THE APPLICATIONS OF HIERARCHICAL FILTER IN THE FLIGHT VEHICLE CONTROL SYSTEM

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ABSTRACT

The flight vehicle control system is a nonlinear, time varying, stochastic and large-scale system. In the system, the state estimation technique is needed to get its control low. A traditional method is changing the nonlinear model into a linear model. Then, Kalman filter can be used to get state estimation. However, the method has many disadvantages, such as low model precision, high computer calculating time and low anti-interference ability, etc. In this paper, a general nonlinear hierarchical filter is introduced. It has a recursive form in subsystems and low computer calculating time. It is especially suitable for the real time filtering in nonlinear large scale system. Moreover it has other advantages yet.

INTRODUCTION

The flight vehicle interception problem is nonlinear, time-varying, high dimensional and stochastic control problem. In practice, some state variables of the system are not certainly known. The state variables needed can only be estimated by means of the sencers and the filters in the control system. Thus, the problems of the filtering method and the filter design applied in the flight vehicle control system have been paid great attention. While the flight vehicle is controlled by way of non-active homing whose cost is lower than active homing, the vehicle-target distance is not known. However this distance is very important to improve the performance of the interception control system. So the study of the estimation of this distance is very important.

In this paper, the problem of the movement state estimation and the vehicle-target distance estimation applied in non-active homing flight vehicle are studied. At first, a hierarchic filtering algorithm is introduced. Then, we presented a hierarchic filter scheme which is used to estimate the movement state of the flight vehicle. After that, the estimation of the vehicle-target distance is studied: a distance estimate model is set up; a hierarchic filtering policy about distance estimate is proposed. At last, the computer simulation of both hierarchic filterings and stochastic control are made. Their estimate results are also given.

HIERARCHIC FILTERING ALGORITHM OF STOCHASTIC LARGE-SCALE NONLINEAR SYSTEMS

Hierarchic filtering theory has been paid attention in the past years. However, most of them are too complex in the computation and the realization. In this paper, a hierarchic filtering algorithm based on the extended Kalman filtering is used.

A stochastic large-scale nonlinear system is described as

$$X_{k+1} = f(X_k, k) + W_k \tag{1}$$

$$Z_{k+1} = h(X_{k+1}, k) + V_{k+1}$$

$$k = 0,\ 1,\ \ldots$$

Where $X_k \in R^n$, $Y_k \in R^m$, the vectors X_o, W_k, V_{k+1}, $k=0, 1, \ldots$ are assumed to be mutually independent Gaussian random variables with known statistical laws:

$$X_o \sim N(\hat{X}_o,\ P_o),\ W_k \sim N(\mu_w, Q_w),\ V_k \sim N(\underline{O},\ R_v)$$

The notation $V \sim N(a, B)$ denote that the random vector V is Gaussion with mean a and covariance B. Here $P_o > 0$, $Q_w \geqslant 0$, $R_v > 0$.

The large-scale system (1) is decomposed into following n_p subsystems:

$$X_{ik+1} = f_i(X_{ik}, T_{ik}, k) + W_{ik}$$

$$Z_{ik+1} = h_i(X_{ik+1}, T_{ik+1}, k) + V_{ik+1} \tag{2}$$

$$i = 1,\ 2,\ \ldots\ n_p \quad (n_p < m)$$

where T_{ik} is cognate variable:

$$T_{ik} = g_i((X_{jk}), k) \tag{3}$$

$$i = 1,2,\ldots n_p,\ j=1,2,\ldots n_p,\ j \neq i$$

Having decomposed the large-scale system (1) into subsystems (2), we can use a hierarchic structure to estimate state in system (1). Because the estimate is made parallell in subsystems, the calculating time can be reduced significantly.

The basic policy in the filtering algorithm is as follows:

1. The second level provides cognate variables to each subsystem filter. In order to get a hierarchic structure, we use improve-coordinating scheme (preestimate-correcting method) to get cognate variable.

$$T_{ik+1} = g_i((X_{jk+1/k}), k+1) \quad (i=1,2,\cdots n_p \quad j=1,2,\cdots n_p \quad j \neq i) \tag{4}$$

2. Since cognate variable is provided by the second level, every subsystem filter in the first level is independent.

Thus, the extended Kalman filter can be used directly to make state estimation.

Above algorithm can be done according to the folloing steps:

Step 1: Initialization, set $X_i = X_{io}$, $T_{io} = g_i((X_{jo}), 0)$, $i=1,2,\cdots n_p$, $j=1,2,\cdots n_p$ $(j \neq i)$.

Step 2: $X_{ik+1/k}$ and T_{ik+1} $(i=1,2,\cdots n_p)$ are calculated in the second level. Then, the results is sent to the first level.

Step 3: $P_{ik+1/k}$, $K_{ik+1/k}$, P_{ik+1} and X_{ik+1} $(i=1,2,\cdots n_p)$ are calculated in the first level. Then, x_{ik+1} is sent to the second level.

Step 4: $k \rightarrow k+1$ return to step 2.

HIERACHIE FILTER OF MOVEMENT STATE OF FLIGHT VEHICLE

The flight vehicle movement can be formulated as following state equation:

$$\dot{X} = f(X, U, t) + W \tag{5}$$

where $X = (x_1, x_2, \cdots x_8)^T$; $U = (u_1, u_2, u_3)^T$;

$W = (W_1, W_2, \cdots W_8)^T$, $W \sim N(0, Q_w)$; $X_o \sim N(X_o, P_o)$

The observation equation used in flight vehicle movement stat estimation is as follows:

$$Z = C^T X + V \tag{6}$$

where: $C = (0,0,K_3,0,0,K_6,0,K_8)^T$; $V=(V_1,V_2,V_3)^T$, $V \sim N(0,R_v)$.

Eight state variables are contained in the stochastic system which is formed by Eq.(5) and Eq.(6). If extend Kalman filter is used to estimete state directly. Large amounts of computer time and capacity are required. This can not satisfy the requirement of real-time flight vehicle inteception problem. Thus, the large system is divided into 3 subsystems, which is consistent with pitch, yaw and roll canals in the flight vehicle. The hierarchic filter model can be presented as follows:

Subfilter 1:

$$\dot{x}_1 = p_1x_1 - p_1x_3 + p_2u_1 + p_3 + W_1$$

$$\dot{x}_2 = p_4x_1 - p_4x_3 + p_5x_2 + p_6u_1 + t_{11} + W_2$$

$$\dot{x}_3 = t_{12}x_2 + t_{13} + W_3$$

$$Z_1 = K_3X_3 + V_1$$

Cognate variable; $T_1 = (t_{11}, t_{12}, t_{13})^T$

$$t_{11} = p_7x_7 + p_8x_5x_3$$

$$t_{12} = 57,3\cos x_8$$

$$t_{13} = 57.3x_5 \operatorname{Sin} x_8$$

Subfilter 2 :

$$\dot{x}_4 = p_9x_4 - p_9x_6 + p_{10}u_2 + p_{11} + W_4$$

$$\dot{x}_5 = p_{12}x_4 - p_{12}x_6 + p_{13}x_5 + p_{14}u_2 + t_{21} + W_5$$

$$\dot{x}_6 = t_{22}x_5 + t_{23} + W_6$$

$$Z_2 = K_6x_6 + V_2$$

Cognate variables: $T_2 = (t_{21}, t_{22}, t_{23})^T$

$$t_{21} = p_{15}x_7 + p_{16}x_2x_7$$

$$t_{22} = 57.3\cos x_8/\cos x_3$$

$$t_{23} = -57.3x_2\sin x_8/\cos x_3$$

Subfilter 3:

$$\dot{x}_7 = p_{17}x_7 + p_{18}u_3 + p_{21} + t_{31} + w_7$$

$$\dot{x}_8 = 57.3x_7 + t_{32} + w_8$$

$$z_3 = K_8x_8 + w_8$$

Cognate variables: $T_3 = (t_{31}, t_{32})^T$

$$t_{31} = p_{19}x_5 + p_{22}x_2x_5 + p_{20}x_2$$
$$t_{32} = -57.3x_5 tg x_3$$

where $p_1, p_2 \dots p_{22}$ are known functions of the state vector, which are derived according to the requirement of the extended Kalman filter and calculated by use of preestimate-correcting method as cognate variables.

Now, the flight vehicle movement state estimation can by means of the above hierarchic filter algorithm.

VEHICLE-TARGET DISTANCE ESTIMATOR OF NON-ACTIVE HOMING FLIGHT VEHICLE

At first, the mathematic model of distance estimator should be set up. So, a coordinate system $O_T x_9 y_9 z_9$ is defined as that the origin O_T of $O_T x_9 y_9 z_9$ is fixed in the center of target; axis $O_T x_9$ is parallel with axis OX; which is in the earth surface coordinate system OXYZ and the direction of the axis $O_T x_9$ is inverse direction of the axis OX; axis $O_T y_9$ is vertical to the horizontal plane and its direction is upward; axis $O_t z_9$ is defined according to the right-hand principle. The coordinate system $O_T x_9 y_9 z_9$ will move following the target. Assume the flight vehicle position in coordinate system $O_T x_9 y_9 z_9$ is (x_9, y_9, z_9). Then, the vehicle-target distance estimate equation can be given as:

$$\begin{aligned} \dot{x}_9 &= x_{10} \\ \dot{x}_{10} &= -a_{Vx} + a_{Tx} \\ \dot{y}_9 &= y_{10} \\ \dot{y}_{10} &= a_{Vy} - a_{Ty} \\ \dot{z}_9 &= z_{10} \\ \dot{z}_{10} &= -a_{Vz} + a_{Tz} \end{aligned} \tag{7}$$

where a_{Vx}, a_{Vy}, a_{Vz} (a_{Tx}, a_{Ty}, a_{Tz}) are projections of vehicle (target) acceleration along axes: OX, OY and OZ in the earth-surface coordinate system. Here a_{Vx}, a_{Vy}, a_{Vz} have following relationship:

$$\begin{aligned} a_{Vx} &= \cos x_6 \cos x_3 a_1 + (-\sin x_3 \cos x_6 \cos x_8 + \sin x_6 \sin x_8) a_2 \\ &\quad + (\cos x_6 \sin x_3 \sin x_8 + \sin x_6 \cos x_8) a_3 \qquad (8) \\ a_{Vy} &= \sin x_3 a_1 + \cos x_3 \cos x_8 a_2 - \cos x_3 \sin x_8 a_3 \end{aligned}$$

$$a_{Vz} = -\sin x_6 \cos x_1 a_1 + (\sin x_3 \sin x_6 \cos x_8 + \cos x_6 \sin x_8) a_2 + (-\sin x_3 \sin x_6 \sin x_8 + \cos x_6 \cos x_8) a_3$$

where a_1, a_2, a_3 are projections of vehicle acceleration along three axes of the vehicle coordinate system.

The state equations of the seeker used for the homing can be deriven from its angle following dynamic equations as:

$$\begin{aligned} \dot{u}_y &= -K_1K_2u_y + K_1(y_9x_{10} - x_9y_{10})/(x_9^2 + y_9^2) + W_9 \\ \dot{u}_z &= -K_1K_2u_z + K_1(-x_{10}z_9 + x_9z_{10})/(x_9^2 + z_9^2) + W_{10} \end{aligned} \quad (9)$$

The observation equations of the seeker can be presented as:

$$\begin{aligned} z_4 &= K_yu_y + V_4 \\ z_5 &= K_zu_z + V_5 \end{aligned} \quad (10)$$

The system, which is formed by Eq.(7) and (9), can be simlified with the help of following assumption: (1) Compared with the flight vehicle acceleration, the target acceleration is small and can be regarded as a white Gaussian noise with zero mean; (2) flight vehicle movement state has been estimated, thus, a_{Vx}, a_{Vy}, a_{Vz} can be got from the vehicle acclerometer and the Eq.(8); (3) both the acceleration noise and the vehicle movement state estimate error is summed up as model noise; (4) the target will move only on the OXZ plane.

Under the above assumptions, hierarchic filter model can be obtained by means of introducing cognate variables. The model is:

Subfilter 4:

$$\begin{aligned} \dot{x}_9 &= x_{10} \\ \dot{x}_{10} &= -\hat{a}_{Vx} + W_{11} \\ \dot{y}_9 &= \hat{y}_{10} \\ \dot{u}_y &= -K_1K_2u_y + K_1(y_9x_{10} - \hat{y}_{10})/(x_9^2 + y_9^2) + W_9 \\ Z_4 &= K_yu_y + V_4 \end{aligned}$$

cognate variable: $T_4 = (\hat{a}_{Vx}, \hat{y}_{10})^T$

Subfilter 5:

$$\begin{aligned} \dot{z}_9 &= z_{10} \\ \dot{z}_{10} &= \hat{a}_{Vz} + W_{12} \\ \dot{u}_z &= -K_1K_2u_z + K_1(-x_{10}^o z_9 + x_9^o z_{10})/(x_9^{o2} + z_9^{o2}) + W_{10} \\ Z_5 &= K_zu_z + V_5 \end{aligned}$$

cognate variable: $T_5 = (\hat{a}_{Vz}, x_9^o, x_{10}^o)^T$

where $\hat{a}_{Vx}$ and $\hat{a}_{Vz}$ are got by substituting ($\hat{x}_1, \hat{x}_3, \hat{x}_4, \hat{x}_6, \hat{x}_8$). into Eq.(8); $\hat{y}_{10} = V\sin\hat{x}_1$; $x_9^o = x_9$, $x_{10}^o = x_{10}$.
with the help of the hierarchical filter algorithm mention above, a distance estimator of non-active homing flight vehicle can be realized

COMPUTER SIMULATION STUDIES OF HIERARCHIC FILTER

The main parameters of hierarchic filter used to estimate flight vehicle movement state are as follows: $x_o = \underline{0}$,

P_o = diag(1.0, 0.01, 5.0, 0.01, 0.01, 0,01, 0.01, 0.01)

R_V = diag(0.01, 0.01, 0.01)

Q_W = diag(0.02, 0.015, 0.02, 0.03, o.o15, o.01,0.01)

The evaluation of the estimate performance can be done by means of the mean and variance of the state estimation error, which is shown in the table 1.

TABLE 1
The mean and variance of the state estimation error

	Δx_1	Δx_2	Δx_3	Δx_4
mean	0.0196	0.00325	0.00646	0.0366
variance	0.0380	0.00178	0.0493	0.00647
	Δx_5	Δx_6	Δx_7	Δx_8
mean	0.00149	0.0305	-0.0107	-0.0259
variance	0.00143	0.0427	0.000808	0.0189

Note: The mean and variance in table 1 is defined as :

$$\text{mean} = \frac{1}{T}\sum_{j=1}^{T} \Delta x_i(j) \quad ; \quad \text{variance} = \frac{1}{T}\sum_{j=1}^{T} \Delta x_i^2(j);$$

(i=1,2,...8), $T = (t_f - t_o)/\Delta t$. Where t_o and t_f are control initial time and terminal time, Δt is the sampling interval

The main parameters of hierarchic filter used to estimat flight vehicle-target distance are as follows:

$$\hat{x}_o = (x_9(t_o), x_{10}(t_o), y_9(t_o), 0, z_9(t_o), z_{10}(t_o), 0)^T$$

P_0 = diag(5, 10, 0.1, 0.01, 10, 183, 0.01)

Q_0 =diag(0.1, 60, 0.05, 0.01, 0.1, 65, 0.01)

R_0 = diag(0.01, 0.01)

Table 2 shows the distance estimation error under the worse condition.

TABLE 2
Distance estimation error

t(s)	1.8	3,6	5.4	7.2	9.0	10.8
x_9(m)	4.5	8.55	4.05	1.8	0.6	0.0
y_9(m)	-1.4	-2.7	-2.5	-1.4	-0.5	-0.2
z_9(m)	22.5	19.0	14.0	2.5	-3.5	0.0

From table 1 and table 2, it can be seen that both movement state and distance estimation are satisfied. In estimaing flight vehicle movement state, all state estimates are basically un-bias and have only a very small estimate error variance. In the vehicle-target distance estimation, all state estimate errors are decreased to zero gradually. And the distance estimates have a high precision near terminal point.

Stochastic Control Simulation of the flight vehicle interception problem has been done by combining the hierarchic filter presented in this paper and controller presented in ref.(3). Simulation results show that the flight vehicle can hit the target precisely under various conditions.

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