A Generalized Forecasting Compensatory Control Strategy and Its Applications in Vibration Control

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Abstract

A generalized Forecasting Compensatory Control (GFCC) strategy is developed using the receding forecasting horizon approach and a simplified i-step ahead forecasting model. The characteristics and robustness of this GFCC algorithm are presented and analyzed. This algorithm has been successfully applied to the vibration control of an experimental beam subjected to impulse as well as random excitations. Both simulation and experimental results show the new GFCC strategy very effective.

1. Introduction

Error compensation techniques have been used frequently in precision machining so that moderately accurate machine tools can be used cost-effectively to produce higher precision parts. Among many compensation techniques, a family of innovative control strategy makes use of in-process sensing and stochastic modeling of machining errors, to predict in real time the impending machining errors and then to compensate for the forecasted errors. This strategy is called Forecasting Compensatory Control (FCC) and has been applied to many machining processes [2,5,6].

Recently, this strategy is applied to the application of structural vibrations control, where vibrations are sensed, modeled and forecasted in real time and a counter excitation is exerted to cancel the unwanted structural vibrations. However, this FCC strategy exhibits some limitations with regard to the saturation of actuators and the guarantee of controlled system stability. Therefore, a generalized FCC (GFCC) strategy is developed.

In this short paper, a simplified i-step ahead forecasting model for the GFCC strategy is first presented and its property is discussed. A control law is then developed and its robustness is analyzed. To demonstrate its efficiency, this GFCC strategy has been applied to the active control of structural vibrations and experimental results are presented.

2. GFCC Strategy

For simplicity in the derivation, an ARMAV model is transformed into a state space equation [1,3]:

$$X(k+1) = AX(k)+BU(k)+GW(k)$$
$$Y(k+1) = CX(k)+BU(k)+V(k+1) \tag{1}$$

where the vectors $\{X(0), W(k), V(k+1), k=0,1,2,..\}$ are assumed to be Guassian random variables with $X(0) \sim N[0,\sigma_X^2(0)]$, $W(k) \sim N[0,\sigma_W^2(k)]$, and $V(k) \sim N[0, \sigma_V^2(k)]$.

2.1 i-Step Ahead Forecasting Model

By assuming the future control action is zero$\{U(j)=0, j=k+1,...k+i-1\}$ in an optimal i-step ahead forecasting model [1], we get the i-step ahead forecasting model for the GFCC strategy:

$$\hat{Y}(k+i|k) = CA^i\hat{X}(k)+CA^{i-1}BU(k) \tag{2}$$

Theorem 1: As long as the closed-loop control system (1) is stable, the i-step ahead forecasting model (2) is stable and its variance matrix of forecasting errors convergences to the optimal forecasting error variance.

2.2 GFCC Control Law

Consider a receding-horizon control index of the form:

$$J(k)=\frac{1}{2}E\left\{\sum_{j=N_1}^{N_2} \|\hat{Y}(k+i|k)-Y^*(k+i)\|_Q+\|U(k)\|_R\right\} \tag{3}$$

where $\|D\|_Q = D^TQD$, N_1 (N_2) is the minimum (maximum) forecasting horizon. Q and R are weight matrices and $Q \geq 0$, $R>0$. $\{Y^*(k+i);\ i=1,2,...\}$ is the desired output sequence.

By minimizing the control index (3), the GFCC control law can be expressed as:

$$U(k)=-\left\{\sum_{j=N_1}^{N_2} \|CA^{i-1}B\|_Q+R\right\}^{-1}$$
$$\left\{\sum_{j=N_1}^{N_2}\left\{CA^{i-1}B\right\}^T Q\left\{CA^i\hat{X}(k)-Y^*(k+i)\right\}\right\} \tag{4}$$

3 Stability Robustness Analysis

Assume all eigenvalues of (A-BK) be inside a disk of radius $(1+\alpha)^{-1/2}$, $\alpha>0$. K is the feedback vector in GFCC controller. The system is affected by deterministic but unknown perturbations, i.e.,

$$X(k+1)=(A-BK)X(k)+\Delta X(k) \tag{5}$$

where

$$\Delta = \begin{bmatrix} \Delta\phi_1 & \Delta\phi_2 & \cdots & \Delta\phi_n \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Delta\phi_i$ is the parameter estimation confidence interval in ARMAV modeling [3].

Theorem 2: The closed-loop system is globally uniformly stable if

$$\|\Delta\| = \sqrt{\Delta\phi_1^2+\Delta\phi_2^2+\bullet\bullet\bullet+\Delta\phi_n^2} \le \mu_1$$

$$\mu_1 \le [(1+\alpha^{-1})^{-1} - \alpha\rho^2(A-BK)]^{\frac{1}{2}}$$

where $\rho(A-BK)$ is the spectral radius of (A-BK).

4. Experiment Results

In order to demonstrate the effectiveness of this GFCC strategy, the GFCC strategy has been implemented to control the vibrations of an experimental cantilever beam. Fig. 1 shows the experimental setup.

The experiment was performed in three steps: system modeling, simulation of the controller and implementation of the controller. Two-DOF (Degree of Freedom) problem was studied in the experiment. An ARMAV(4,4,3) model was fitted using the DDS methodology [3]. A controller based on the GFCC strategy is then developed. The experiment results of controlling the vibrations of both the first and second natural modes are shown in Fig.2.

The robustness boundary of the developed GFCC controller is calculated for different α ($0.0<\alpha<1.0991$) and the results are shown in Fig.3. If using 2σ confidence interval of parameter estimates in ARMAV model, we have $\|\Delta\|=0.0002<\mu_1=0.1626$. Thus, the developed GFCC controller is quite robust.

5. Conclusion

Based on above analysis and experiment study, the following conclusions can be drawn:

1.A new computer control strategy - the GFCC strategy, has been developed. By using a receding-horizon approach and the GFCC i-step ahead forecasting model, an analytical control law is obtained. Several control methods such as FCC, MV, and GMV can be derived as subsets of this GFCC strategy by choosing different parameters in the GFCC. Compared with GPC[4], the GFCC can be easily used in MIMO system.
2.The GFCC has been successful in the control of a beam vibration subject. The vibrations of both first and second natural modes are reduced.
3.From the robustness analysis, it is shown that the GFCC can tolerate the maximum parameter perturbation caused by the ARMAV modeling errors.

6. References

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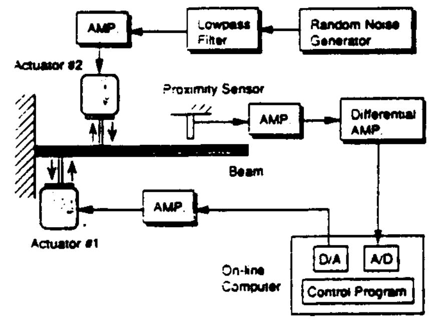
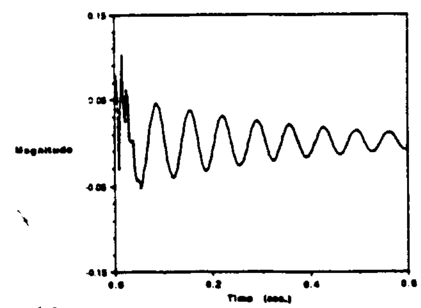
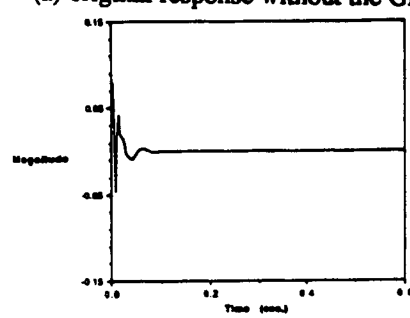


Fig.1 Experimental setup for beam vibration control



(a) original response without the GFCC



(b) response with the GFCC using a Two-DOF model
Fig. 2 Actual response of the beam to a pulse impact

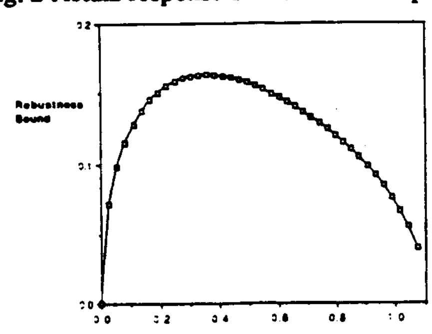


Fig. 3 Maximum robustness bound with different α