \cdots\ \ \hat{n}_p]^{\mathrm{T}}\end{aligned}, \quad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mp} \end{bmatrix} = [\mathbf{b}_1\ \ \mathbf{b}_2\ \ \cdots\ \ \mathbf{b}_p]$$

Recalling the quality margin concept, if $\hat{y}_{\text{new}}(\mathbf{x}_c^*, \hat{\mathbf{n}}_{\text{new}})$ in (6) does not go beyond the specified tolerance limits, $\mathbf{x}_c^*$ can be continuously used to control the process which will still ensure the acceptable quality (i.e. within tolerance). Hence, the quality margin specified by the tolerance is transferred to a noise margin associated with the current $\mathbf{x}_c^*$. The margin of the ith noise variable when other noise variables do not change is calculated by

$$\Delta\hat{n}_i = \hat{n}_{\text{new}}^i - \hat{n}_c^i = \pm\frac{\delta}{(\beta_2^i + \mathbf{b}_i^{\mathrm{T}}\mathbf{x}_c^*)} \quad \text{with } \hat{n}_{\text{new}}^j = \hat{n}_c^j \quad i, j = 1, 2, \ldots, p \text{ and } j \neq i \tag{7}$$

where $\pm\delta$ represents the tolerance. Using (7), we can determine how far a *single* noise variable can deviate from its current level without making the response out of quality specification under the current setting of controllable variables. It is noted that the quality margin will be further discussed and classified later in Section 3.1.

From (6), it can also be found that there is a phenomenon named *self-compensation of noise change* (SCNC). This phenomenon occurs when $\Delta\hat{n}$'s satisfy the condition $\sum_{i=1}^{p} k_i \Delta\hat{n}_i = 0$. In this situation, $\Delta\hat{y}$ is equal to zero which indicates no deterioration in responses even though noises do change. For example, suppose that the response y follows the model

$$y = 8 + 2x - n_1 + 0.5n_2 + xn_1 - 0.5xn_2 + \varepsilon \tag{8}$$

At the moment, suppose $x_c = -\frac{1}{7}$ and $(n_c^1, n_c^2) = (-0.5, -0.5)$ which control y_c at the specified mean target 8. At any other time, once $(\Delta\hat{n}_1, \Delta\hat{n}_2)$ satisfies the relationship of $\Delta\hat{n}_2 = 2\Delta\hat{n}_1$, the impact of those noise variables on the response is compensated by those noises themselves; thus, no adjustment is needed. This phenomenon is called SCNC. Apparently, some unnecessary adjustments will be avoided if such phenomenon can be considered in the control law design.

The concept and effectiveness of SCNC and quality margin are further illustrated with example (8) as shown in Figures 1(a)–(d).

Figure 1(a) shows the traditional control law design where $x_c^* = -\frac{1}{7}$ is only valid for the current noise level $(\hat{n}_c^1, \hat{n}_c^2) = (-0.5, -0.5)$. In other words, any changes in the noise variables will trigger the re-computation of a x_{new}^* and the operation of adjustment (i.e. 'always adjust' policy).

Figure 1(b) shows the case that the quality margin specified by the tolerance is considered in the control law design. Suppose that the tolerance is 8 ± 0.08, then $\hat{y}_{\text{new}}$ will not go beyond the tolerance limits when $-0.57 < \hat{n}_{\text{new}}^1 < -0.43$ or $-0.64 < \hat{n}_{\text{new}}^2 < -0.36$ alone. Hence, considering the quality margin in a

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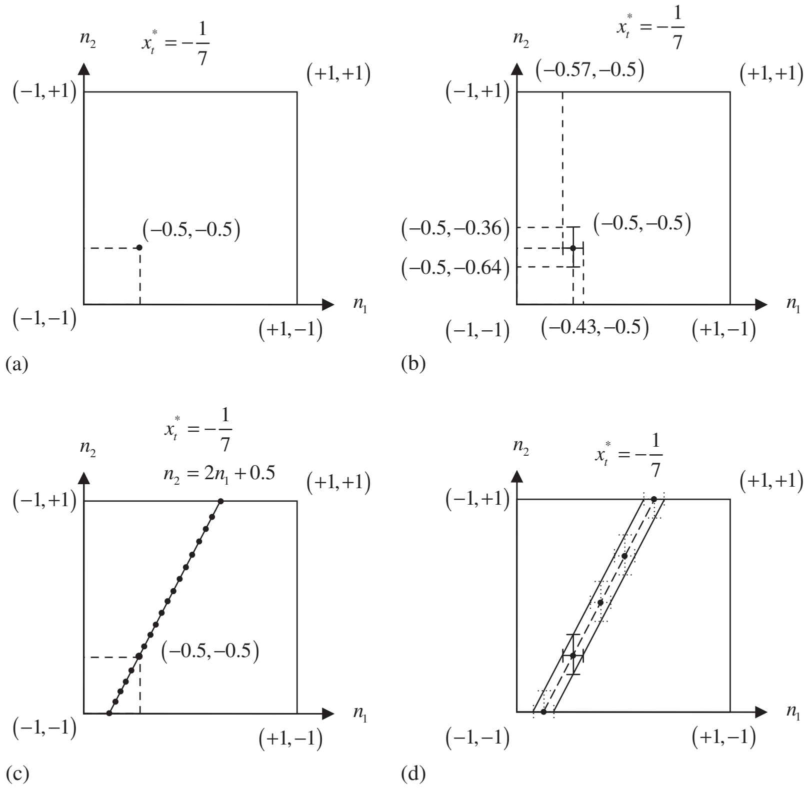

Figure 1. Valid noise region for the current x_c^* under different considerations in control law design

control law design expends the valid noise region for the current x_c^* from a single point to two orthogonal line segments.

Figure 1(c) shows the case when the SCNC is considered in the control law design. Using $k_1\Delta\hat{n}_1 + k_2\Delta\hat{n}_2 = 0$ and the noise observation at point $(-0.5, -0.5)$, a linear equation $\hat{n}_2 = 2\hat{n}_1 + 0.5$ is calculated. Under the current x_c^*, any pair of $(\hat{n}^1_{\text{new}}, \hat{n}^2_{\text{new}})$ at any other time on that line will not drive the response out of the quality tolerance. In this case, the valid noise region for the current x_c^* is expended from a single point to a straight line.

Figure 1(d) shows the control law design when both the quality margin and the SCNC are considered simultaneously. By combining 1(b) and 1(c), it can be found that the valid noise region for current x_c^* becomes a straight strip. At other times, any pair of $(\hat{n}^1_{\text{new}}, \hat{n}^2_{\text{new}})$ in this strip region will not make the response go beyond the quality tolerance under the current x_c^*. This example indicates that the frequency of real-time adjustment can be significantly reduced when both the quality margin and the SCNC are considered simultaneously.

3. RM-APC DESIGN WITH REDUCED ADJUSTMENT FREQUENCY

This section discusses how to fully utilize both the quality margin and the SCNC in the control law design to minimize the control frequency with an ensured product quality. With those considerations in mind, the proposed design approach is different from the existing RM-APC designs in optimization objective, optimization sequence, and traversal strategy. The discussion in this section is organized in two cases: single noise variable case and multiple noise variables case.

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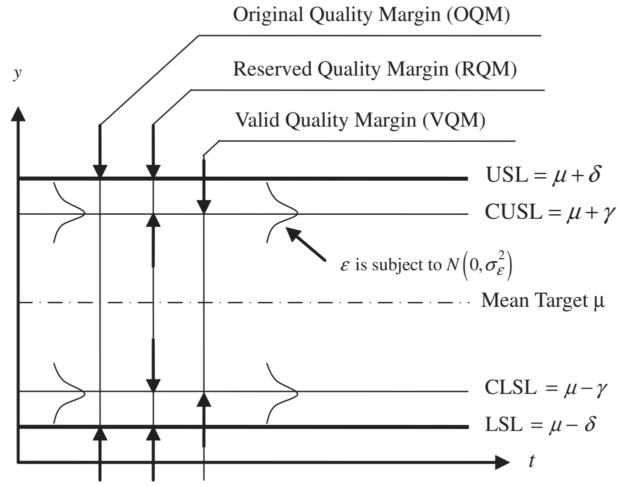

Figure 2. Boundaries in the proposed approach

3.1. Single noise variable case

In single noise variable case, only quality margin can be utilized for the adjustment frequency reduction purpose. In this case, the response deterioration in (6) is re-written as:

$$\Delta\hat{y} = (\beta_2 + \mathbf{b}_1^{\mathrm{T}}\mathbf{x}_c^*)(\hat{n}_{\text{new}} - \hat{n}_c) = k_1(\mathbf{x}_c^*)\cdot\Delta\hat{n} \tag{9}$$

From (9), it can be seen that $\Delta\hat{y}$ is a linear function of $\Delta\hat{n}$ with a variable slope $k_1(\mathbf{x}_c^*)$. Here, $k_1(\mathbf{x}_c^*) = \beta_2 + \mathbf{b}_1^{\mathrm{T}}\mathbf{x}_c^*$. The valid noise region for the current $\mathbf{x}_c^*$ is given by

$$\{\Delta\hat{n}_{\max} : |\Delta\hat{y}|\mathbf{x}_c^*, \Delta\hat{n}_{\max}| \leq \gamma\} \tag{10}$$

Here, γ determines the conservative upper specification limit (CUSL) and conservative lower specification limit (CLSL) for the process response. The relationship between CUSL/LUSL and USL/LSL is expressed in Figure 2. Use of valid quality margin (VQM) rather than original quality margin (OQM) in calculating (10) reflects that the modeling error is explicitly considered. A portion of OQM (i.e. reserved quality margin, RQM) is pre-reserved for disturbance of ε due to model uncertainties. The VQM is the actual quality margin which is utilized for adjustment reduction afterwards.

A distance of $3\sigma_\varepsilon$ between CUSL and USL (also CLSL and LSL) is suggested which ensures 99.73% of the controlled $\hat{y}$ that hits CUSL or CLSL will not make $y = \hat{y} + \varepsilon$ go beyond the specified tolerance limits. From Figure 2, it can be observed that the existence of ε contributes the probability that a response goes beyond the OQM. A larger σ_ε^2 provides more limitations on the performance of adjustment frequency reduction. Quantitative study on how ε impacts the performance of the proposed method is given in Section 4.1.

The control law design procedure can be illustrated as a traversing process in the noise domain so that for any level of noise, a corresponding $\mathbf{x}^*$ is available. The design procedure may start from the low level of the noise or from the high level of the noise. In this paper, traversing process starts from the low level of the noise. The main steps of the control law design are summarized below, and further illustrated in Figure 3.

Step 1: Set the current noise level n_c in (3). The original value of n_c is set as -1.

Step 2. Set $T = \text{CLSL}$ in (3) and calculate an $\mathbf{x}_{\text{temp}}^*$. Then check if $(\beta_2 + \mathbf{b}_1^{\mathrm{T}}\mathbf{x}_{\text{temp}}^*) > 0$: if yes, set $\mathbf{x}_c^* = \mathbf{x}_{\text{temp}}^*$; if not, set $T = \text{CUSL}$ in (3) and calculate another $\mathbf{x}_{\text{temp}}^*$ and check if $(\beta_2 + \mathbf{b}_1^{\mathrm{T}}\mathbf{x}_{\text{temp}}^*) < 0$, if yes, set $\mathbf{x}_c^* = \mathbf{x}_{\text{temp}}^*$. If both trials fail, then set $T = \mu$ in (3) and calculate the $\mathbf{x}_c^*$.

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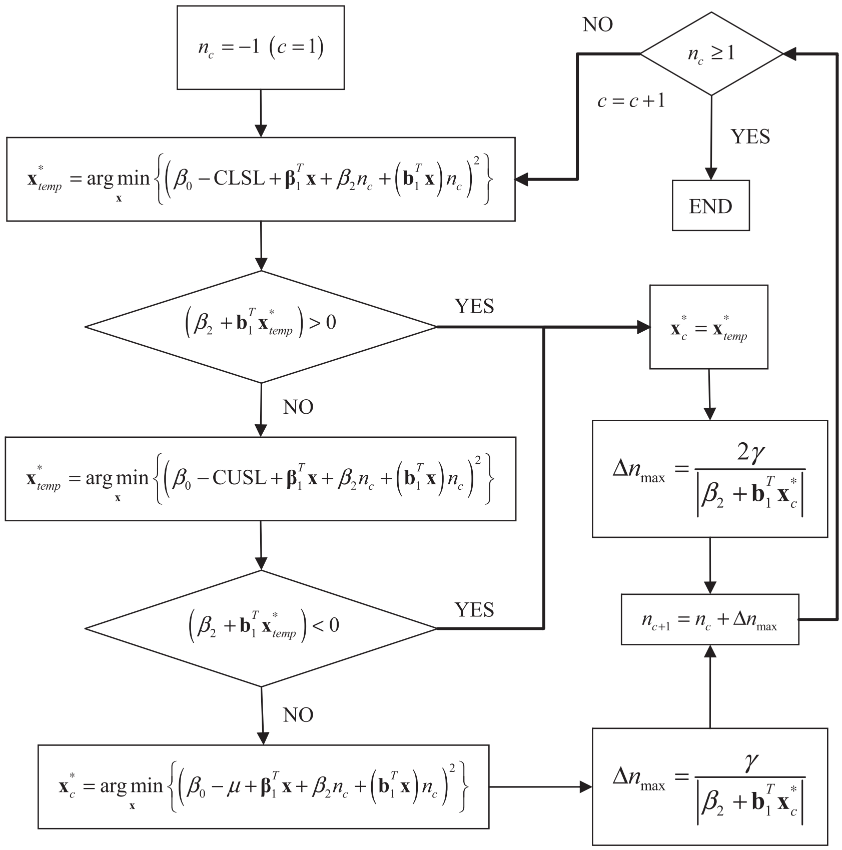

Figure 3. The flow chart for the proposed method in single noise case

Step 3: Based on (9), calculate the valid noise region for the current $\mathbf{x}_c^*$ using

$$\Delta n_{\max} = \begin{cases} \dfrac{2\gamma}{|\beta_2 + \mathbf{b}_1^{\mathrm{T}} \mathbf{x}_c^*|} & \text{if } \mathbf{x}_c^* \text{ obtained by setting } T = \text{CUSL(or } T = \text{CLSL)} \\ \dfrac{\gamma}{|\beta_2 + \mathbf{b}_1^{\mathrm{T}} \mathbf{x}_c^*|} & \text{if } \mathbf{x}_c^* \text{ obtained by setting } T = \mu \end{cases} \tag{11}$$

Step 4: Update the current noise level by $n_c = n_c + \Delta n_{\max}$ and repeat the Steps 1–4 until $n_c \geq 1$.

The rules presented in Step 2 aim to make sure that the direction of the response deterioration (i.e. the sign of $\Delta\hat{y}$) is towards the inner region of the quality margin. For example, if the current $\mathbf{x}_c^*$ controls the response to the CLSL, a positive $\Delta\hat{y}$ is expected. Because Δn is always positive during traversal, $(\beta_2 + \mathbf{b}_1^{\mathrm{T}} \mathbf{x}_c^*) > 0$ ensures a positive $\Delta\hat{y}$.

The resulting control law with reduced adjustment frequency in single noise variable case can be described by

$$\mathbf{x}^* = \begin{cases} \mathbf{x}_1^* & \text{if } \hat{n} \in [-1, n_2) \\ \mathbf{x}_2^* & \text{if } \hat{n} \in [n_2, n_3) \\ & \vdots \\ \mathbf{x}_F^* & \text{if } \hat{n} \in [n_F, +1] \end{cases} \tag{12}$$

where F represents the number of $\mathbf{x}^*$ in the proposed control law.

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3.2. Multiple noises case

In a multiple noises case, both the quality margin and SCNC can be utilized in the control law design. As shown in Figure 1(d), the central idea of the proposed method is to determine those hyper-strips corresponding to various $\mathbf{x}^*$'s and fill the multi-dimensional noise domain with them. The main procedures of the proposed method in the case of $p(p>1)$ noise variables are given as follows:

Step 1: Set the current noise level $\mathbf{n}_c$ in the center of the multi-dimensional noise domain.

Step 2: Substitute $T=\mu$ and the current noise level in (3) and calculate the current $\mathbf{x}_c^*$.

Step 3: Determine the valid noise region Π_c for the current $\mathbf{x}_c^*$ by two hyper-constraints $\{\mathbf{n}:(\boldsymbol{\beta}_2+\mathbf{B}^{\mathrm{T}}\mathbf{x}_c^*)(\mathbf{n}-\mathbf{n}_c)\le \mathrm{CUSL}\}$ and $\{\mathbf{n}:(\boldsymbol{\beta}_2+\mathbf{B}^{\mathrm{T}}\mathbf{x}_c^*)(\mathbf{n}-\mathbf{n}_c)\ge \mathrm{CLSL}\}$, together with the unit hypercube $\{\mathbf{n}:\|\mathbf{n}\|_\infty\le 1\}$.

Step 4: Update the current noise level in the geometrical center of the unexplored noise domain Ω (i.e. Ω is the complement of Π's with respect to the unit hypercube).

Step 5: Repeat the Steps 1–4 until the whole noise domain is traversed.

In Step 3, the valid noise region Π_c for the current $\mathbf{x}_c^*$ can be calculated and expressed as

$$\Pi_c:\ \mathbf{n}\ \text{subject to}\ \begin{cases}(\boldsymbol{\beta}_2+\mathbf{B}^{\mathrm{T}}\mathbf{x}_c^*)(\mathbf{n}-\mathbf{n}_c)\le \mathrm{CUSL}\\ (\boldsymbol{\beta}_2+\mathbf{B}^{\mathrm{T}}\mathbf{x}_c^*)(\mathbf{n}-\mathbf{n}_c)\ge \mathrm{CLSL}\\ \|\mathbf{n}\|_\infty\le 1\end{cases} \tag{13}$$

In step 4, suppose that the vertexes of the unexplored noise domain Ω are denoted by $\mathbf{D}_c=[D_1\ D_2\ \ldots\ D_V]$ where $D_v(d_1,d_2,\ldots,d_p)$, $v=1,2,\ldots,V$. V is the number of the vertexes. Here, the geometrical center of Ω is defined by

$$S_c\left(\frac{\sum_{v=1}^{V}d_1}{V},\frac{\sum_{v=1}^{V}d_2}{V},\ldots,\frac{\sum_{v=1}^{V}d_p}{V}\right) \tag{14}$$

The coordinates of the centric point are the arithmetic averages of the corresponding coordinates of vertexes. This treatment provides two useful properties in the proposed design approach: one is that (14) guarantees S_c to locate inside the unexplored noise domain Ω; another is that S_c is close to the centric area of Ω. The first property ensures that the determination of valid noise region in the next cycle occurs inside the unexplored noise domain. The second property contributes to the efficient traversing strategy. Recalling Figure 1(d), a centric current noise level in Ω provides a superior chance for the valid noise strip to occupy the unexplored noise domain farthest. Incidentally, setting the initial current noise in the center of the unit hypercube is also based on this consideration. Finally, for the next cycle, we have $\mathbf{n}_c=S_c$.

The resulting control law in multiple noises case can be described as

$$\mathbf{x}^*=\begin{cases}\mathbf{x}_1^* & \text{if } \hat{\mathbf{n}}\in\Pi_1\\ \mathbf{x}_2^* & \text{if } \hat{\mathbf{n}}\in\Pi_2\\ \vdots & \\ \mathbf{x}_F^* & \text{if } \hat{\mathbf{n}}\in\Pi_F\end{cases}\quad \text{and}\quad \sum_{i=1}^{F}\Pi_i=\{\mathbf{n}:\|\mathbf{n}\|\le 1\} \tag{15}$$

where F represents the number of $\mathbf{x}^*$ in the proposed control law.

The practical adjustment frequency is simultaneously impacted by both the value of F and the noise change pattern. Here, noise change pattern refers to both the change of noise frequency and the change of noise magnitude. On the condition of a moderate noise change pattern, the adjustment frequency is directly proportional to the value of F. Different noise change patterns determine different proportionality coefficients. That is, for example, significant changing noises will limit the reduction performance of a control law with F, and vice versa. The impact of noise change pattern on adjustment frequency reduction is further discussed via a case study in Section 4.1.

4. CASE STUDIES

Two case studies are presented to illustrate the proposed control law design, applicability, and performances. The first case study is based on the heat treatment process of leaf spring (Pignatiello and Ramberg[12]). A comprehensive comparison study is provided with regard to design procedures, control performance, and adjustment frequency between the proposed approach and the RM-APC (Jin and Ding[4]). The second case is an implementation study in a steel rolling process, which shows the applicability and effectiveness of the proposed approach.

4.1. *Simulation study with a heat treatment process of leaf spring*

Pignatiello and Ramberg[12] reported a leaf spring experiment. A forming process followed by a heat treatment process creates the curvature in leaf springs and should theoretically generate an unloaded spring with a height of 7.6 in. The objective of the process control is to make the free spring height as close to the target 7.6 in as possible. We use the same process model as that in Jin and Ding[4] for the comparison purpose.

The reduced response model with the significant factors is

$$\begin{aligned} y = {} & 7.6360+0.1106B+0.0881C-0.1298Q+0.0519H \\ & -0.0827CQ+0.0423BQ+\varepsilon \end{aligned} \tag{16}$$

where B, C, Q, and H denote high heat temperature, heating time, quench oil temperature, and hold down time, respectively; ε is the model error and subject to $N(0,\sigma_\varepsilon)$. Among these factors, Q and H are observable noise factors; B and C are controllable factors. All process factors are coded as a value in $[-1,+1]$. The variation patterns of noise factors Q and H are assumed to randomly change in a smooth and continuous pattern. For illustration, a section of a smooth time series model of noise variable Q generated for the simulation study is presented in Figure 4. Figure 4 also shows a section of a roughly changing Q model hypothetically generated for comparison purpose. In the leaf spring forming process, the quench oil temperature Q and hold down time H appear to vary smoothly and continuously.

Based on (3), the optimization function is given as:

$$\begin{aligned} & \min_{B,C}(7.6360-7.6+0.1106B+0.0881C-0.1298\hat{Q}+0.0519\hat{H}-0.0827C\hat{Q}+0.0423B\hat{Q})^2 \\ & s.t.\ B,C\in[-1,+1] \end{aligned} \tag{17}$$

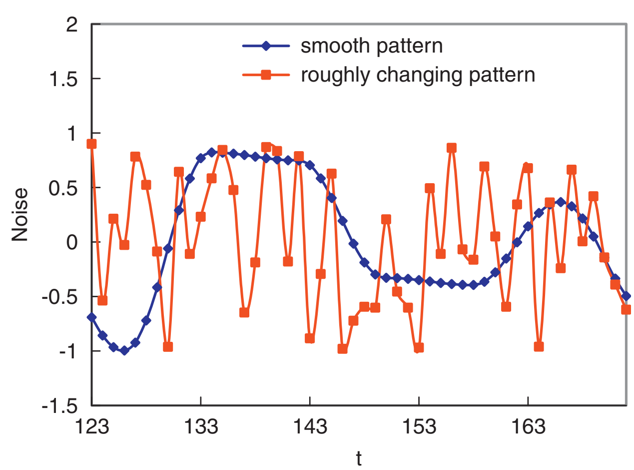

Figure 4. Illustration for the variation patterns of noise factors. This figure is available in colour online at www.interscience.wiley.com/journal/qre

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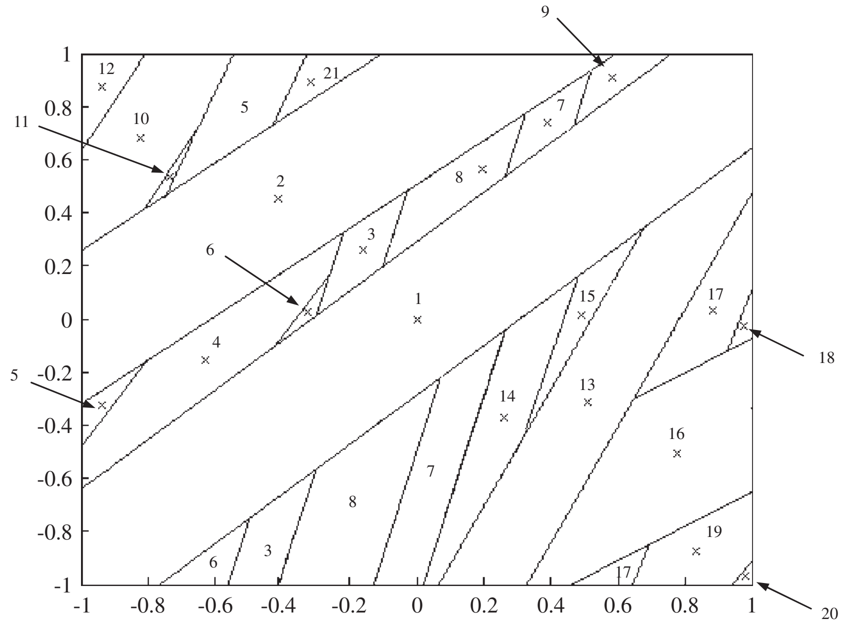

Figure 5. The graphic illustration of the proposed control law in a leaf spring case

Table I. The numerical result of the proposed control law in leaf spring case

Π_i	(B_i^*, C_i^*)	Π_i	(B_i^*, C_i^*)	Boundaries defining Π_i
1	$(0.33, -0.82)$	11	$(-1, -0.54)$	we present the boundaries of Π_8 for example:
2	$(0.09, -1)$	12	$(-1, -0.8)$	$-0.015 \leq -0.1058Q + 0.0519H - 0.0087 \leq 0.015$
3	$(-1, 0.33)$	13	$(0.35, 0)$	$-0.0483Q + 0.0519H \geq 0.015$
4	$(-0.23, -0.65)$	14	$(-0.21, 0.63)$	$-0.0483Q + 0.0519H \leq -0.015$
5	$(-1, -0.42)$	15	$(-0.14, 0.96)$	$-0.0429Q + 0.0519H - 0.0412 \leq -0.015$
6	$(-0.16, -0.55)$	16	$(0.76, -0.77)$	$H \geq -1$
7	$(-0.47, 0.65)$	17	$(0.45, 0.63)$	
8	$(-0.12, -0.35)$	18	$(0.65, -1)$	
9	$(-0.01, -0.14)$	19	$(0.83, -0.19)$	
10	$(-0.78, -0.76)$	20	$(0.9, 0.65)$	
		21	$(-0.36, -0.76)$	

Following the RM-APC design procedures presented in Section 2.1, the RM-APC control law is given as:

$$\begin{aligned}&(0.1106 + 0.0423\hat{Q})B^* + (0.0881 - 0.0827\hat{Q})C^* + (0.036 - 0.1298\hat{Q} + 0.0519\hat{H}) = 0\\&s.t.\ B^*, C^* \in [-1, +1]\end{aligned} \tag{18}$$

In this study, the tolerance for response y is $7.6^{+0.03}_{-0.03}$ (i.e. $\delta = 0.03$) and $\sigma_\varepsilon = 0.005$. A control law with reduced adjustment frequency is obtained following the design procedures depicted in Section 3.2. Both the graphic illustration and numerical result are given in Figure 5 and Table I, respectively.

The numbers in Figure 5 show the traversing sequence in the proposed design procedures and also represent the serial number of Π_i's. In each Π_i, the corresponding optimal setting of factors B and C in Table I guarantees the response within the specified tolerance. Totally 21 sets of optimal settings for B and C constitute the proposed control law.

In order to conduct a comparison study on control performance and reduction of adjustment frequency, the RM-APC control law in (18) and the proposed control law in Table I are used to control the simulated

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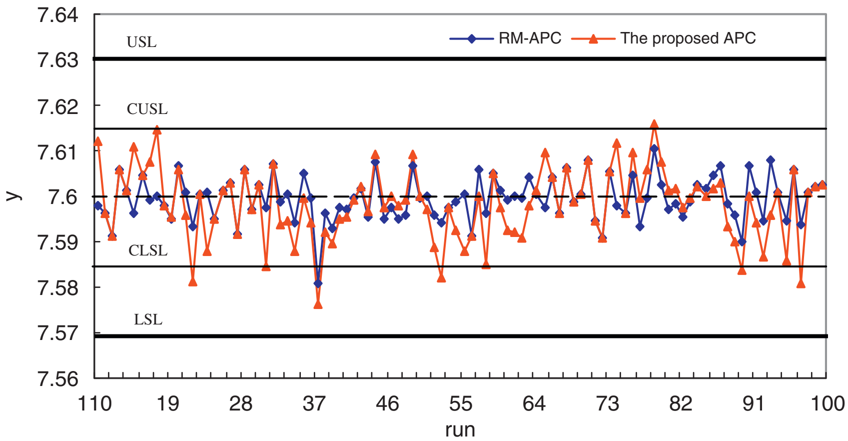

Figure 6. Controlled response and deviation of y. This figure is available in colour online at www.interscience.wiley.com/journal/qre

Table II. Comparison of the RM-APC control law and the proposed control law for the leaf spring forming process

Approach	Process mean	Process standard deviation	Adjustment frequency
RM-APC	7.6001	0.0051	16 000
The proposed control law	7.6002	0.0090	4040

leaf spring forming process. Sixteen thousand noise values are generated for Q and H, respectively. Thus, 16 000 simulated runs are conducted. A section of controlled responses under different control laws is shown in Figure 6. The comparison in some numerical indexes is given in Table II.

Both Figure 6 and Table II evidence the property of the proposed control law: it sacrifices the variability of responses to reduce the number of online adjustments. At the same time, the proposed approach still ensures the production within engineering specifications. This comparison study shows that the proposed control law is capable of sharply reducing the adjustment frequency while ensuring a qualified controlled process performance.

As we mentioned in Section 3.1, another comparison study is conducted to have an insight on how model error ε impacts the performance of the proposed control law. Replacing the value of $\sigma_\varepsilon = 0.005$ with 0.003, 0.004, 0.006, and 0.007, respectively, the calculation of the proposed control law and the process simulation is repeated. For comparability, the randomly generated Q and H series are kept the same in different simulation runs. The results are summarized in Figure 7.

By loss value sacrifice (LVS), we mean the difference between the total loss values of the processes controlled by the RM-APC and the proposed control law, respectively, i.e.:

$$LVS = LV_{RM-APC} - LV_{reduced-APC}$$

From Figure 7, it can be found that for the proposed approach, with the shrink of VQM (i.e. increased model error) the performance of adjustment frequency reduction is imprisoned and the performance of the process control approaches that of the RM-APC. Hence, in terms of adjustment frequency reduction, the proposed approach works better when there is a larger quality margin contributed by a more accurate RM. Nevertheless, the proposed approach can always deliver a control law with reduced adjustment frequency if a tolerance is available.

It is noted that the variation pattern of noise has an impact on the performance of adjustment frequency reduction. A comparison study is conducted by introducing noises with different changing smoothness into

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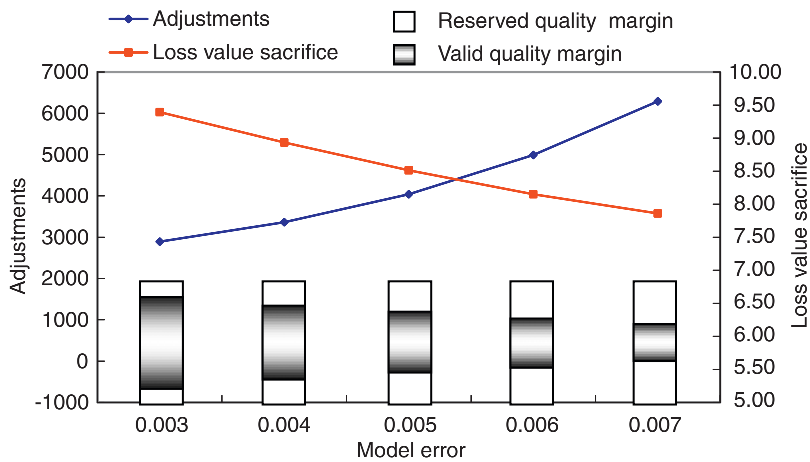

Figure 7. Comparison results on the impact of model error. This figure is available in colour online at www.interscience.wiley.com/journal/qre

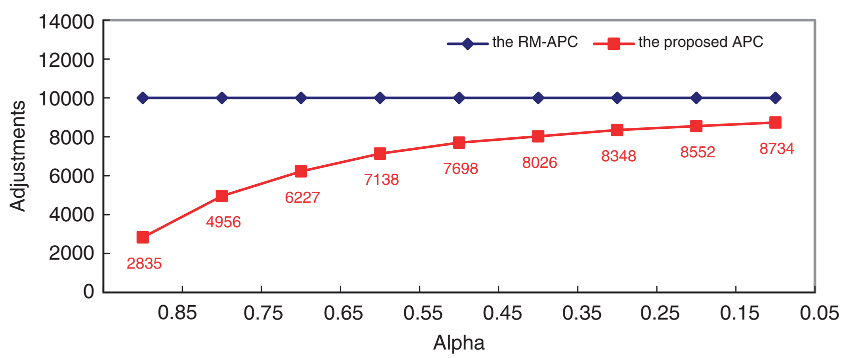

Figure 8. The practical adjustments under different noise patterns. This figure is available in colour online at www.interscience.wiley.com/journal/qre

the simulated leaf spring process. Here, an exponential-smoothing-like method is used for generating noises with different changing patterns, from roughness to smoothness. The noise generators for Q and H are given by

$$Q_t = \alpha_Q \times Q_{t-1} + (1-\alpha_Q) \times w_t \quad \text{and} \quad H_t = \alpha_H \times H_{t-1} + (1-\alpha_H) \times u_t \tag{19}$$

where w_t and u_t are white noises subject to $N(0, 0.36)$. By regulating the value of α, different changing patterns of Q and H can be simulated. Large α_Q makes Q_t close to Q_{t-1} with a small part of variation from w_t, which produces a smooth noise pattern. On the contrary, small α_Q produces a roughly changing noise pattern. Nine series of (Q, H) with nine different α's are applied to the simulated process and the results from 10 000-runs are organized in Figure 8.

From Figure 8, it can be seen that the proposed control law can greatly reduce the adjustments compared with the RM-APC when $\alpha = 0.85$ (i.e. very smooth pattern). And the difference in adjustments is largely narrowed when $\alpha = 0.05$ (i.e. very rough pattern). A trend is clearly observed that the increasing roughness of noise changing pattern gradually weakens the power of adjustments reduction possessed by the proposed approach. Hence, in terms of adjustment frequency reduction, the proposed approach works better when the noises change in a smoother manner during production.

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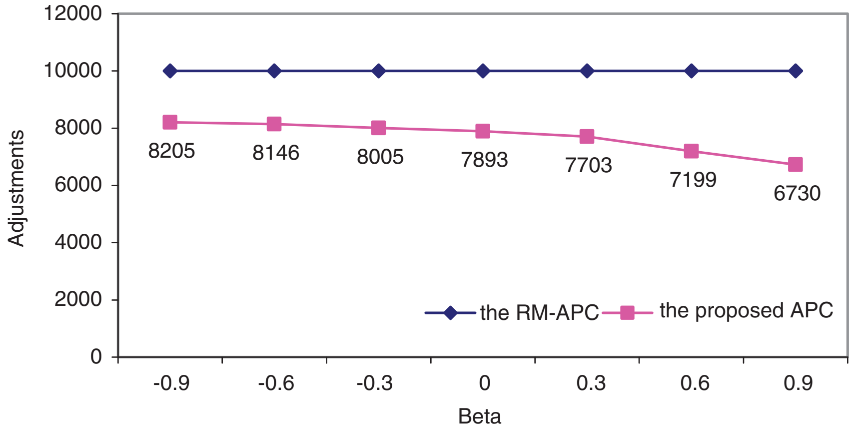

Figure 9. The practical adjustments under different noise correlations. This figure is available in colour online at www.interscience.wiley.com/journal/qre

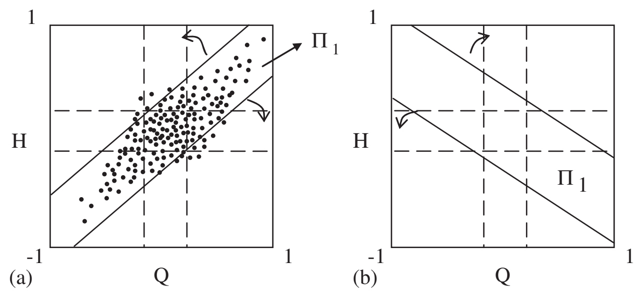

Figure 10. The impact of noise correlation on the performance of the proposed method (arrows indicate that the orientation of Π_1 could rotate clockwise or counter-clockwise to the status denoted by the dotted area). (a) The case that positive correlation of noises makes performance better and (b) the case that negative correlation of noises makes performance better

Although the noise factors are assumed to be independent of each other in the paper (refer to Section 2.1), one may be interested in how much the performance of the proposed method is affected by having correlation among noise factors. Hence, an explicit analysis is conducted. Here, noise factor Q and H are generated by

$$Q_t \text{ is subject to } N(0, 0.36) \quad \text{and} \quad H_t = \beta \times Q_t + \sqrt{1-\beta^2} \times \upsilon_t \tag{20}$$

where υ_t is an auxiliary random variable subject to $N(0, 0.36)$, and β is the correlated coefficient of Q and H and within $[-1, +1]$. This generator guarantees that the dependent noise factor H is still subject to $N(0, 0.36)$. By regulating the value of β, different degrees of dependency between H and Q can be simulated. Seven series of (Q, H) with seven different β's are applied to the simulated process and 10 000-runs results are organized in Figure 9.

From Figure 9, it can be seen that the performance of the proposed method gets better as the positive correlation of Q and H becomes stronger. This is because the orientation of Π_1 matches the plot of Q and H when they are positively correlated (see Figure 10(a)) in the leaf spring case. Compared with the other Π_i's, the orientation of Π_1 poses the majority of the impacts that noise correlations put on the performance. If there is a stronger positive correlation between Q and H, there are more chances that Q and H will be located in Π_1. Recall that if the noise pair (Q, H) is located in the same Π_i, no adjustment is needed. Thus, fewer adjustments are required in this case. On the contrary, if there is a stronger negative correlation

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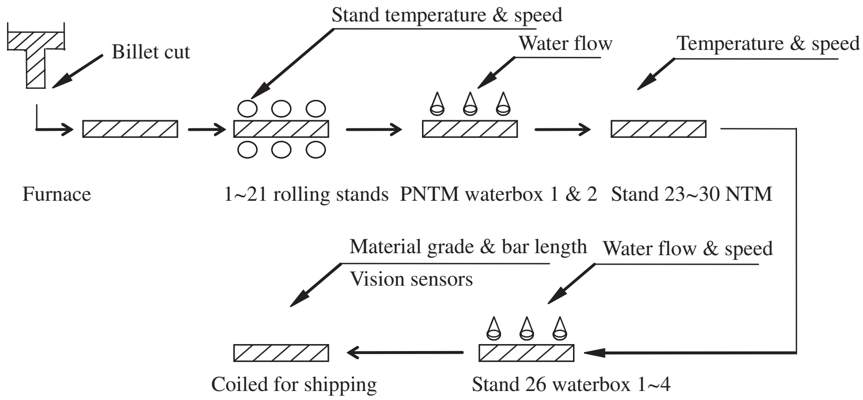

Figure 11. Hot rolling process and distributed sensing system

between Q and H, there are more chances that Q and H will scatter across various Π_i's. Then, more adjustments are needed. Moreover, it is noted that the orientation of Π_1 (i.e. slope) is determined by

$$-\frac{-0.1298-0.0827C_1^*+0.0423B_1^*}{0.0519}$$

where B_1^* and C_1^* are the optimal solution for Π_1 (i.e. when $(Q,H)=(0,0)$). Finally, the impact of noise correlation on the performance of the proposed method can be illustrated in Figure 10.

4.2. *Quality control in hot steel rolling process*

4.2.1. *Hot rolling process*

A hot steel rolling process follows the previous casting process and consecutively reduces the diameter of the billet through several stands. In the casting process, ingots and scrapes are charged into a bowl-shaped ladle and is heated in the furnace. The melted steel is then poured into a tundish and ready for continuous casting. The hot steel from each stand of mold will be cut into 10–12 billets, which are then transferred to progressive rolling containing 14–48 stands. The billets with reduced diameter coming from the final stand become the rolled products called 'steel bars'. At the end of the rolling process, the steel bars are coiled for shipping.

Numerous sensing data are available from different types of sensors installed along the rolling production line. An exemplary layout of the rolling process is given in Figure 11. The data for material grade and the length of steel bars, although not acquired through automatic sensing devices, are also available through off-line measurement. Besides, vision sensors are located at the last few stands of the rolling to take images of the surface of steel bars. With the development of the seam detection algorithms[13], the images are processed and the number of seams per steel bar is recorded as the measurement of the product quality.

4.2.2. *A logistic RM for rolling process*

The process response (i.e. the number of seams per steel bar) is transformed into a binary quality index Y (i.e. 'good' and 'bad' qualities). This is done by identifying the inherent clusters in data through principle component analysis (PCA). Then, a logistic RM is obtained by Jin *et al.*[14] expressing the relationship between a binary response variable and the predictors (i.e. the process variables), being:

$$\text{logit}(p)=\ln(p/(1-p))=29.24-0.09N-0.11X \tag{21}$$

where $p=\Pr(Y=1)$ is the probability of bad products; N is the average water flow (Gallon/Min) over two PNTM water boxes; X is the average water flow (Gallon/Min) over four water boxes after the last stand. N is practically difficult to be manipulated online so that it is treated as a noise factor. X is considered

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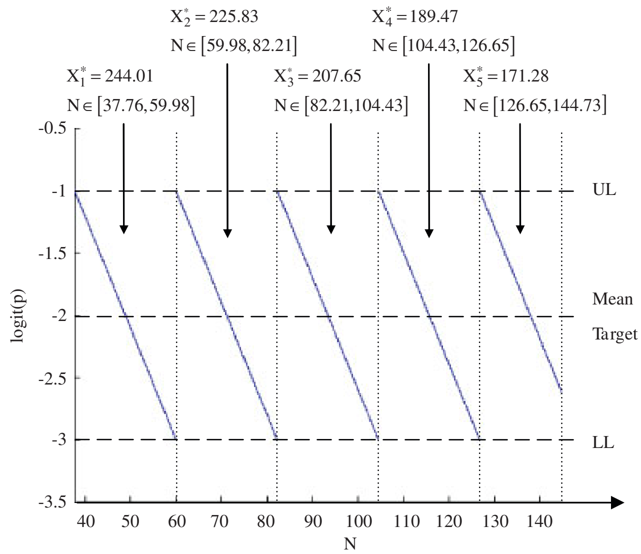

Figure 12. The proposed control law in a hot rolling process case. This figure is available in colour online at www.interscience.wiley.com/journal/qre

as an online controllable factor. This process model is valid within the ranges of $N \in [37.76, 144.73]$ and $X \in [130.19, 397.89]$.

4.2.3. Control strategy

The specified quality target is set by $\text{logit}(p) = -2^{+1}_{-1}$, which aims to control the probability of bad products within [4.74%, 26.89%]. Following the design procedures in Figure 3, substituting $\text{logit}(p) = -3$ in (21) and starting from the low level of N results in $X^*_{\text{temp}} = 262.19$. Then, calculating $\beta_2 + \mathbf{b}_1^{\text{T}} \mathbf{x}^*_{\text{temp}} = -0.09 < 0$ suggests that setting target to -3 is not correct. Substituting $\text{logit}(p) = -1$ and repeating the calculation result in $X^*_{\text{temp}} = 244.01$ and $\beta_2 + \mathbf{b}_1^{\text{T}} \mathbf{x}^*_{\text{temp}} = -0.09 < 0$. This solution meets the criteria depicted in Figure 3 and is set to be ultimate solution $X^*_1 = 244.01$. Then, update the current N value by $N = 37.76 + 2/0.09 = 59.98$ and repeat the calculation procedures until the whole range of N is traversed. The final control law is illustrated in Figure 12.

From Figure 12, it can be seen that the slopes of all response lines are consistent because there happens to be no X by N interaction term in the logistic RM. If there is an interaction term, the slope will change as X^* changes.

5. CONCLUSION

Process control is an important issue in manufacturing systems. Considering adjustment cost reduction, this paper proposed an APC design approach which is able to sharply reduce the adjustment frequency while ensuring a specified process performance. The central idea of the proposed design approach is to fully utilize the quality margin specified by tolerance and the self-compensation property of noise change. Through searching and determining the valid noise region for the current optimal settings of controllable factors, some unnecessary adjustments can be avoided. Therefore, the proposed design method expands the RM-APC by considering both process control performance and adjustment frequency. A fixed adjustment cost could also be added to the quadratic function so that the proposed approach could also benefit from the deadband adjustment policy by Box and Jenkins[7]. One may like to do so especially when the adjustment costs and the quality gain due to adjustments can be quantified.

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The case study of a leaf spring forming process shows that the proposed control law will reduce the adjustment frequency during production but still ensure the specified performance. Roughly 75% adjustments are avoided with the proposed control law when it is compared with the RM-APC approach. The simulation study suggests that a large product tolerance and a small modeling error will lead to more effective results in the adjustment reduction when using the proposed approach. The study also indicates that the proposed method is more efficient in the adjustment reduction when the process noises change in a smooth pattern. However, in terms of adjustment reduction, the worst performance for the proposed approach is that the practical adjustment frequency is equal to that of the RM-APC. In this case, the control performances of the two control laws are also the same. Therefore, the design of RM-APC can be considered to be a special case of the proposed approach when the VQM shrinks to zero. A real case study of the hot rolling process demonstrates the applicability of the proposed method.

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