

A FAULT DETECTION, ISOLATION, AND IDENTIFICATION TECHNIQUE FOR COMPLEX MISO LINEAR SYSTEMS

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ABSTRACT

This paper presents a method, based on a generalized likelihood ratio test (GLRT), for combining fault detection, isolation, and identification in complex multi input - single output (MISO) linear systems. By "complex" it is meant that the system is comprised of a number of subsystems, connected through series, parallel, feedback, etc. relations. The method applies to systems corrupted by multiple noise terms with additive faults entering the system at various locations, and requires measurement of the process output and inputs. In addition to detecting faults, the method determines which subsystem experienced the fault, determines the type of fault (if more than one type is hypothesized), estimates the time of occurrence of the fault, and estimates the magnitude of the fault (shown to be both unbiased and efficient). A simulation example illustrating the performance of the GLRT method is also provided.

1. INTRODUCTION

The concept of automatic fault detection and diagnosis has long been of critical importance in certain high performance applications in, for example, the aerospace industry. With the increases in microprocessor availability and computing power, fault detection and diagnosis techniques can now be used economically in less high performance applications such as failure detection in centrifugal pumps and leak detection in pipelines [10] and machine tool diagnosis [11].

Fault detection and diagnostics generally consist of the following three tasks [9]:

- (1) *Fault detection*: determining when something is wrong with the system,
- (2) *Fault isolation*: determining the exact location of the fault (e.g., in complex systems, determining which subsystem has experienced a fault), and
- (3) *Fault identification*: determining the type of fault, fault magnitude, time of occurrence of the fault, etc.

In this paper an approach is developed that, given the input and output measurements and system model, combines the above three tasks and does so in an easily implementable and computationally efficient manner. The form of the system analyzed in this paper is assumed to be a complex MISO linear system consisting of a number of interconnected subsystems, some of which may experience an additive deterministic fault. The inputs and output of the system are assumed to be measurable, but not the input and output of the individual subsystems. The technique used to accomplish the three detection and diagnostic tasks is based on a generalized likelihood ratio test (GLRT) applied to the innovations of a filter designed to whiten the system output. In essence, the GLRT tests the innovations for the presence of the fault signatures,

which can be calculated given the system model and the hypothesized faults.

GLRTs, as well as a number of their properties, are discussed in some detail in a general context in [13]. The application of GLRTs in fault detection gained attention in the mid seventies [15], and a survey of the early work is given in [14]. The majority of the research regarding GLRTs in fault detection has used a state-space form of the system model, as opposed to an input/output form used in this paper. The underlying concept of the state-space GLRT approach is to estimate the process states using a Kalman filter, and apply a GLRT to test for the presence of a fault signature in the Kalman filter innovations.

More recent fault detection surveys have been given in [3], which analyzes more general signals, as well as dynamic systems, [10], which seeks to integrate parameter estimation with fault detection, and [8], which focuses on the analytic redundancy method. In one of the original works on fault detection using a GLRT [15] a finite set of jump directions in the state vector are hypothesized, and a GLRT is used to detect the jump and estimate jump time, magnitude, and direction. The probability of detection and false alarm are also discussed. Relaxing the assumption that the faults are step changes in the state vector, a GLRT for detecting piecewise step changes is developed in [12]. The resulting test, being subject to lags in the detection speed, is used in conjunction with the single step hypothesis GLRT. [4] covers many issues both in the GLRT approach and fault detection in dynamic systems in general. [6] consider faults in systems that can be represented as state-space and ARMA models, as well as pure regression models. [5] compares several jump detection algorithms and designs a test combining a GLRT with a cusum test on the Kalman filter innovations.

For complex systems, i.e. systems composed of a number of interconnected subsystems, the complete system description can be of high order, requiring a state-space description with a large number of state variables. In such a case, the state-space GLRT method can result in a computationally expensive algorithm, since a high order Kalman filter is needed for estimating the states. It has been pointed out, though [3], that in situations where the GLRT implementation is complex it may at least serve as a benchmark for other algorithms because of its theoretical optimality, albeit in an off-line setting [7]. Here, the optimality is in reference to the tradeoff between probability of detection and probability of false alarm. The GLRT developed in this paper is based on an input/output model of the system and requires no state estimation. As a result, it is more computationally efficient than state-space GLRT methods for complex systems. An additional advantage of the input/output approach over the state-space approach is that fault representation is more straightforward. For complex systems the state-space approach groups the dynamics of the individual subsystems together in one high order state-space model. As a result, representing the faults in terms of the state variables may be difficult. With the input/output representation of the system,

however, the subsystems remain separated conceptually, making the fault representation more straightforward.

The purpose of this paper is to present an easily implemented, computationally efficient method for combining fault detection, isolation, and identification in complex MISO linear systems. The system model is fairly general and applies to a wide variety of processes. Section 2 provides a description of the assumed form of the system and faults. In section 3 the design of the whitening filter is discussed, and in section 4 the GLRT is developed. Section 5 outlines both the off-line and on-line aspects of the procedure for implementing the GLRT and provides a simulation example illustrating the GLRT performance.

2. SYSTEM DESCRIPTION

The system under consideration in this paper is assumed to be of the form

$$y(t) = \sum_{i=1}^{n_u} G_{u_i}(z)u_i(t) + \sum_{i=1}^{n_a} G_{a_i}(z)a_i(t) + \sum_{i=1}^{n_f} G_{f_i}(z)K_i f_{j_i, \mu_i}(t), \quad (1)$$

where $u_i(t)$ ($i = 1, 2, \dots, n_u$) are the system inputs, $y(t)$ is the system output (scalar), and $a_i(t)$ ($i = 1, 2, \dots, n_a$) are the noise sequences entering the system. $a_i(t)$ is assumed to be zero-mean uncorrelated Gaussian noise, i.e. $a_i(t) \sim \text{NID}(0, \sigma_{a_i}^2)$, and $E[a_k(t)a_l(s)] = 0$ ($k \neq l$). The additive, deterministic faults, $K_i f_{j_i, \mu_i}(t)$, may enter the system at n_f distinct locations. However, it is assumed that only one fault occurs at any given time and that the occurrence of the faults are spaced far enough apart that their effects do not overlap. K_i is the magnitude of the fault at the i th fault location, and μ_i is the time of occurrence. Thus, $\mu_i = \infty$ is equivalent to no fault occurring at the i th fault location. $f_{j_i, \mu_i}(t)$ is the unit magnitude fault, and j_i indicates the type of fault occurring at the i th fault location. As an example, suppose that a fault of type 1 is a step function, and a fault of type 2 is an impulse. Then, according to the previous definitions

$$f_{1, \mu}(t) := \begin{cases} 0: t < \mu \\ 1: t \geq \mu \end{cases} \quad \text{and} \quad f_{2, \mu}(t) := \begin{cases} 0: t \neq \mu \\ 1: t = \mu \end{cases}$$

In addition, in equation (1) each of the $G_{u_i}(z)$, $G_{a_i}(z)$, and $G_{f_i}(z)$ are assumed to be rational transfer functions in the shift operator z relating the inputs, noise, and faults to the system output. It is assumed that the poles of each $G_{u_i}(z)$, $G_{a_i}(z)$, and $G_{f_i}(z)$ lie strictly inside the unit circle in the complex z -plane.

Figure 1 shows an example of a type of system described by equation (1). There are n subsystems, each subsystem being modeled as an ARMAX process plus an

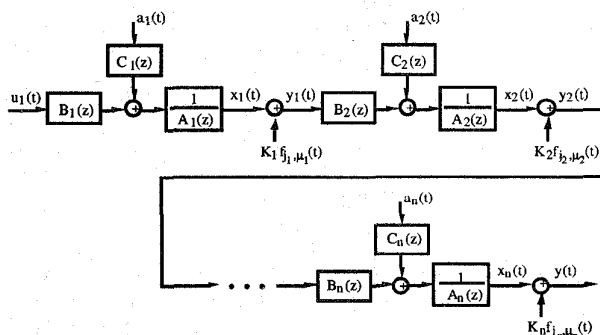


Figure 1 Example (sequential manufacturing process) of a system described by equation (1).

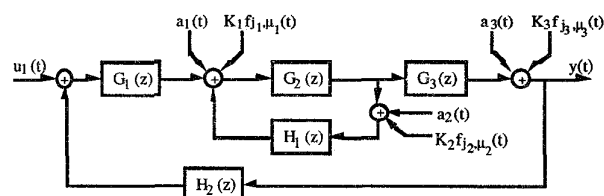


Figure 2 Example (servo system) of a system described by equation (1).

additive deterministic fault. The subsystems are connected in series so that the input to a subsystem is the output of the previous subsystem. The type of system depicted in Figure 1 could represent a sequential manufacturing process such as, for example, an automobile body assembly process or a paper making process. Using linear system theory, the entire system can be expressed as in equation (1), assuming the model of each subsystem is known. In actuality, equation (1) represents a very general class of MISO linear systems consisting of combinations of series and parallel connected subsystems with feedforward loops, feedback loops, etc. Consider, for example, the system depicted in Figure 2. This type of system could be used to model a servo system with position and velocity control loops, where $K_1 f_{j_1, \mu_1}(t)$ could represent an actuator fault and $K_2 f_{j_2, \mu_2}(t)$ and $K_3 f_{j_3, \mu_3}(t)$ sensor faults.

3. WHITENING FILTER AND FAULT SIGNATURE

To simplify the derivation and implementation of the GLRT, in this section a whitening filter will be designed, the output of which is uncorrelated Gaussian innovations that are used directly in the fault detection test.

Consider the transfer function description of equation (1), and define

$$y'(t) = y(t) - \sum_{i=1}^{n_u} G_{u_i}(z)u_i(t), \quad (2)$$

where it is assumed that the $u_i(t)$ are measured inputs. Then, under no-fault conditions,

$$y'(t) = \sum_{i=1}^{n_a} G_{a_i}(z)a_i(t). \quad (3)$$

Since the $a_i(t)$ are assumed uncorrelated, from linear stochastic system theory it is known that the power spectral density of $y'(t)$ is given by

$$P_y(\omega) = \left\{ \sum_{i=1}^{n_a} |G_{a_i}(z)|^2 \sigma_{a_i}^2 \right\} \Big|_{z=e^{j\omega}} \quad (4)$$

Each of the $|G_{a_i}(z)|^2$ in equation (4) is a rational function of $e^{j\omega}$, and, thus, $P_y(\omega)$ is a rational function of $e^{j\omega}$. Consequently, the spectral factorization theorem [2] applies, stating that there exists a stable, invertible linear filter which, when driven by white noise, gives an output whose power spectral density is $P_y(\omega)$. (Note that to ensure invertibility we must assume that the numerator of equation (4) has no zeros exactly on the unit circle, although they may lie either inside or outside of it). Denoting this filter by $G(z)$, we have

$$P_y(\omega) = |G(z)|^2 \Big|_{z=e^{j\omega}}. \quad (5)$$

Furthermore, $G(z)$ will have the form of a ratio of two polynomials in z , i.e.

$$G(z) = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots + g_s z^{-s}}{1 + h_1 z^{-1} + h_2 z^{-2} + \dots + h_d z^{-d}} \quad (6)$$

By the spectral factorization theorem, $G(z)$ is invertible. Thus, we can define its inverse, denoted $G^{-1}(z)$, by simply interchanging the numerator and denominator of equation (6).

Under the more general condition in which a fault may have occurred, filtering $y'(t)$ by $G^{-1}(z)$ and denoting the output by $e(t)$ gives

$$e(t) = G^{-1}(z)y'(t) = v(t) + \sum_{i=1}^{n_f} K_i \tilde{f}_{ji,\mu_i}(t), \quad (7)$$

where $v(t) := G^{-1}(z) \sum_{i=1}^{n_a} G_{a_i}(z) a_i(t)$ and

$\tilde{f}_{ji,\mu_i}(t) := G^{-1}(z) G_{f_i}(z) f_{ji,\mu_i}(t)$. $\tilde{f}_{ji,\mu_i}(t)$, the response of the unit magnitude fault when propagated through the system and the whitening filter, will be referred to as the fault signature of $f_{ji,\mu_i}(t)$. From the discussion preceding equation (5) it is clear that $v(t)$ is zero-mean, white, Gaussian noise. $G^{-1}(z)$ will therefore be referred to as the whitening filter. Let the variance of $v(t)$ be denoted σ_v^2 .

Equation (7) is conceptually significant in that it shows $e(t)$ is the sum of the NID(0, σ_v^2) sequence $v(t)$ and the deterministic fault signature. If we assume that only one fault has occurred, say in subsystem i , then $e(t)$ is an uncorrelated Gaussian sequence with mean $K_i \tilde{f}_{ji,\mu_i}(t)$ and variance $\sigma_e^2 = \sigma_v^2$ independent of the occurrence of a fault.

4. GENERALIZED LIKELIHOOD RATIO TEST FOR FAULT DETECTION

In this section a generalized likelihood ratio test (GLRT) for detecting the presence of a fault signature in the uncorrelated innovations of equation (7) is developed. A GLRT, as opposed to a likelihood ratio test (LRT), is used because the fault magnitude is unknown and must be estimated from the data.

The first step in developing the GLRT is to formalize the detection problem by defining the statistical hypotheses to be tested. For implementation purposes, instead of testing for faults occurring at all previous times, only faults occurring in the interval $\{t-N, t-N+1, \dots, t\}$ will be tested for, where t is the current time and $N+1$ is the window length. In general, when selecting N there is a tradeoff between computational complexity and probability of detection.

The convention behind the hypotheses definitions is as follows. With n_f possible fault locations (see equation (1)), suppose that any one of m_i different types of faults (e.g. a step function, spike, ramp, etc.) may occur at the i th fault location, and note that the number and types of potential faults need not

be the same for each fault location. Define $M := \sum_{i=1}^{n_f} m_i$. Thus,

there are M different faults to be tested for at each time, which, for convenience, are enumerated {fault 1, fault 2, ..., fault M }. Furthermore, suppose that the window length is set at $N+1$. Then, at each time t the following hypotheses would be tested. The null hypothesis, $H_0(t)$, that no fault has occurred, and the alternative hypotheses:

$$\begin{cases} H_1(t): \text{fault 1 occurred at time } t \\ H_2(t): \text{fault 1 occurred at time } t-1 \\ \vdots \\ H_{N+1}(t): \text{fault 1 occurred at time } t-N \end{cases}$$

$$\begin{cases} \text{N+1 hypotheses associated with fault 2} \\ \vdots \\ \text{N+1 hypotheses associated with fault M} \end{cases} \begin{cases} H_{N+2}(t): \text{fault 2 occurred at time } t \\ H_{N+3}(t): \text{fault 2 occurred at time } t-1 \\ \vdots \\ H_{2(N+1)}(t): \text{fault 2 occurred at time } t-N \\ \vdots \\ H_{M(N+1)-N}(t): \text{fault M occurred at time } t \\ H_{M(N+1)-N+1}(t): \text{fault M occurred at time } t-1 \\ \vdots \\ H_{M(N+1)}(t): \text{fault M occurred at time } t-N \end{cases}$$

The fault detection, isolation, and estimation task then becomes determining which of the above $M(N+1)+1$ hypotheses is most likely. To simplify the analysis, we now introduce the following notation.

Definition (1): At any given time t define the unit magnitude fault function $f_i^t(\bullet)$ as that occurring under $H_i(t)$ in the above hypotheses definitions and $\tilde{f}_i^t(\bullet)$ as its corresponding fault signature (see equation (7)). Also define K_i as the magnitude of the fault occurring under $H_i(t)$. Note that the notation for K_i has been altered slightly from that of equation (1). In equation (1) the subscript refers only to the fault location, while in definition (1) it refers to the location, as well as the time of occurrence, of the fault.

As mentioned earlier in this section, we are looking only for faults occurring in the interval $\{t-N, t-N+1, \dots, t\}$. Since, if μ is the time of occurrence of the fault $e(t)$ ($t < \mu$) are distributed independently of which hypothesis is true, for the detection problem we need only consider the $N+1$ length innovations vector defined as

$$\underline{e}(t) := [e(t-N) \ e(t-N+1) \ \dots \ e(t)]^T \quad (8)$$

Determining which of the $M(N+1)+1$ hypotheses is most likely is a multiple hypotheses testing problem. If the fault magnitudes were known *a priori* then an LRT testing procedure, which is known to be optimal in the Neyman-Pearson sense [13], could be used. At each time t , and for $i = 1, 2, \dots, M(N+1)$, the likelihood ratios

$$\Lambda_i := \frac{P_{\underline{e}|H_i}(\underline{e}|H_i)}{P_{\underline{e}|H_0}(\underline{e}|H_0)} \quad (9)$$

could be calculated and mapped onto a decision space. In equation (9) the numerator and denominator are the conditional probability densities of the given residual vector under H_i and H_0 , respectively. Note that the argument t in $\underline{e}(t)$ and $H_i(t)$ has been dropped for convenience.

Since Λ_i depends on K_i , and K_i is, in general, not known, a GLRT is used for the fault detection and isolation. The idea is to find the maximum likelihood estimate (MLE) of K_i under H_i , denoted \hat{K}_i , and use that estimate in place of K_i in equation (9). The hypothesis selected is then the one that maximizes equation (9). To calculate Λ_i the conditional probability densities of \underline{e} under each hypothesis must be found. Since \underline{e} is a jointly Gaussian random vector, it is sufficient to find its mean and covariance matrix. From equation (7) and the fact that the faults are assumed to be deterministic, the mean and covariance matrix of \underline{e} under H_i are

$$\underline{m}_i = E[\underline{e}|H_i] = K_i \tilde{f}_i \quad \text{and} \quad R_i = E[(\underline{e}-\underline{m}_i)(\underline{e}-\underline{m}_i)^T | H_i] = \sigma_v^2 I, \quad (10)$$

where $\tilde{f}_i := [\tilde{f}_i^t(t-N) \ \tilde{f}_i^t(t-N+1) \ \dots \ \tilde{f}_i^t(t)]^T$, the $\tilde{f}_i^t(\bullet)$ are as in definition (1), and I is the identity matrix of dimension $N+1$. These results show that \underline{m}_i is dependent on H_i but R_i is not. Substituting the mean and covariance matrix of \underline{e} into the

multivariate Gaussian probability density, after simplification equation (9) becomes

$$\Lambda_i = \exp \left\{ \frac{1}{2\sigma_v^2} \left[2\mathbf{e}^T \mathbf{K}_i \tilde{\mathbf{f}}_i - \mathbf{K}_i^2 \tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i \right] \right\}. \quad (11)$$

The MLE of \mathbf{K}_i under H_i can be easily shown [1] to be

$$\hat{\mathbf{K}}_i := \operatorname{argmax}_{\mathbf{K}_i} \left\{ p_{e|H_i}(\mathbf{e}|H_i, \mathbf{K}_i) \right\} = \frac{\mathbf{e}^T \tilde{\mathbf{f}}_i}{\tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i}. \quad (12)$$

Using this estimate in place of \mathbf{K}_i in equation (11), maximizing Λ_i is equivalent to maximizing

$$S_i := 2\mathbf{e}^T \hat{\mathbf{K}}_i \tilde{\mathbf{f}}_i - \hat{\mathbf{K}}_i^2 \tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i = \frac{(\mathbf{e}^T \tilde{\mathbf{f}}_i)^2}{\tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i} = \left(\frac{\mathbf{e}^T \tilde{\mathbf{f}}_i}{\tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i} \right)^2 \hat{\mathbf{K}}_i^2. \quad (13)$$

Thus, the GLRT becomes:

- choose H_i such that S_i is maximized
- choose H_0 if $S_i < \gamma \quad \forall i \in \{1, 2, \dots, M(N+1)\}$ (14)

Remark (1): The threshold γ in the test of equation (14) is introduced because, from equation (13), it is clear that $S_i \geq 0 \quad \forall i \in \{1, 2, \dots, M(N+1)\}$. This is a consequence of the maximization involved in finding the MLE of \mathbf{K}_i . In general, when selecting γ there is a tradeoff between probability of detection and probability of false alarm. Guidelines for selecting γ are given in [1].

Remark (2): It is shown in [1] that $\hat{\mathbf{K}}_i$ of equation (12) is both an unbiased and efficient estimate of \mathbf{K}_i . In addition, the variance of $\hat{\mathbf{K}}_i$ is given by

$$\operatorname{Var}[\hat{\mathbf{K}}_i | H_i] = \frac{\sigma_v^2}{\tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i}. \quad (15)$$

Remark (3): The GLRT of equations (13) and (14) is a correlation receiver. The residual vector is "correlated" with each of the hypothesized fault signatures and scaled, and the hypothesis whose fault signature is most correlated with the residual vector is chosen.

5. IMPLEMENTATION AND SIMULATION RESULTS

In this section the implementation procedure is summarized and simulation results are presented. In more general settings the implementation of GLRTs may be computationally expensive, but for the process and faults considered in this paper the implementation is relatively simple. The off-line and on-line portion of the procedure is distinguished and summarized as follows.

Off-line: Given that the transfer function description of the system (equation (1)) is known (including the noise variances and the hypothesized forms of the faults), the following steps must be taken.

- 1) Calculate the power spectral density of $y(t)$ using equation (4). Then, find $G(z)$ such that equation (5) is satisfied. The existence of $G(z)$ is guaranteed by the spectral factorization theorem, and it may be calculated by equating coefficients with the aid of a numerical software package. The whitening filter is then $G^{-1}(z)$.
- 2) After selecting the window length N , using the hypothesized faults define the appropriate set of hypotheses to be tested for (see the beginning of section 4). For each hypotheses H_i , using the transfer function

description of the system and the whitening filter, calculate the fault signature vector $\tilde{\mathbf{f}}_i$ and the inner products $\tilde{\mathbf{f}}_i^T \tilde{\mathbf{f}}_i$.

- 3) Select the threshold γ to be used in the GLRT of equation (14).

On-line:

At each time t in the on-line implementation, the following steps are required.

- 1) With measurements of the system input and output, filter $y(t)$ with $G^{-1}(z)$ to obtain the innovation $e(t)$. Update the innovations vector of equation (8).
- For $i = 1, 2, \dots, M(N+1)$ repeat steps 2) and 3).
- 2) Calculate $\hat{\mathbf{K}}_i$, the MLE of the fault magnitude under H_i , using equation (12).
- 3) Calculate the test statistic S_i using equation (13).
- 4) Select H_i based on equation (14), giving the estimated fault type, fault location, and fault time.

To illustrate the GLRT method, simulation results are now presented. In the example, step faults are added to each of two ARMAX subsystems connected in series, and the GLRT method is used to detect the faults.

Example:

The system used in this simulation is that of Figure 1 with $n = 2$ and no input excitation (the system is driven by noise alone). The ARMAX polynomials and noise statistics are as follows:

$$\begin{aligned} A_1(z) &= 1 - 1.8z^{-1} + 0.9z^{-2}, & C_1(z) &= 1 - 1.2z^{-1} + 0.45z^{-2}, \\ A_2(z) &= 1 - 1.0z^{-1} + 0.74z^{-2}, & B_2(z) &= 1 - 0.7z^{-1}, \\ C_2(z) &= 1 - 0.5z^{-1}, & \sigma_{a_1}^2 &= 1, \quad \sigma_{a_2}^2 = 2, \quad \text{and } u(t) = 0 \quad \forall t \end{aligned}$$

For this system it can be easily shown that, under no fault conditions, the variance of $x_1(t)$ and $x_2(t)$ are 3.61 and 7.08, respectively. The following two faults were added to the process: 1) a step fault in subsystem 1 of magnitude 3.9 ($= 2\sigma_{x_1}$) from timesteps 201 through 250, and 2) a step fault in subsystem 2 of magnitude 4.0 ($= 1.5\sigma_{x_2}$) from timesteps 501 through 550. Here, σ_{x_i} is defined as the square root of the variance of $x_i(t)$. A window length of $N = 20$ and a threshold $\gamma = 45$ were used. In the simulation only step faults were tested for.

The output of the system, $y(t)$, is shown in Figure 3. The whitened innovations $e(t)$, obtained by filtering $y(t)$ with $G^{-1}(z)$, are shown in Figure 4, which shows the entire sequence of innovations, and Figure 5, which shows the innovations only during the early stages of the faults. In Figure 5 the solid line represents the fault signature of the true faults, and the dotted line the actual innovations. Table 1 summarizes the simulation results during the periods in which each of the faults occurred. The first column gives the simulation timestep, column (a) the subsystem in which the fault was detected, column (b) the estimated time of occurrence of the fault, and column (c) the

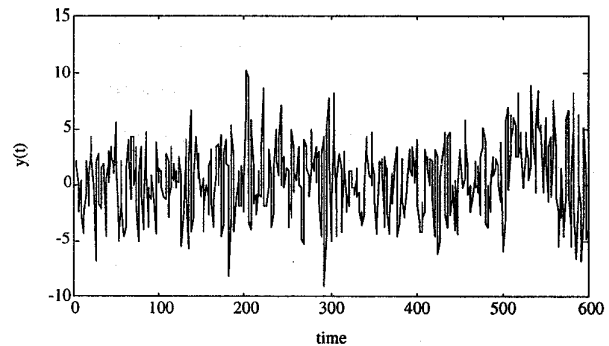


Figure 3 Simulated process output with step faults present from timesteps 201-250 and 501-550.

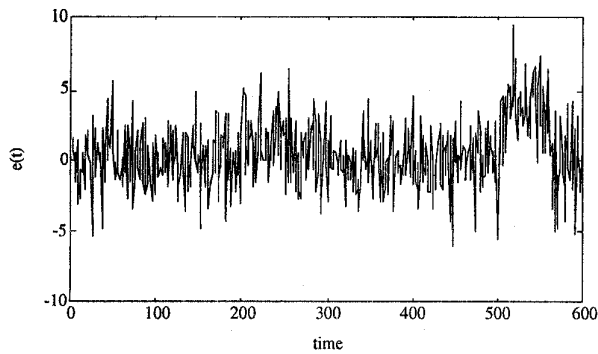


Figure 4 Whitened innovations

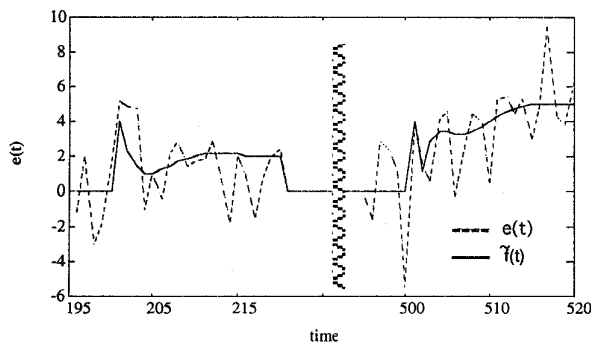


Figure 5 Whitened innovations at the onset of the faults and the corresponding fault signatures.

estimated fault magnitude. The first fault was detected with a delay of 1 timestep and the second with a delay of 7 timesteps. The time of occurrence of the faults were estimated correctly, and approximately 20 timesteps after the occurrence of the faults the estimates of the fault magnitudes were close to the actual values of 3.9 and 4.0 for the first and second faults, respectively. Furthermore, the threshold of $\gamma = 45$ was chosen conservatively enough so that false alarms were completely avoided during the simulation.

Subsystem 1 Fault				Subsystem 2 Fault			
timestep	(a)	(b)	(c)	timestep	(a)	(b)	(c)
201	none	-	-	501	none	-	-
202	1	201	5.94	502	none	-	-
203	1	201	6.55	503	none	-	-
204	1	201	6.08	504	none	-	-
205	1	201	5.99	505	2	501	4.05
206	1	201	5.59	506	none	-	-
207	1	201	5.60	507	none	-	-
208	1	201	5.70	508	2	501	3.51
209	1	201	5.41	509	2	501	3.62
210	1	201	5.21	510	2	501	3.17
211	1	201	5.03	511	2	501	3.42
212	1	201	5.04	512	2	501	3.60
213	1	201	4.76	513	2	501	3.63
214	1	201	4.11	514	2	501	3.71
215	1	201	4.08	515	2	501	3.57
216	1	201	3.94	516	2	501	3.59
217	1	201	3.54	517	2	501	3.95
218	1	201	3.43	518	2	501	3.91
219	1	201	3.44	519	2	501	3.84
220	1	201	3.51	520	2	501	3.92
221	1	201	3.91	521	2	501	4.05

Table 1 Simulation results during the initial stages of the faults. column (a): estimated fault type; column (b): estimated fault time; column (c): estimated fault magnitude. True magnitudes were 3.9 for subsystem 1 fault and 4.0 for subsystem 2 fault.

6. SUMMARY AND CONCLUSIONS

In this paper an easily implementable, computationally efficient method for fault detection and diagnosis in complex MISO linear systems has been developed. The complex system may be comprised of a number of interconnected subsystems experiencing, potentially, additive faults at various locations. The method, based on a GLRT, combines the tasks of fault detection, isolation, and identification. The faults are isolated in the sense that the subsystem experiencing the fault, or, more specifically, the location of the fault is determined. The identification takes the form of determining the type of fault that occurred, estimating the time of occurrence, and estimating the magnitude of the fault. In its final form, the GLRT has an interpretation as a correlation receiver, correlating the residuals of a whitening filter with the hypothesized fault signatures.

The implementation procedure, both the off-line design and the on-line algorithm, has been summarized. A simulation example illustrating the performance of the GLRT has been included. In the simulation, step faults were added to each of two subsystems connected in series to form the complete system. The simulation results indicate the GLRT is capable of good detection, isolation, and estimation performance.

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