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AN ADAPTIVE CAUTIOUS PREDICTIVE CONTROLLER FOR REAL-TIME IMPLEMENTATION

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ABSTRACT

An adaptive cautious predictive (ACP) control algorithm is presented for a class of nonlinear stochastic systems, which include linear adaptive control systems. By introducing a new long-range predictive control loss function, an easily implemented control law is derived in closed-form. The resulting control law is not only suitable for real-time implementation, but it also posesses caution in the sense that the error covariance matrix of the estimated states are taken into consideration. Consequently, the ACP controller exhibits good robustness with respect to system uncertainties. Simulation results are provided to demonstrate these properties.

1 INTRODUCTION

In the past decade, predictive control techniques have been the focus of much research, due to their relative ease in implentation and their success in many industrial applications (Richalet et al., 1978), (De Keyser et al., 1988). Although their foundation lies in the closely related optimal stochastic control, there are significant differences[1]. In general, the main difference between the two techniques is the form of the loss function that is minimized. In optimal stochastic control the loss function takes the form of the statistical expectation of some, say positive definite quadratic, function of the future states and inputs of the system (Aoki, 1967). The expectation is over the basic underlying random variables of the system and can be unconditional or conditioned upon the available measurements at the current time. In contrast, in predictive control, the loss function is generally of the form of a similar quadratic function of the predicted value of the future states and input, based on a suitable model of the system (De Keyser et al., 1988). Most commonly, the predicted future states are their conditional means, conditioned upon all available data at the current time, and the inputs considered deterministic as a result of the "open-loop feedback" assumption (Aoki, 1967), (Tse and Athans, 1972), (Bar-Shalom and Tse, 1974). Thus, a significant difference between optimal stochastic control and predictive control results from the difference between the expressions $E[X^TQX]$ (optimal stochastic control) and $E[X]^TQE[X]$ (predictive control), where $E[\cdot]$ denotes an expectation, Q is a weighting matrix, and X is a random vector. Another significant difference is that optimal stochastic control usually incorporates a fixed horizon control index, while predictive control laws are usually based on a receding horizon one. However, in this paper we focus more on the former difference.

Most of the popular predictive control strategies were first developed for linear systems. Recent surveys/comparative studies of such methods are provided in Kramer and Unbehauen (1992), De Keyser et al. (1988), Garcia et al. (1989), and Scattolini and Bittanti (1990). Among the more widely used methods are Dynamic Matrix Control (Cutler and Ramaker, 1980), Generalized Predictive Control (Clarke, et. al., 1987), and Model Algorithmic Control (Rouhani and Mehra, 1982). The majority of these algorighms were originally derived using an input/output representation of the system, but, as pointed

[1]The authors of this paper have not encountered any precise, widely accepted definitions of "optimal control" and "predictive control" that illustrate their differences. The distinctions made between the two control approaches in the introductory paragraph do, however, seem to reflect a fundamental difference between the various controllers labeled as either "optimal" or "predictive" in the control literature. In spite of the obvious abuse of language (predictive controllers are also optimal in the sense that they, too, minimize a loss function), we adopt this terminology as a convenient means of distinguishing between two different control approaches.

out in Li et al. (1989), most have equivalent state-space forms. The main differences between the various predictive control algorithms are 1) the type of model used for the i-step ahead prediction, whether it be a step-response, impulse response, or transfer function model; 2) whether or not the minimization is a constrained one, with equality or inequality constraints placed on the future states and input; and 3) specific assumptions regarding the control and prediction horizons. It is interesting to note that, although the loss functions are quite different, for certain systems where certainty equivalence applies the receding horizon optimal control law turns out to be identical to an appropriately defined predictive control law. Mohtadi and Clarke (1986) have shown that receding horizon LQ control can be represented as a specific case of Generalized Predictive Control, and, thus, some of the well-developed stability results for state-space LQ control can be applied.

For more general nonlinear stochastic systems, the situation is much more complicated. The optimal stochastic controller differs considerably from the predictive controller, with both control approaches having their respective advantages. In light of the discussion in the first paragraph, it would seem that predictive control would result in a much simpler controller structure. This is, in fact, the case and explains to a large extent the popularity of predictive control in nonlinear and adaptive control systems. The optimal stochastic control problem requires the solution of complicated stochastic dynamic programming equations (Aoki, 1967) which is, in general, infeasible and even more so in a real time situation. On the other hand, stochastic optimal control has a major advantage over predictive control in that the very definition of optimality that defines the problem is widely accepted and results in a controller with a number of desirable properties. If the exact solution were known, the resulting controller would be both *cautious* and *dual*. Somewhat detailed descriptions of these properties are given in Wittenmark (1975a), Jacobs and Patchell (1972), Bar-Shalom (1981), and Astrom and Wittenmark (1989). Loosely speaking, a controller is dual if it compromises between a control action and a probing action: not only does a dual controller attempt to drive the system to the desired state, but it also determines the control input such that, in the future, state estimation will be more accurate (and, thus, future control will be more accurate). A controller is cautious if the control input is a function of the covariance of the current state estimation error. In this case the controller is aware of the errors in the estimates and takes more cautious action if the uncertainties are large. A mathematically rigorous definition of caution and dual control is given in Bar-Shalom and Tse (1974).

However, due to the inherent complexities of the exact optimal stochastic control solution for nonlinear systems, there have been a number of suboptimal approximations proposed which attempt to retain the dual and cautious properties of the optimal controller. A wide-sense dual controller is obtained in Tse et al. (1973) by linearizing the system about a nominal trajectory and using a second order Taylor expansion of a fixed horizon loss function about the nominal trajectory. The controller is referred to as wide-sense because, in order to avoid excessive complexity, the closed-loop control is restricted to being a function of the estimated state and error covariariance matrix. This method was further analyzed in Bar-Shalom (1981), where the loss function is expanded in terms of its certainty equivalence, cautious, and probing parts, and in Dersin et al. (1981), where it was compared to the optimal controller for a simple scalar example. Although the wide-sense suboptimal controller represents a tremendous reduction in complexity from the optimal controller, it is still too complicated to be implemented in real-time in many dynamic systems. Mookerjee and Bar-Shalom (1989) use a similar perturbation model expanded about a nominal trajectory to obtain a closed-form adaptive dual controller for MIMO ARMA systems, but with the restriction that a 2-step horizon be used in the loss function. Ku and Athans (1973) develop, for adaptive control of linear systems, an open loop feedback controller that posesses caution but not dual control properties. The algorithm is fairly complicated and not well suited for real-time implementation. Wittenmark (1975b) and Milito et al. (1980) take a different approach to dual control by minimizing a 1-step ahead criteria augmented explicitly by a term which penalizes for poor estimation. The former has no analytic solution for the optimal control input, while the latter is relatively simple in form. Although the dual controllers of Mookerjee and Bar-Shalom (1989) and Milito et al. (1980) are feasible for real-time implementation, they suffer from the restriction of a 1-step or 2-step horizon, and, thus, may not be suitable for controlling non-minimum phase systems.

The control of nonlinear or linear adaptive control system has been approached using predictive control concepts also. As in the optimal control problem, exact solutions are extremely difficult. A standard approach for nonlinear systems is to linearize about the current state estimate or some nominal trajectory, and apply the well known linear predictive control techniques to resulting perturbation models. Lee and Ricker (1993) use such a technique with Dynamic Matrix Control. Predictive control is much more commonly applied to linear, unknown sytems in a parameter adaptive control framework in which certainty equivalence is enforced. Kramer and Unbehauen (1992) and De Keyser et al. (1988) provide comparative studies of some of the more popular predictive control methods used for adaptive control. However, with any of the standard predictive control indices the resulting controllers are not cautious, and, because of their open-loop feedback characteristic, are not dual either.

The purpose of this paper is to introduce a new predictive control loss function for nonlinear and linear adaptive control systems that combines some of the desirable properties of both optimal control and predictive control. The resulting adaptive cautious predictive (ACP) controller posesses cautious properties like the optimal stochastic controllers and, at the same time, has a closed-form solution that is easily implemented in most real-time dynamic systems. Unlike the previous cautious dual controllers of Mookerjee and Bar-Shalom (1989) and Milito et al. (1980) that are suitable for on-line implementation, the ACP controller developed here has no restrictions on the length of the predictive control horizon. The format of the remainder of the paper is as follows. In section 2 the ACP loss function is introduced and the control law derived. Section 3 discusses implementation concerns and provides simulation results.

2 THE ADAPTIVE CAUTIOUS PREDICTIVE CONTROLLER

Consider a discrete-time nonlinear stochastic system, affine in the input, modeled as

$$X(k+1) = f(X(k), k) + g(X(k), k)u(k) + W(k) \quad (1)$$

$$Y(k+1) = C(X(k+1), k+1) + V(k+1) \quad (2)$$

where $X(k)\in \mathbf{R}^n$, $Y(k)\in \mathbf{R}^1$, $W(k)\in \mathbf{R}^n$, $V(k)\in \mathbf{R}^1$, and $u(k)\in \mathbf{R}^1$ are the state, output, sytem noise, observation noise, and input, respectively, and k is the time index. X(0) and W(k), V(k+1) (k=0,1,2...) are assumed to be independent Guassian random vectors with known mean (W(k) and V(k) are assumed to be zero mean) and covariance matrices. Furthermore, f(X(k),k), g(X(k),k), and C(X(k),k) are assumed to be continuously differentiable in X(•) for all k.

Before we derive the ACP law, let us assume that the present time is indexed by k, the control sequence $U^{k-1}:=\{u(0), u(1), ..., u(k-1)\}$ has been applied to the system and the observation sequence $Y^k:=\{y(1), y(2), ..., y(k)\}$ has been obtained. The state estimate and its error covariance matrix are assumed available from the estimates

$$\hat{X}(k|k) := E[X(k)|Y^k,U^{k-1}] \text{ and}$$

$$P(k) := E[[X(k)-\hat{X}(k|k)][X(k)-\hat{X}(k|k)]^T|Y^k,U^{k-1}], \quad (3)$$

where E[•] denotes an expectation. The state estimation can be obtained by one of the following approximate methods: 1) extended Kalman filter (Stengel, 1986), 2) adaptive filter with tuning (Jazwinski, 1970), and 3) second-order filter (Athans et al., 1968). Depending on the specifics of the problem, one of these methods may be more appropriate than the others.

By the continuous differentiability assumpution, in a neighborhood of $\hat{X}(k|k)$, f(X(k),k) and g(X(k),k) can be expressed as:

$$f(\xi,k) = f(\hat{X}(k|k),k) + \left.\frac{\partial f(\xi,k)}{\partial \xi^T}\right|_{\xi=\hat{X}(k|k)}\left(\xi-\hat{X}(k|k)\right) + O'(\xi,k), \quad (4)$$

$$g(\xi,k)=g(\hat{X}(k|k),k)+\left.\frac{\partial g(\xi,k)}{\partial \xi^T}\right|_{\xi=\hat{X}(k|k)}\left(\xi-\hat{X}(k|k)\right) + O''(\xi,k), \quad (5)$$

where $O'(\xi,k)$ and $O''(\xi,k)$ are such that $\frac{\|O'(\xi,k)\|}{\|\xi-\hat{X}(k|k)\|}, \frac{\|O''(\xi,k)\|}{\|\xi-\hat{X}(k|k)\|}$ $\to 0$ as $\|\xi-\hat{X}(k|k)\| \to 0$. Here, $\|\cdot\|$ is the standard Euclidean norm on $\mathbf{R}^n$. We now assume that the system and state varies slowly enough (i.e., is sampled fast enough) so that over the predictive control time horizon N (defined below in equation (7)) the system can be approximated by a linearization about $\hat{X}(k|k)$ at time index k. Thus, substituting equations (4) and (5) into equation (1) and ignoring the $O'(\xi,k)$ and $O''(\xi,k)$ terms, gives the following approximation of X(k+i) (i = 1, 2, ..., N):

$$X(k+i) \approx A(k)X(k+i-1) + \{b(k)+B(k)[X(k+i-1)-\hat{X}(k|k)]\} *u(k+i-1) + D(k) + W(k+i-1) \quad (6)$$

where

$$A(k) = \left.\frac{\partial f(\xi,k)}{\partial \xi^T}\right|_{\xi=\hat{X}(k|k)}, \quad B(k) = \left.\frac{\partial g(\xi,k)}{\partial \xi^T}\right|_{\xi=\hat{X}(k|k)},$$

$$b(k) = g(\hat{X}(k|k),k), \text{ and } D(k) = f(\hat{X}(k|k),k) - A(k)\hat{X}(k|k).$$

Consider now the predictive control index defined as:

$$J(u,k)=\frac{1}{2}E\left[\sum_{i=1}^{N}\|\hat{X}(k+i|k)-X^*(k+i)\|_Q^2+ru^2(k+i-1)\Big|Y^k,U^{k-1}\right], \quad (7)$$

subject to

$$u(k+i) = \begin{cases} 0 \text{ (random vibration control)} \\ u(k) \text{ (trajectory tracking)} \end{cases} : i=1,2,...,N, \quad (8)$$

where N is the prediction horizon, $X^*(k+i)$ is the desired value of the state at time k+i, and Q and r are weighting parameters with Q a positive semi-definite, symmetric matrix and r > 0. Here $\|X\|_Q$ is defined as $(X^TQX)^{1/2}$ for any vector X. $\hat{X}(k+i|k)$ is the i step ahead prediction of the state at time k. The main difference between the above loss function and standard predictive control loss functions lies in the definition of $\hat{X}(k+i|k)$:

$$\hat{X}(k+i|k) := E[X(k+i)|Y^k, U^{k-1}, \tilde{X}(k)]: \ i = 1, 2, ..., N, \quad (9)$$

where $\tilde{X}(k) := X(k) - \hat{X}(k|k)$ is the state estimation error at time k, and $\hat{X}(k|k)$ is as in equation (3). Thus, $\hat{X}(k+i|k)$ is allowed to be a function of the unknown random variable $\tilde{X}(k)$, which accounts for the need for the conditional expectation in equation (7). Note that no such expectation is needed in standard predictive control loss functions, where the i-step ahead prediction of the state is not allowed to depend on any unknown random variables. The dependence of $\hat{X}(k+i|k)$ on $\tilde{X}(k)$ is essentially what results in the cautious property of the ACP controller, since, after making appropriate substitutions and taking the expectation in equation (7), P(k) will be present in the control law.

The ACP control strategy is a receding horizon predictive control problem. At time k, one minimizes the criterion J(u, k) to solve for the control u(k), and applies u(k) to the system. Then, at time k+1, J(u,k+1) is minimized to solve for u(k+1) etc... For the sake of simplicity, the remainder of the paper will focus on the random vibration control problem, for which the constraint is u(k+i) = 0 (i = 1, 2, ..., N), i.e. that the future control actions are assumed to be zero within the prediction horizon while calculating the present control u(k). This strategy is commonly used in predictive control studies (De Keyser et. al., 1988). A physical interpretation is that one assumes u(k), and no other control action, will applied to bring the system back to the desired value within the prediction horizon. The following ACP control results can be extended to the trajectory tracking problem for which the constraint is u(k+i) = u(k) (i=1, 2, ..., N).

Taking the conditional expectation of equation (6) gives

$$\hat{X}(k+i|k)=\begin{cases} A(k)\hat{X}(k|k)+A(k)\tilde{X}(k)+\left[b(k)+B(k)\tilde{X}(k)\right]u(k)+D(k): & i=1 \\ A(k)\hat{X}(k+i-1|k)+D(k): & i\geq 2 \end{cases} \tag{10}$$

The expression for i = 1 results from the definition of $\tilde{X}(k)$ and the fact that $\hat{X}(k|k)$ is a function of $\{Y^k, U^{k-1}\}$, and thus $E[X(k)|Y^k, U^{k-1}, \tilde{X}(k)] = X(k)$. Equation (10) can be solved explicitly to give, for i = 1, 2, ..., N

$$\hat{X}(k+i|k) = A^i(k)\hat{X}(k|k) + A^i(k)\tilde{X}(k) + A^{i-1}(k)\left[b(k)+B(k)\tilde{X}(k)\right]u(k) + \sum_{j=1}^{i} A^{i-j}(k)D(k). \tag{11}$$

Substituting (11) into the predictive control performance index (7) and defining

$$A_{ik} = A^i(k)\hat{X}(k|k) + \sum_{j=1}^{i} A^{i-j}(k)D(k) - X^*(k+i),$$
$$b_{ik} = A^{i-1}(k)b(k), \text{ and } B_{ik} = A^{i-1}(k)B(k) \tag{12}$$

we obtain

$$J(u,k) = \frac{1}{2}E\left[\sum_{i=1}^{N}\left\|A_{ik}+A^i(k)\tilde{X}(k)+\left[b_{ik}+B_{ik}\tilde{X}(k)\right]u(k)\right\|_Q^2 + ru^2(k) \mid Y^k, U^{k-1}\right]. \tag{13}$$

Since the state estimation is assumed unbiased, setting $\partial J(u,k)/\partial u(k)=0$ to solve for the optimal input gives

$$u(k) = -\left\{\sum_{i=1}^{N}\left(\|b_{ik}\|_Q^2+E\left[\|B_{ik}\tilde{X}(k)\|_Q^2\right]\right) + r\right\}^{-1} \left\{\sum_{i=1}^{N}\left(b_{ik}^TQA_{ik}+E\left[\tilde{X}^T(k)B_{ik}^TQA^i(k)\tilde{X}(k)\right]\right)\right\} \tag{14}$$

Now introduce the following notation: Let $a_{ts}^{(i)}$ be the t-th row, s-th column element in the matrix $B_{ik}^TQB_{ik}$; $b_{ts}^{(i)}$ be the t-th row, s-th column element in the matrix $B_{ik}^TQA^i(k)$; and p_{ts} be the t-th row, s-th column element in the matrix $E[\tilde{X}(k)\tilde{X}^T(k)]$. Then, after some algebra, we have

$$E\left[\tilde{X}^T(k)B_{ik}^TQA^i(k)\tilde{X}(k)\right] = \sum_{t=1}^{n}\sum_{s=1}^{n} b_{ts}^{(i)}p_{ts} \quad \text{and}$$
$$E\left[\|B_{ik}\tilde{X}(k)\|_Q^2\right] = \sum_{t=1}^{n}\sum_{s=1}^{n} a_{ts}^{(i)}p_{ts}. \tag{15}$$

Substituting (15) into (14), the adaptive cautious predictive control law becomes

$$u(k) = -\left\{\sum_{i=1}^{N}\left(b_{ik}^TQb_{ik}+\sum_{t=1}^{n}\sum_{s=1}^{n} a_{ts}^{(i)}p_{ts}\right)+r\right\}^{-1} \left\{\sum_{i=1}^{N}\left(b_{ik}^TQA_{ik}+\sum_{t=1}^{n}\sum_{s=1}^{n} b_{ts}^{(i)}p_{ts}\right)\right\} \tag{16}$$

The ACP control strategy can be summarized as follows. At each sampling instant, first, the effects of the control variables on the state are predicted. Then, the control action is determined by minimizing a receding quadratic loss function of the predicted trajectory error and the control effort. The obtained control action is then applied to the system, and, at the next sample instant, the above steps are repeated.

Based on the developed ACP control algorithm, following observations are made:

Remark 2.1: Using the ACP control, the adaptive system takes not only the instantaneous state estimates but also the associated confidence level into account, i.e

$$u(k) = u(\hat{X}(k|k), P(k))$$

Further investigation of (15) and (16) reveals that the term $E[\|B_{ik}\tilde{X}(k)\|_Q^2] \geq 0$ is contained in the denominator in the ACP control law. This seems to have the effect of reducing the input magnitude when P(k) is large. On the other hand, if P(k) is small, the state estimation error is neglected and the control law of equation (16) approaches that of certainty equavalence control (see Remark 2.3). This cautious property, which automatically adjusts the control action according to the quality of the state estimation, is a key feature of ACP control. In adaptive control problems, this cautious property is especially important during the transient period of state estimation and/or when the system has large uncertainties.

Remark 2.2: It should be noted that in considering the state estimation quality in ACP control, the computational expense is not significantly increased. This is one of the major advantages of ACP control over the other multi-step horizon cautious controllers outlined in the introduction. The arbitrary time horizon N in the cost function of equation (7) allows greater flexibility in controller design than in cautious control approaches where the prediction horizon is restricted to only one or two steps (Jacobs, 1981), (Mookerjee and Bar-Shalom, 1989), (Wittenmark, 1975b), and (Milito et al., 1980).

Remark 2.3: If certainty equivalence (CE) (Stengel, 1986) is enforced, the corresponding CE predictive control law can be obtained by simply ignoring the state estimation error in the state prediction equation (10). Setting $\tilde{X}(k) := 0$ in equation (10), the CE control law would be:

$$u(k) = -\left\{\sum_{i=1}^{N}\left(b_{ik}^TQb_{ik}\right)+r\right\}^{-1} \left\{\sum_{i=1}^{N}\left(b_{ik}^TQA_{ik}\right)\right\}. \tag{17}$$

3 IMPLEMENTATION CONCERNS AND SIMULATION RESULTS

Before we begin the simulation studies, we first mention an important implementation concern. The ACP algorithm requires the state estimate $\hat{X}(k|k)$ at each time k. If, for example, an extended Kalman filter (EKF) is used for state estimation, one of the implementation requirements is that the initial covariance matrix P(0) (see equation (3)) is known. For most practical applications, this is an unrealistic requirement. As a result, to aviod instability and to allow fast convergence of the state estimation, it is common practice to overestimate

P(0) for use in the state estimation. While this, in general, has beneficial effects on the state estimation, in the ACP control algorithm it has the effect of causing the controller to be "overly cautious" during the initial timesteps. In other words, the magnitude of the control input during the initial stages will be lower than what the optimal input, in terms of minimizing equation (8), would be if the true P(0) were used. To aviod this situation, it is recommended that two initial covariance matrices be used: 1) P(0), chosen to fine-tune the EKF, to be used in the state estimation algorithm; and 2) P'(0), chosen to more accurately reflect the initial state estimation error covariance matrix, to be used in the ACP algorithm. During the course of the experiment both P(k) and P'(k), to be used in the EKF and ACP control algorithms, respectively, should be updated by the EKF. If the state estimation is stable this should only effect the input selection during the transient period of the experiment, with P(k) and P'(k) both converging to the same steady state value.

The objectives of the following simulation studies are 1) to illustrate the cautious property of ACP control; 2) to compare ACP control with a widely used certainty-equivalence control; and 3) to demonstrate the ACP controller's robustness towards high system noise levels and uncertainties in the state estimate.

We shall consider specifically the second order linear mass/spring/damper system illustrated in Fig. 1, where the parameters M (mass), K (spring constant), and C (damping constant) are unknown. Here, y(t) and u(t) are the displacement and force, respectively, at time t.

In state-space form, the system is modeled as

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{C}{M} \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} X(t)$$

Assigning the numerical values M=5kg, K=5000kg/s^2, and C=32kg/s and sampling the system with sampling interval τ=.02sec, the discrete time system equations become

$$X(k+1) = \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_3 & \theta_4 \end{bmatrix} X(k) + \begin{bmatrix} \theta_5 \\ \theta_6 \end{bmatrix} u(k) + W(k) \qquad (18)$$

$$= \begin{bmatrix} 0.8145 & 0.3509 \\ -0.8773 & 0.7022 \end{bmatrix} X(k) + \begin{bmatrix} 0.0742 \\ 0.3509 \end{bmatrix} u(k) + W(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} X(k) + V(k).$$

Here, the scaling is such that y(k) is measured in units of 1/20 m, and u(k) in units of 1/100 N. Also, system noise, W(k), and observation noise, V(k), have been added to the discrete time system equations. The assumptions on the noise are as in equations (1) and (2) with K_W and K_V used to denote the covariance matrices of W(k) and V(k), respectively.

Although system (18) is a linear system, if the state is augmented by the unknown parameters as $Z(k) = [X^T(k)\ \theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6]^T$ in an EKF (Stengel, 1986) formulation, then the simultaneous state and parameter estimation problem becomes

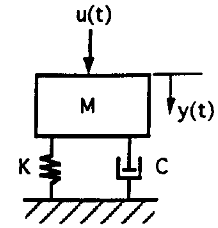

FIGURE 1: MASS/SPRING/DAMPER SYSTEM CONSIDERED IN THE SIMULATION

a nonlinear one that can be modeled by the system of equations (1) and (2).

The problem at hand is to find the control law for random vibration surpression (see equation (8)) with, possibly, an initial disturbance. The predictive control index used in all simulations in this section is

$$J(u,k) = \frac{1}{2} E\left[\sum_{i=1}^{N} \left(\|\hat{X}(k+i|k)\|_Q^2 \right) + ru^2(k) \middle| Y^k, U^{k-1} \right], \qquad (19)$$

where Q=diag({5, 0.5, 0, 0, 0, 0, 0, 0}), r=0.1, and N=5.

In the following simulations, the ACP control developed in this paper is compared to both certainty equivalence control and no control. Various parameters in the simulations (e.g. K_W, X(0) and the initial guess for θ_6) are varied to represent different conditions, with the controller performances being compared under each set of conditions. The criteria used for controller performance evaluation is

$$J_{M,tr} = \sum_{k=1}^{100} y^2(k), \text{ and } J_{M,tot} = \sum_{k=1}^{300} y^2(k), \qquad (20)$$

where $M \in \{ACP, CE, 0\}$. The subscripts ACP, CE, and 0 indicate that the type of control used is ACP, CE, and no control, respectively. The subscript "tr" indicates the controller performance during the transient period, i.e. the first 100 timesteps, and the subscript "tot" indicates the controller performance over the whole 300 timestep simulation.

Table 1 presents a summary of the simulation results. In it, the performance indices of equation (20) have been averaged over 20 trials for each set of conditions. σ_w^2 denotes the variance of the system noise, i.e. $E[W(k)W(k)^T] = \sigma_w^2 I$, where I is the 2x2 identity matrix. $\hat{\theta}_6(0)$ is the initial guess for the input parameter θ_6 of equation (18). X(0) is the initial state, so that a "1.5,1.5" in that row indicates that the initial state was $[1.5\ \ 1.5]^T$. A simulation for which a larger initial state was used represents the random vibration control problem with an initial disturbance. For all simulations, the observation noise variance was $K_V = 0.01$.

As an example, Figs. 2 and 3 show typical simulation results using the conditions of system 2 in Table 1: $\sigma_w^2 = 0.1$, $\hat{\theta}_6(0) = 0.1$, and $X(0) = [5\ \ 5]^T$. The controled output response using ACP control is compared with that using no control and CE control in Fig. 2(a) and 2(b), respectively, and Fig. 3 shows the calculated input command for both CE control and ACP control during the first 50 timesteps of the simulation. Based

system:	1	2	3	4	5	6
σ_w^2	0.1	0.1	0.1	0.1	0.5	0.5
$\hat{\theta}_6(0)$	0.1	0.1	0.5	0.5	0.1	0.1
$X(0)$	1.5, 1.5	5, 5	1.5, 1.5	5, 5	1.5, 1.5	5, 5
$J_{ACP,tot}$	88	186	104	191	446	747
$J_{CE,tot}$	127	381	127	276	907	1550
$J_{0,tot}$	191	350	191	343	855	1317
$J_{ACP,tr}$	38	130	44	131	177	441
$J_{CE,tr}$	69	327	66	225	614	1288
$J_{0,tr}$	73	225	67	218	272	697

TABLE 1: SUMMARY OF SIMULATION RESULTS UNDER VARIOUS CONDITIONS

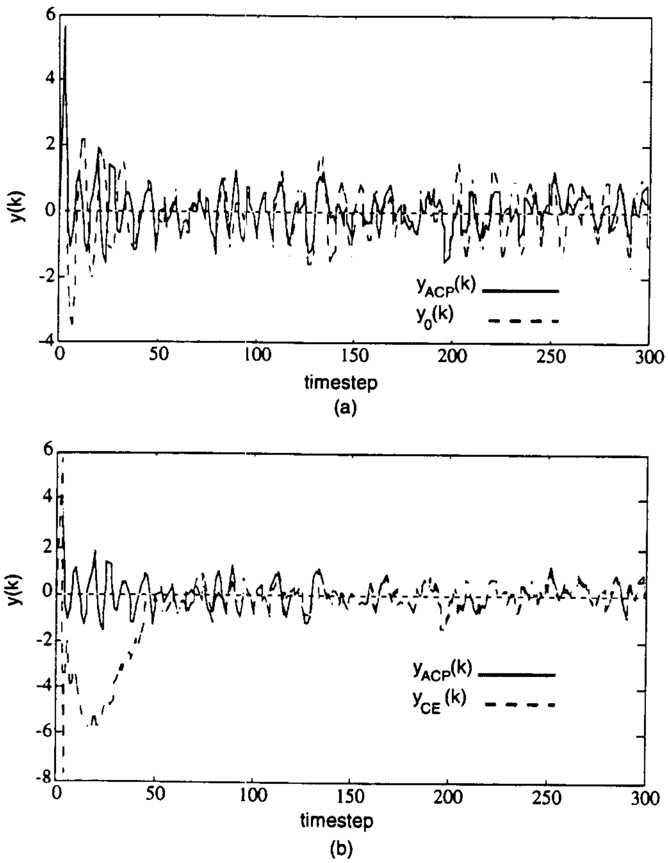

FIGURE 2: SIMULATION RESULTS COMPARING THE CONTROLLED OUTPUT USING: (a) ACP VS. NO CONTROL, AND (b) ACP VS. CE CONTROL. $y_0(k)$ IS THE OUTPUT USING NO CONTROL.

on the results in Table 1 and Figs. 2 and 3, the following observations are made:

1) In general, the ACP control was much more effective than CE control during the transient period of the simulations. This was expected, considering the cautious nature of the ACP controller: Initially, when there is much uncertainty in the parameter estimates, the control input is automatically chosen more conservatively, with, in general, a much smaller magnitude (see Fig. 3). As seen in Table 1, using CE control during the transient period is risky, with, in some cases, its performance actually worse than using no control at all. Note, however, that the performance of CE control in the steady state (which can be found by simply subtracting $J_{CE,tr}$ from $J_{CE,tot}$) is still much better than no control.

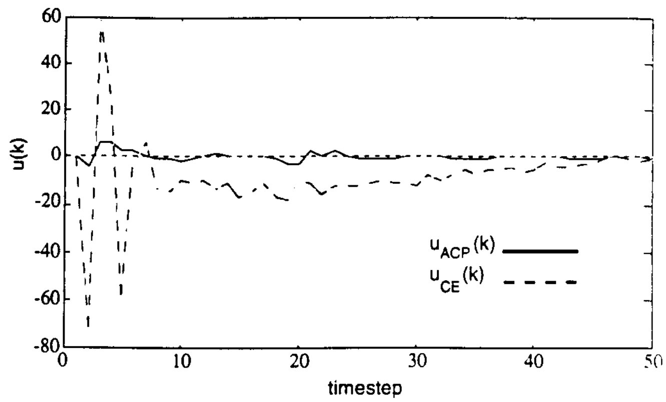

FIGURE 3: SIMULATION RESULTS COMPARING THE CONTROL INPUT USING ACP AND CE CONTROL

2) The more uncertain the state and parameter estimates (i.e. the higher σ_w^2), the better the relative performance of ACP control over CE control. This, again, is a consequence of the cautious nature of the ACP controller.

3) With larger initial disturbances, good control is more crucial during the transient period in order to reduce the effects of the disturbances. In these situations (systems 2, 4, and 6) ACP control is much more effective than CE control. Fig. 2 illustrates this situation well. Both the settling time and the magnitude of the steady state vibration is significantly reduced using ACP control, whereas with CE control, although the steady state performance is roughly equivalent to that of ACP control, its transient performance is much worse than using no control.

4) While changing $\hat{\theta}_6(0)$ seems to have little effect on ACP control, when $\hat{\theta}_6(0)$ is smaller than the true value (θ_6 = 0.3509) the performance of CE control is worse. This can be explained as follows: With the input parameter θ_6 overestimated, this, in general, reduces the magnitude of the control input. Thus, choosing $\hat{\theta}_6(0)$ large provides, in a sense, a type of caution for CE control. Note, however, that even with the overestimated $\hat{\theta}_6(0)$ the ACP controller performs much better than the CE controller.

5) ACP control achieves much better transient performance largely by reducing the magnitude of the control input during this period of system uncertainty. This, in itself, is a desirable side effect. As illustrated in Fig. 3, which shows the control input for both ACP and CE control during the first 50 timesteps of the simulation, the peak magnitude of the input is more than ten times smaller using the ACP control law. This characteristic of ACP

control greatly reduces the possibility of saturating the actuators during actual implementation.

6) Although, as the simulations have demonstrated, ACP control is much more effective in the transient period, for the particular system used in these simulations its performance is roughly equivalent to that of CE control in the steady state, after the parameter estimates have converged and the state estimation error covariance matrix approaches its steady state value. A detailed inspection of equations (12) and (16) for the system used here would reveal that the only elements of P(k) that are a factor in the control law of equation (16) are those in the seventh and eighth rows. Here, P(k) is that of the extended state in the EKF formulation, as defined in the paragraphs following equation (18). In the simulations conducted, even for the cases when $\sigma_w^2 = 0.5$, the elements of the seventh and eighth rows of P(k) become very small in the steady state with, for example, magnitudes smaller that roughly 0.05 (in most cases much smaller than that even). Thus, in the steady state, the ACP control law for the system used in these simulations is nearly identical to the CE control law of equation (17).

As a final note, the following remarks concerning implementation of the ACP controller are made.

Remark 3.1: For the EKF formulation of the joint state/parameter estimation problem used in these simulations, use of two different initial covariance matrices, P(0) and P'(0), as outlined in the first paragraph of this section, is especially crucial. The reason for this is that if the over-estimated P(0) is used in both the EKF and the ACP control algorithms, the controller will be overly cautious during the initial stages, resulting in very little input excitation to the system. This could very likely cause poor estimation results for the parameters θ_5 and θ_6, which would result in poor controller performance. This phenomena is known as controller "turn-off" and is discussed in, for example, Astrom and Wittenmark (1989).

Remark 3.2: As in most adaptive control schemes, ACP control requires tuning for optimal performance (e.g. selection of P(0), P'(0), Q, r, and N). Thus, if no a priori knowledge is available for selection of the controller parameters, it is advisable to use ACP control in a supervisory environment with backup controllers to avoid instability. The authors are currently working on implementing ACP control in such a supervisory environment.

4 CONCLUSIONS

In this paper, an adaptive cautious predictive (ACP) control algorithm has been developed for a class of nonlinear stochastic systems. However, it is also applicable to the joint parameter estimation and adaptive control problem for linear systems, which becomes nonlinear in an extended Kalman filter formulation. Through the introduction of a new long range predictive control loss function, the ACP control algorithm posesses caution (the state estimation error covariance matrix is considered in the control law) and, as a result, achieves better robustness towards state and parameter estimation errors. Simulation results have demonstrated that it can significantly improve adaptive control performance, especially in the transient period. Not only does ACP control posess caution, like more complicated optimal and suboptimal stochastic controllers, but it also posesses the desirable properties of long range predictive controllers. The predictive control horizon is arbitrary, and, most importantly, the control law is in closed-form and requires only slightly more computational expense than the analogous enforced certainty equivalence controller. As a result, implementation of ACP control in real-time dynamic systems is highly feasible.

REFERENCES

Aoki, M., 1967, *Optimization of Stochastic Systems*, Academic Press, New York.

Astrom, K. J., and Wittenmark, B., 1989, *Adaptive Control*, Addison-Wesley, New York.

Athans, M., Wishner, R. P., and Bertolini, A., 1968, "Suboptimal State Estimation for Continuous-time Nonlinear Systems From Discrete Noisy Measurements,". *IEEE Transactions on Automatic Control*, Vol. AC-13, pp. 504-514.

Bar-Shalom, Y., 1981, "Stochastic Dynamic Programming: Caution and Probing," *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 5, pp. 1184-1195.

Bar-Shalom, Y. and Tse, E., 1974, "Dual Effect, Certainty Equivalence, and Separation in Stochastic Control," *IEEE Transactions on Autumatic Control*, Vol. AC-19, No. 5, pp. 494-500.

Clarke, D.W., Mohtadi, C., and Tuffs, P. S., 1987, "Generalized Predictive Control–Part I. The Basic Algorithm," *Automatica*, Vol.23, No.2, pp.137-148.

Cutler, C. R., and Ramaker, B. L., 1980, "Dynamic Matrix Control–a Computer Control Algorithm," JACC, San Francisco, Ca., paper WP5-B.

De Keyser, R. M. C., Van De Velde, G. A., and DuMortier, F. A. G., 1988, "A Comparative Study of Self-Adaptive Long-range Predictive Control Methods," *Automatica*, Vol. 24, pp.149-163.

Dersin, P. L., Athans, M., and Kendrick, D. A., 1981, "Some Properties of the Dual Adaptive Stochastic Control Algorithm," *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 5, pp. 1001-1008.

Garcia, C. E., Prett, D. M., and Morari, M., 1989, "Model Predictive Control: Theory and Practive–a Survey," *Automatica*, Vol. 25, No. 3, pp. 335-348.

Jacobs, O. L. R., 1981, "Introduction to Adaptive Control," in Harris, C.J. and Billings, S. A. (eds.), *Self-Turning and Adaptive Control: Theory and Applications*, Peter Peregrinus Ltd.

Jacobs, O. L. R., and Patchell, J. W., 1972, "Caution and probing in stochastic control," *International Journal of Control*, Vol. 16, No. 1, pp. 189-199.

Jazwinski, A., 1970, *Stochastic Processes and Filtering Theory*, Academic, New York.

Kramer, K., and Unbehauen, H., 1992, "Aspects in Selecting Predictive Adaptive Control Algorithms," *American Control Conference*, pp. 2396-2401.

Ku, R., and Athans, M., 1973, "On the Adaptive Control of Linear Systems Using the Open-Loop-Feedback-Optimal Approach," *IEEE Transactions on Automatic Control*, Vol.AC-18, No. 5, pp. 489-493.

Lee, J. H., and Ricker, N. L, 1993, "Extended Kalman Filter Based Nonlinear Model Predictive Control," *American Control Conference*, San Francisco, pp. 1895-1899.

Li, S., Lim, K. Y., and Fisher, D. G., 1989, "A State Space Formulation for Model Predictive Control," *AIChE Journal*, Vol. 35, No. 2, pp. 241-249.

Milito, R., Padilla, C. S., Padilla, R. A., and Cadorin, D., 1980, "Dual Control Through Innovations," *IEEE Conference on Decision and Control*, Vol. 1, pp. 341-345.

Mohtadi, C., and Clarke, D. W., 1986, "Generalized Predictive Control, LQ, or Pole-placement: A unified approach," *IEEE Conference on Decision and Control*, pp. 1536-1541.

Mookerjee, P. and Bar-Shalom, Y., 1989, "An Adaptive Dual Controller for a MIMO-ARMA System," *IEEE Transactions on Autumatic Control*, Vol. 34, No. 7, pp. 795-800.

Richalet, J. A., Rault, A., Testud, J. L., and Papon, J., 1978, "Model Predictive Heuristic Control: Applications to an Industrial Process," *Automatica*, Vol. 14, pp. 413-428.

Rouhani, R., and Mehra, M., 1982, "Model Algorithmic Control (MAC); Basic Theoretical Properties," *Automatica*, Vol. 18, No. 4, pp. 401-414.

Scattolini, R., and Bittanti, S., 1990, "Model Predictive Control: Theory and Practive–a Survey," *Automatica*, Vol. 26, No. 5, pp. 915-917.

Stengel, R.F., 1986, *Stochastic Optimal Control: Theory and Application*, Wiley, New York.

Tse, E., and Athans, M., 1972, "Adaptive Stochastic Control for a Class of Linear Systems," *IEEE Transactions on Autumatic Control*, Vol. AC-17, No. 1, pp. 38-52.

Tse, E., Bar-Shalom, Y., and Meier, L., 1973, "Wide-Sense Adaptive Dual Control for Nonlinear Stochastic Systems," *IEEE Transactions on Automatic Control*, Vol. AC-18, No. 2, pp. 98-108.

Wittenmark, B., 1975a, "Stochastic adaptive control methods: a survey," *International Journal of Control*, Vol. 21, No. 5, pp. 705-730.

Wittenmark, B., 1975b, "An Active Suboptimal Dual Controller for Systems with Stochastic Parameters," *Automatic Control Theory and Applications*, Vol. 3, No. 1, pp. 13-19.