TOLERANCE ANALYSIS FOR SHEET METAL ASSEMBLY USING A BEAM-BASED MODEL

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ABSTRACT: The tolerance study is one of the most critical and neglected issues in sheet metal assembly manufacturing research. The fundamental shortcoming of currently existing tolerancing methods is the lack of physical significance of the model. This paper presents a new tolerance analysis methodology for sheet metal assembly based on physical/functional modeling of the fabrication error using the developed beam-based model. The modeling method includes principles of decoupling automotive parts into beam members, modeling of beam-to-beam joint geometry, and identification of part locating points. Tolerance analysis is conducted in three steps: (1) input of part and assembly tooling variability based on the statistical distribution; (2) calculation of accumulated tolerances for key process characteristics using statistical models; and (3) calculation of resultant tolerances based on the flexible beam-based model.

1. INTRODUCTION

Sheet metal assembly manufacturing is widely used in the automotive, aerospace, and appliance industries. Geometrical accuracy and dimensional variation are two of the most important quality and productivity factors in sheet metal assembly processes. Predictability and estimation of these factors during early product design stages is a key goal of any leading manufacturer. Traditionally, tolerance analysis is used during the design stage to evaluate the effect of dimensional variation on the cost and performance of the manufactured products. It is well-known that tolerance stacking in assemblies controls the critical clearances and interferences in design. Therefore, the specification of tolerance limits on each dimension and feature is considered by many to be a vital design function (Chase and Parkinson, 1991).

In general, tolerance analysis deals with the problem of finding cumulative tolerances of the final product for a given set of component tolerances. The upper and lower limits for each dimension are specified to ensure that tolerance build-up will not result in unexpected interference or minimum clearance violations between mating parts. Tolerance analysis research can be summarized in a simplified form as:

1. Representation of tolerances and tolerance accumulation, such as: (a) geometric tolerances (Requicha, 1983); (b) variational geometry (Light and Gossard, 1982); (c) triplet representation (Juster et al., 1993).
2. Mathematical models of tolerance accumulation (Shapiro and Gross, 1981), such as: (a) worst case models; (b) statistical models—RSS model, Spotts' modified and mean shift model, Hasofer-Lind model, method of moments, and six-sigma method; and (c) sample-based models (Monte Carlo simulation).
3. Functional models of variation propagation (Voelcker, 1997), such as: (a) tolerancing vs. dimensioning; (b) kinematic adjustments; and (c) 1-D mechanistic model (Liu et al., 1996).

The previous research conducted in the area of tolerance analysis significantly advanced the knowledge base and enhanced the link between design and manufacturing. However, designers still often assign tolerances arbitrarily or base their decisions on insufficient data or deficient models. Chase and Greenwood (1988) stated that today's high-technology products require knowledgeable design decisions based on realistic models, which include producibility requirements. The short review of the literature and aforementioned summary of the current research efforts show that major advancements were accomplished in the area of mathematical representation of tolerances and mathematical models of tolerance accumulation. However, very little research was conducted in the area of developing physical/functional models of the product and process for tolerancing purposes (Takezawa, 1980; Liu et al., 1996). The inclusion of realistic physical/functional models of integrated product and manufacturing processes is especially important for the current technology of manufacturing complex products (Chase and Greenwood, 1988). The currently used common models for assembly tolerance accumulation have distinct limitations when applied to tolerance analysis of sheet metal assemblies. Some of these limitations can be listed as follows:

1. Information about part design is limited to part geometry. It does not include the influence of part flexibility on tolerance accumulation. The only exception to this is the work by Liu et al. (1996), which demonstrated the importance of part flexibility in tolerance analysis by analyzing the impact of welding gun misalignment on final assembly dimensions by using a 1-D model. However, they did not analyze the other aspects of the assembly process, nor did they present a model beyond the 1-D case.
2. Information about product design is limited to inclusion of kinematic constraints. It does not analyze the influence of part-to-part joint geometry. Current tolerance analysis models are built based on the assumption that no interactions exist between parts.
3. Information about manufacturing process design is limited to information about the distribution of part variation, e.g., normal/generalized distributions or distributions estimated based on previous experience. Neither the influence of part locating layouts nor the joining processes are taken into consideration.

This paper presents a new modeling technique which tries to resolve the above limitations. The proposed modeling, when applied to tolerance analysis, allows development of a much more realistic model of the manufacturing process. It is based on the beam-based modeling of the product and allows inclusion of the product fabrication error affected by assembly process, parts dimensional faults, and part-to-part joints.

This enhances the current research in the area of tolerance analysis by presenting principles which include part and assembly process fabrication error modeling for tolerancing study. The other major advantage of flexible beam-based modeling is that the geometry is reduced to only those parameters required to perform a tolerance analysis. The concept of using flexible beam-based modeling allows, on one side, inclusion of a sufficient amount of information about the product and process, and on the other side, still makes the modeling procedure computationally efficient. Additionally, the presented modeling can be used in combination with any statistical model of tolerance accumulation, such as worst case/RSS.

2. THE PRINCIPLES OF BEAM-BASED MODELING FOR SHEET METAL ASSEMBLY

The validity of the tolerance analysis methodology strongly depends on the physical significance of the modeling, underlying assumptions, and principles. The presented analytic modeling of assemblies provides a quantitative basis for evaluation of design tolerances. The objective of the proposed modeling is to analyze the product dimensional response under assembly process conditions. The modeling procedure is developed by creating a simplified beam-based model of the automotive body (Fig. 1), including the critical characteristics of the assembly processes: (1) parts locating layout; (2) location of part-to-part joints; and (3) geometry of part-to-part joints (Shiu et al., 1997). The selection of beam-based modeling is based on the assumptions that (1) the most rigid parts have the biggest impact on the structural/dimensional response (dimensional variation) of the product; (2) dimensional variation is propagated through part-to-part joints; and (3) only selected critical points/features of the assembly are important for tolerancing. Therefore, the simplified, yet representative, model of the automotive body has to contain all the major load-carrying elements to correctly the model dimensional response of the product caused by part and process discrepancies.

The main concept of integrating the design of the product and the manufacturing assembly process is presented in the form of four principles: (1) final product decomposition modeling; (2) beam connectivity modeling; (3) component locating layout modeling; and (4) part-to-part joint geometry modeling. They are based on the key characteristics of the automotive body structure, assembly process, and types of part-to-part joints. The following sections describes the notation used as well as the aforementioned four principles.

Short Review of Structure Analysis

The analyzed structure can be divided into nodes (part-to-part joints) and beams (members) (Figs. 1b and 2). At each joint, the loading condition is specified by six generalized force components, P_1 through P_6, and the response is described by six displacement components, Δ_1 through Δ_6. These forces and displacements are shown in their positive directions at joint i (Fig. 2). P_1, P_2, and P_3 represent the forces and P_4, P_5, and P_6 represent the moments. In a similar way, Δ_1, Δ_2, and Δ_3 represent the translational displacement components, while Δ_4, Δ_5, and Δ_6 represent the rotational displacement components. The right-hand rule is used to establish the positive directions between axes. In matrix form, the forces and displacement vectors can be expressed as: $\{P\}_i=[P_1\ P_2\ P_3\ P_4\ P_5\ P_6]$ and $\{\Delta\}_i=[\Delta_1\ \Delta_2\ \Delta_3\ \Delta_4\ \Delta_5\ \Delta_6]$, where i indicates that each element of the array is associated with joint i. The forces and displacements are applied to all joints located on the structure, which represent the structural loads applied to the structure and their responses. Figure 2b shows a typical beam member ij joining the two joints, i and j. At joint i of the beam member ij, the forces and displacements are shown in reference to the local coordinate system xyz. The relationship between the structural forces and the displacements of a selected structure is given by:

$$\{\mathbf{P}\}_{nx1} = [\mathbf{K}]_{nn}\{\Delta\}_{nx1} \tag{1}$$

where [**K**] is defined as the total structure stiffness matrix, {**P**} and {Δ} represents the total structural forces and displacements respectively, n is the total number of nodes in the structure, and $[\mathbf{K}]_{ii}$ is the direct structure stiffness matrix. $[\mathbf{K}]_{ij}$ is the cross-stiffness matrix ($i=1,2,...,n$; $j=1,2,...,n$; $i \neq j$) (West, 1989).

Final product decomposition modeling

The product decomposition principle describes the method of decoupling the automotive body into beam elements. The different automotive body subassemblies are classified, based on their geometry, into five basic representative element types. Their decomposition is described by dividing typical sheet metal assemblies into simple elements:

(1) Straight beams are the simplest elements of more complicated structures. The joints with other parts, reaction forces, and moments can be considered as the loadings, end forces, and end moments (Fig. 3a).
(2) Angled beams, which are modeled by multiple connected straight beams, are used when the straight beam simplification does not apply (Fig. 3b).
(3) Planar frame structures can be modeled into a single plane. This structure can be identified by its flexibility and large deformable shape in one particular axis (Fig. 3c).
(4) Multi-planar structures can be modeled by multiple single planes.
(5) Finally, non-planar structures can be identified as complicated 3-D multiple planar closed structures that cannot be simplified to a multi-planar structure.

Having defined the basic types of beam model structures, we can represent the product and its subassemblies using a hierarchical structure with all subassemblies organized in the order of assembly sequence (Shiu et al., 1997).

Beam connectivity modeling

The beam-to-beam connectivity model for a single joint is based on the structure stiffness matrix formulation of any number of beams that are connected to a single node (Shiu et al., 1997). The structure stiffness matrix is represented for each beam-to-beam connection.

Component locating layout modeling

Part assembly requires a proper location for each part/subassembly. Each set of fixture points for a particular part is used to determine the orientation and location of that part. Fixture points are selected as part dimensional constraints realized by locators during the assembly process. These constraints are considered as the reduced stiffness matrix by the preset values on the boundaries.

For example, Fig. 1b shows an example of the fixture points where the underbody, the aperture, and the roof are located and then assembled. The structure in Fig. 1b has a 4-2-1 scheme to locate the structure at points a, b, c, and d. Point a is controlled in six directions as shown by the six arrows in Fig. 1b. In a similar way point c receives a four-directional control and points b and d receive two-directional controls each. These points do not receive any direct applied forces but the reaction forces from the locator of the assembly stations. The structural forces and displacement relationship is given by Eq. (1). The matrix from Eq. (1) represents a number of 6n simultaneous equations relating n structure forces to n structure nodal displacements. As shown in Fig. 1b, there are some points on the component that are constrained by the fixture points (locators)—points a, b, c, and d. These constrained displacements can be defined as vector $\{\Delta\}_{II}$. Defining the remaining free displacements of the component as vector $\{\Delta\}_{I}$, we get: $\{\Delta\}_I = [\textit{All free displacements}]^T$, $\{\Delta\}_{II} = [(\Delta_1)_a\ (\Delta_2)_a\ (\Delta_3)_a\ (\Delta_3)_b\ (\Delta_2)_c\ (\Delta_3)_c\ (\Delta_3)_d]^T$ where $(\Delta_1)_a$, $(\Delta_2)_a$, and $(\Delta_3)_a$ represent the x, y, and z directional constraints on point a respectively. In a similar way, the load vector $\{\mathbf{P}\}$ can be separated into the applied loads $\{\mathbf{P}\}_I$, which correspond to the free displacements, and the reaction forces vector $\{\mathbf{P}\}_{II}$, which are associated with the constrained boundary displacements. Finally, we have:

$$\begin{Bmatrix} \{\mathbf{P}\}_I \\ \{\mathbf{P}\}_{II} \end{Bmatrix} = \begin{bmatrix} [\mathbf{K}]_{I,I} & [\mathbf{K}]_{I,II} \\ [\mathbf{K}]_{II,I} & [\mathbf{K}]_{II,II} \end{bmatrix} \begin{Bmatrix} \{\Delta\}_I \\ \{\Delta\}_{II} \end{Bmatrix} \tag{2}$$

Because $\{\Delta\}_{II} = \{\mathbf{0}\}$ for no displacements at the fixture points (locators), Eq. (2) can be reduced to:

$$\{P\}_I = [\mathbf{K}]_{I,I}\{\Delta\}_I \tag{3}$$

Eq. (3) gives the relationship between the applied forces and free displacements where $[\mathbf{K}]_{I,I}$ is the reduced stiffness matrix. In a similar way, the matrices are developed for all other stages of the process.

Part-to-part joint geometry modeling

Part-to-part joint interaction has a significant impact on the dimensional integrity of the part. Currently, in the automotive body assembly process the most often-used joints can be classified into lap-to-lap, butt-to-butt, and lap-to-butt joints (Ceglarek and Shi, 1995). Each joint represents a distinct set of interaction conditions or force loadings, which can be modeled using kinematic constraints. Detailed modeling of part-to-part joint geometry is presented in Shiu et al. (1997).

3. TOLERANCE ANALYSIS METHODOLOGY

Accurate tolerance analysis methodology depends on the physical significance of the model, which allows estimation of the process error (fabrication error) that contributes to the final accumulated tolerances. This section briefly describes fabrication error, then presents the developed tolerance analysis methodology.

3.1 Fabrication Error

Assembly of parts/subassemblies deviating from design nominals creates internal stresses, which affect the geometry of the final product. The deviations from the design nominals can be described as fabrication error, which can be further characterized as a product self-straining problem. Fabrication error increases interactions between subassemblies due to internal strains and stresses in the absence of externally applied forces. To solve this problem, all equilibrating vertical reactions have to be determined in order to impose the zero-rotation boundary condition at the joint points of the structure, as described in the example below.

Figure 4 shows a typical beam member ij with an initially fixed span between the ends. The occurrence of fabrication error can be described as a vector $\{\delta_s\}_{ij}$ in end i (Fig. 4). If these displacements are restrained, a set of fixed-end forces is imposed at joints i and j. These fixed-end forces are given by:

$$\{F_s\}^f_{ij} = -[k]^i_{ii}\{\delta_s\}_{ji} \text{ and } \{F_s\}^f_{ji} = -[k]^j_{ji}\{\delta_s\}_{ij} \tag{4}$$

which are the fixed-end forces induced at ends i and j respectively by the removal of $\{\delta_s\}_{ij}$, and $[k]_{ii}^j$ and $[k]_{ji}$ are the matrices given in the previous section. In order to sustain these fixed-end forces, there must be an equivalent set of structure forces at joints i and j (Fig. 4), which can be described as:

$$\{P_s\}^e_i = [\beta]^T_{ij}\{F_s\}^f_{ij} \text{ and } \{P_s\}^e_j = [\beta]^T_{ji}\{F_s\}^f_{ji} \tag{5}$$

where $[\beta]_{ii}$ is the compatibility matrix. Similar equations can be derived for joint j. Because the given structure of Fig. 4 cannot provide these sustaining forces, they must be removed through a standard joint loaded stiffness analysis, where the loads at joint i are $\{P\}_i=-\{P_s\}^c$, and similarly for joint j.

3.2 Tolerance Analysis

The presented tolerance analysis approach allows incorporation of fabrication error into the stackup model. The proposed tolerance analysis approach is conducted in three steps: (1) determination of the input of the part and assembly tooling variability, based on the statistical distribution; (2) calculation of accumulated tolerances for process characteristic points (key control characteristics—KCCs) using statistical models; and (3) calculation of resultant tolerances for product characteristic points (key product characteristics—KPCs) based on the flexible beam-based model.

Development of the tolerance analysis methodology will be conducted using a simple model, as shown in Fig. 5, where a simple vertical beam is assembled to two horizontal beams. The solid line describes the part (beam) in the case of no process (tooling) and product error. On the other hand, the shadowed line in Fig. 5 describes the case with assembly error. The coefficients c_{11}, c_{12}, c_{21}, and c_{22} are determined from the structure stiffness matrix **[K]**, described in the previous section. x_1 and x_2 represent the magnitude of the fabrication error of the part assembled in a given assembly station. They can be described as the KCCs. The measurement points, KPCs, are marked in Fig. 5 as z_1 and z_2. Thus, a relationship between the KCCs and KPCs can be described by the following governing equation:

$$\begin{aligned} d_{11}z_1 + d_{12}z_2 &= x_1 \\ d_{21}z_1 + d_{22}z_2 &= x_2 \end{aligned} \tag{6}$$

Suppose x_1 and x_2 have specified tolerances of +/-Tol_3 and +/-Tol_4, respectively. Then the absolute value of the part assembly has to be constrained by $x_1 \leq Tol_3$ and $x_2 \leq Tol_4$. Thus, Eq. (6) can be modified to:

$$\begin{aligned} |d_{11}z_1 + d_{12}z_2| &\leq Tol_3 \\ |d_{21}z_1 + d_{22}z_2| &\leq Tol_4 \end{aligned} \Rightarrow \begin{cases} d_{11}z_1 + d_{12}z_2 \leq Tol_3 \\ d_{11}z_1 + d_{12}z_2 \geq -Tol_3 \\ d_{21}z_1 + d_{22}z_2 \leq Tol_4 \\ d_{21}z_1 + d_{22}z_2 \geq -Tol_4 \end{cases} \tag{7}$$

These can form the constraint of an optimization problem shown as $\mathbf{Dz} \leq \mathbf{T}_x$, where $\mathbf{D} = \begin{bmatrix} d_{11} & -d_{11} & d_{21} & -d_{21} \\ d_{12} & -d_{12} & d_{22} & -d_{22} \end{bmatrix}^T$; $\mathbf{z}=[z_1\ z_2]^T$; and $T_x=[Tol_3, Tol_3, Tol_4, Tol_4]^T$.

Worst case and RSS scenarios: The objective of the worst case is to find the overall worst- or RSS-built conditions, based on the given components' tolerances. The following equation calculates the overall build tolerance based on worst-case and RSS conditions respectively (Chase and Parkinson, 1991):

$$T_{z_j} = \sum_{i=1}^{n} \left| \frac{df}{dx_i} \right| T_{x_i} \quad \text{and} \quad T_{z_j} = \sqrt{\sum_{i=1}^{n} \left(\frac{df}{dx_i} \right)^2 T_{x_i}^2} \tag{8}$$

where z_i, i=1,2,...n are the KPC points and x_i, i=1,2,...n are the KCC points. Eq. (8) describes the largest/worst and RSS displacements of all selected control points of the automotive body structure for the maximum allowable fabrication error of the analyzed sheet metal assembly process.

4. EXPERIMENTAL VERIFICATION

The verification of the proposed tolerance analysis method is conducted based on the three-beam three-joint 3-D assembly structure with butt-to-butt joints between beams (Fig. 6). The verification is conducted as follows: (1) model the assembled structure—determine the compatibility matrices and calculate the stiffness matrix **K**; (2) calculate the relations between the KPCs (joint 1) and KCCs (δ_x at joints 2, 3, and 4); and (3) calculate of the resulting tolerance. Based on the methodology developed in the previous sections, the final equation of the structure deformation as a function of fabrication error can be derived as shown below:

$$\begin{Bmatrix} \{\Delta_1\}_1 \\ \{\Delta_2\}_1 \\ \{\Delta_3\}_1 \end{Bmatrix} = \begin{bmatrix} -0.7069 & -0.7069 & 0.0002 \\ -0.7067 & -0.7067 & -1.414 \\ 0.7071 & -0.7071 & 10.000 \end{bmatrix} \begin{Bmatrix} \{\delta_x\}_{21} \\ \{\delta_x\}_{31} \\ \{\delta_x\}_{41} \end{Bmatrix} \tag{9}$$

From Eq. (9), if all KCC points are set to +/- 2.377, the worst build on KPC is:

$$\text{If} \begin{bmatrix} \{\delta_x\}_{21} \\ \{\delta_x\}_{31} \\ \{\delta_x\}_{41} \end{bmatrix} \leq \begin{bmatrix} \pm 2.3776 \\ \pm 2.3776 \\ \pm 2.3776 \end{bmatrix}, \text{ then } \begin{bmatrix} \{\Delta_1\}_1 \\ \{\Delta_2\}_1 \\ \{\Delta_3\}_1 \end{bmatrix} \leq \begin{bmatrix} \pm 3.362 \\ \pm 6.722 \\ \pm 3.362 \end{bmatrix} \tag{10}$$

The experimental verification of the above results is conducted below. The length of assembly denotes the assembly or fabrication error of the part. The length of all beams can be adjusted to simulate the fabrication error of the beam (Fig. 6c). The locking pin simulates the welding process that locks the beam in place with the existing interaction force due to fabrication error, which is directed in a similar way as in the case of the butt joint. The resulting position of joint 1 is measured by a CMM (Fig. 6). The measurement of joint 1 allows verification modeling and effectiveness of the developed tolerance analysis methodology.

In order to perform the experiments, a set of fabrication errors reflecting the assembly process is introduced by changing the length of the beams at joints 2, 3, and 4 on the level of +/-2.38 mm. The resultant displacement (error) from the free state is modeled and measured. The displacement of joint 1 in three axes represents the error of the final product. For the sake of simplicity, each beam has the same length. Table 1 shows the sequence of the possible assembly with listed beam length options and experiment number. Each experiment is repeated five times, with its variation (6-sigma) and mean shown in Table 2. The presented results show good correlation with the modeling data.

5. CONCLUSIONS

In sheet metal assembly, an estimation of accurate manufacturing tolerances during the design stage is one of the most important quality and productivity factors. The fundamental shortcoming of the currently existing tolerance analysis methods is the lack of physical significance of the model. Tolerance analysis of sheet metal assembly needs to take into consideration non-rigid parts, additional constraints caused by the assembly fixture locating scheme, and part-to-part joint geometry.

This paper develops a tolerance analysis methodology based on the physical/functional modeling of the sheet metal assembly process. The proposed modeling technique, when applied to tolerance analysis, allows development of a much more realistic model of the manufacturing process. It is based on flexible beam-based modeling of the product and process, and allows inclusion of the product fabrication error affected by: (1) parts/subassemblies locating layouts; (2) location of part-to-part joining elements; and (3) geometry of part-to-part joints. The concept of using flexible beam-based modeling allows, on one side, inclusion of a sufficient amount of information about the product and process and, on the other side, still makes the modeling procedure computationally efficient. Additionally, the presented modeling can be used in combination with any statistical model of tolerance accumulation, such as worst case or RSS. The experimental verification of the developed approach shows a high accuracy with the model.

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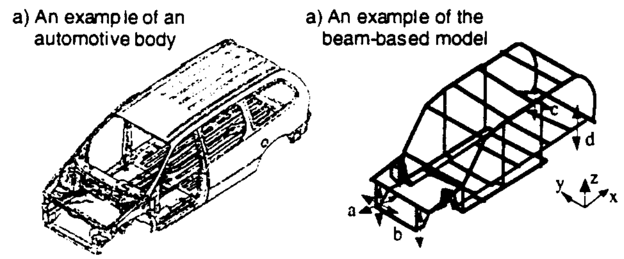
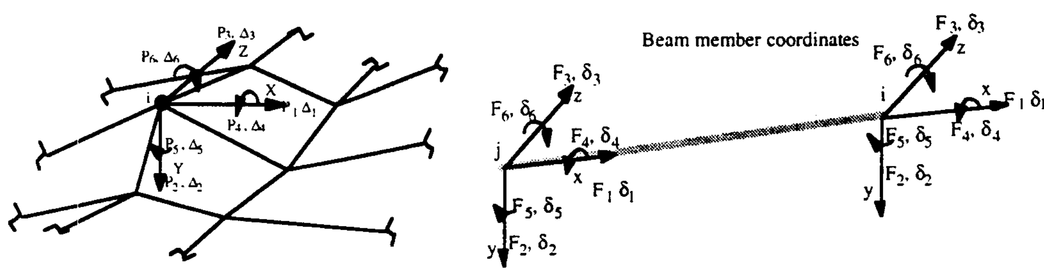
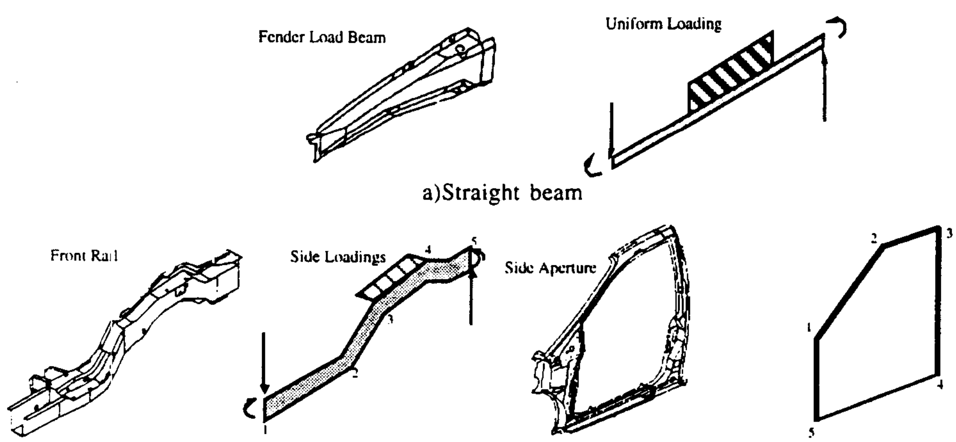


Figure 1. An example of sheet metal assembly of an automotive body using the flexible beam-based model.



a) An example of node (joint) notation b) An example of beam member forces and displacements notation

Figure 2. An example of a node (joint) and beam member forces and displacement for flexible beam-based modeling.



a)Straight beam

b) Angled beam c) Planar frame

Figure 3. Basic types of beam model structures.

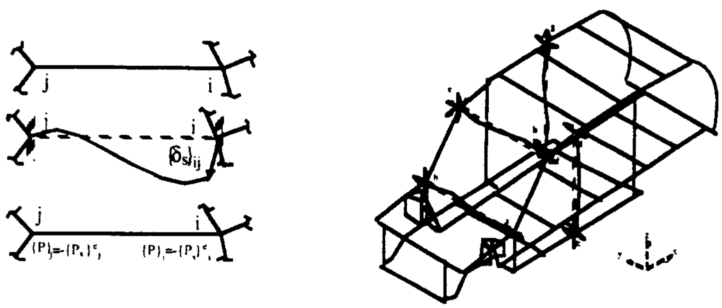


Figure 4. An example of fabrication error.

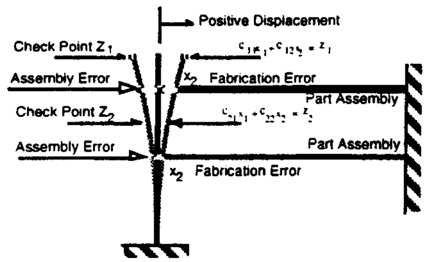


Figure 5. Schematic diagram of the KPC and KCC points with process error.

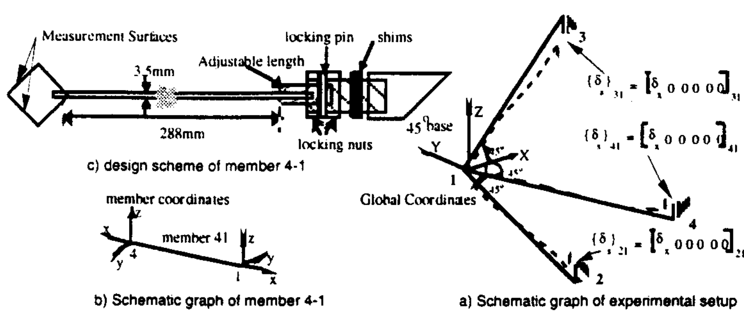


Figure 6. Experimental setup.

Table 1. The design of experiments.

Experiment number	Beam 1-2	Beam 1-3	Beam 1-4
1	Option 1	Option 1	Option 1
2	Option 2	Option 1	Option 1
3	Option 1	Option 2	Option 1
4	Option 2	Option 2	Option 1
5	Option 1	Option 1	Option 2
6	Option 2	Option 1	Option 2
7	Option 1	Option 2	Option 2
8	Option 2	Option 2	Option 2
Beam length	**Beam 1-2** (mm)	**Beam 1-3** (mm)	**Beam 1-4** (mm)
Option 1	+2.377	+2.377	+2.377
Option 2	- 2.377	- 2.377	- 2.377

Table 2. Experimental results.

Experiment number	$\{\Delta_1\}_1$ (mm)	$\{\Delta_1\}_1$ (mm)	$\{\Delta_1\}_1$ (mm)
1	-3.12 0.68	0.04 0.97	-0.01 0.75
2	0.25 0.23	3.56 0.17	***-3.51*** 0.16
3	-0.28 1.61	3.73 0.47	***3.36*** 0.45
4	***3.57*** 0.47	***6.86*** 0.38	-0.08 1.15
5	***-3.24*** 0.57	***-6.50*** 1.21	0.02 0.43
6	0.04 0.52	-3.17 1.00	-3.40 0.45
7	0.13 0.38	-2.94 1.04	3.31 0.31
8	3.41 0.87	0.20 0.89	0.02 0.60
Tolerance	**+/- 3.57**	**+/- 6.86**	**+/- 3.51**