PROCESS-ORIENTED TOLERANCE SYNTHESIS FOR MULTISTAGE MANUFACTURING SYSTEMS

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ABSTRACT

In multistage manufacturing systems, quality of final products is strongly affected not only by product design characteristics but also by key process design characteristics. However, historically, tolerance research has primarily focused on allocating tolerances based on product design characteristics for each component. Currently, there is no analytical approach for multistage manufacturing processes to optimally allocate tolerances to integrate product and process characteristics at minimum cost. One of the major obstacles is that the relationship between tolerances of process and product characteristics is not well understood and modeled. Under this motivation, this paper aims at presenting a framework addressing the *process-oriented* (rather than *product-oriented*) *tolerancing* technique for multistage manufacturing processes. Based on a developed state space model, tolerances of process design characteristics at each fabrication stage are related to the quality of final product. All key elements in the framework are described and then derived for a multistage assembly process. An industrial case study is used to illustrate the proposed approach.

1. INTRODUCTION

Traditional tolerancing research (Bjorke, 1978; Chase and Parkinson, 1991) mainly focused on an assembly that is build up through many mating features of individual components. Given manufactured components are inherently imprecise, tolerance is used to control the quality of final assembly by specifying allowable limits on its components. There are two basic directions in tolerancing research: 1) tolerance analysis or (variation) prediction, 2) tolerance synthesis or allocation. A general idea of tolerance analysis is given as follows. First, a mathematical expression such as geometrical or dimensional tolerance is used to represent the component tolerance according to their properties. Then, based on mathematical model of tolerance accumulation such as the Worst Case (WC) model and Root Square Sum (RSS) model (Shapiro and Gross, 1981), or functional model (Voelcker, 1997) such as kinematics adjustments (Whitney *et al.*, 1994) and mechanistic model (Liu *et al.*, 1996), the variation resulting from component tolerance is computed and predicted for the final product. Tolerance synthesis/allocation, on the other hand, is conducted in the inverse direction: given the quality specification on final product, what tolerance should be assigned to each component? There is always manufacturing cost associated with the tolerance to be assigned: the tighter the tolerance, the higher the cost. Hence, the optimal tolerance with minimum manufacturing cost can be allocated by solving a constrained optimization problem (Wu, *et al.*, 1988; Lee and Woo, 1989, 1990).

Previous research conducted in the area of tolerance analysis and allocation significantly advanced the knowledge base of mechanical design and becomes one of the important tools to improve quality of production system. However, just like being described above, the traditional tolerancing technique is primarily concerned with dimensional or geometrical variables of component in an assembly. These variables are the description of dimensional property and characteristics of product and its component, which are fully determined in design stage. Thus, we call them *product variables*. Correspondingly, we label the traditional toleracning techniques as *product-oriented tolerancing* since the inclusion of process information in the traditional tolerancing scenario is limited, where product variables and manufacturing process are only connected by the associated manufacturing cost. As long as the cost-tolerance function is specified, the tolerance allocation can be conducted based on the mathematical/functional model without concerning any other effects from the manufacturing process.

Recently researchers come to realize that the traditional product-oriented approach overlooked the impact of *process variables* (such as variables describing status of fixtures and machine tools) on the product quality in complex multistage manufacturing processes. We present the following examples to materialize the meaning of process variables and address the new challenges.

Example 1. Automotive body assembly process (as shown in Fig. 1), which is designed of 150 - 250 sheet metal parts assembled in 55 - 75 assembly stations. The variation of the autobody is not only contributed by dimensional imperfection of each part but also affected by fixturing errors at each individual station and imprecise welding force and location. The dimension/geometry of fixture, welding force, and position of welding spot are considered as process variables. Furthermore, the variation of process variable at each station propagates along the assembly stream and accumulates on the final product. The tolerance of process variables has direct impacts on the precision of autobody assembly.

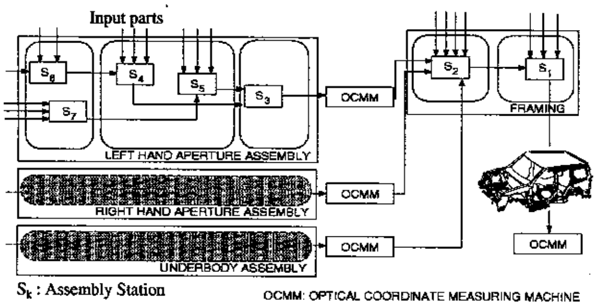


Figure 1. Automotive body assembly process.

Example 2. An engine head machining process (Fig. 2). The engine head is involved in multi-operation process such as cutting, milling, drilling, and taping. The engine head will be transferred to different working stations and positioned by different sets of fixture. Process variables such as fixture position, machine tool position, vibration, and cutting force are the major contributions to the variation of finished engine head. The final product quality is a function of tolerances of process variables at each machining stage.

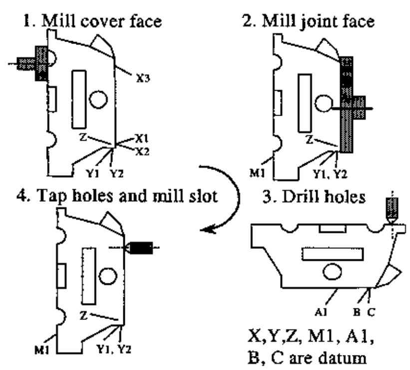


Figure 2. Multistage machining process

Example 3. A transfer progressive die process (Fig. 3).

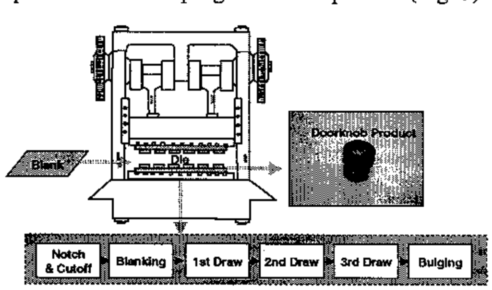


Figure 3. Multistage transfer die process

One stamped part is actually fabricated through multiple die stations including blanking, several drawing operations, and bulging. The tonnage force, nitrogen cushion pressure, and shut height at each station as well as punch speed are treated as process variables. Variability of those process variables contribute at individual stations and propagate throughout the stamping process, eventually impacting the dimensional accuracy and surface quality of the final stamped part. Controlling the variation of those process variables can greatly improve the quality of final product. A systematic approach needs to be developed.

The above three cases exemplify multistage manufacturing processes that can be defined as processes involving multiple stations or operations to produce a product. The quality of final product from a multistage manufacturing process is not only determined by the tolerance of each individual component but also affected by variations of many process variables such as fixturing error, force, and tooling vibration during different manufacturing stages. Process variables cover a very broad category and includes rather diverse quantities associated with manufacturing process. They are not part of product information but indicators of process status. One of the major challenges in analyzing process variables is that the relationship between the variation of process variables and product quality is of very complex fashion depending on process design, tooling layout, and manufacturing sequence. Moreover, additional complexity is introduced because process variables can change over time due to factors such as tooling degradation. Hence, they cannot be represented by simple WC/RSS stack-up or product functional model with only kinematics relationship.

Determining the influence of process variable on the product quality and finding a way to improve it fall into the field of design of experiments (DOE) and robust design represented by Taguchi methods (Taguchi, 1986). Nevertheless, the experimental approaches are difficult to conduct in studying variational behavior of large-scale multistage manufacturing systems. Researchers started developing engineering models to analyze impact of variation of process variables on product quality. Rong and Bai (1996) studied the effect of dimensional tolerance of locators on the geometrical accuracy of a machined workpiece by using datum-machining surface relationship graph (DMG). Choudhuri and DeMeter (1999) analyzed the impact of geometric tolerances of the locator on the variation of machined feature. Although both work advanced knowledge on impacts of process variables on product quality, there are two limitations of their work: 1) focused on a single workstation (rather than multiple station/stage process), 2) only tolerance analysis is discussed and no tolerance allocation scheme proposed.

In our opinion, tolerancing for multistage manufacturing processes relies on the development of model representing variation propagation in the process. There exist several models describing propagation of variation in multistage manufacturing processes. Lawless *et al.* (Lawless *et al.*, 1999; Agrawal *et al.*, 1999) adoptes an AR(1) model to investigate variation accumulation in manufacturing processes. Mantripragada and Whitney (1999) developed a state transition model to describe propagation of variation in assembly processes. Suri and Otto proposed an Integrated System Model (ISM) and allocated tolerance in a process by using the ISM (Suri *et al.*, 1999). Nevertheless, their focus is still on the tolerance of product variables since the variations of process variables are assumed to be known. The problem of tolerance allocation integrating product and process variables is still not comprehensively addressed.

Therefore, in this paper we would like re-formulate tolerance allocation problem as follows: *for a multistage manufacturing process, given the quality specification of final product, how to optimally allocate the tolerances of process variables such that the given quality* *criteria can be achieved at minimum cost.* To differentiate this problem from the conventional product-oriented tolerance synthesis, we call it *process-oriented tolerance synthesis*.

This paper will focus on the development of process-oriented tolerance synthesis method, using as a case in point sheet metal assembly process. The development requires understanding and modeling of variation and its propagation in multistage manufacturing processes, information from product and process design, as well as information regarding quality requirements. Such kinds of model did not exist until recent developments (Jin and Shi, 1999; Jin *et al.*, 1999; Ding *et al.* 2000). The lack of analytical process model is one of the major obstacles causing that process-oriented tolerance synthesis technique is underdeveloped and falls behind the industrial needs. The outline of the paper is as follows. In Section 2, the general framework of process-oriented tolerance synthesis technique is developed for automotive body assembly process. Section 3 illustrates the proposed technique using an industrial case study. Finally, the developed methodology is summarized in Section 4. We would like to indicate, that despite the research presented in the specific context of sheet metal assembly process, the methodology is applicable to generic multistage manufacturing processes.

2. FRAMEWORK OF PRCOESS-ORIENTED TOLERANCING

2.1 Overview

A schematic diagram is shown in Fig. 4 to demonstrate tolerance, quality, and cost interwining in a multistage manufacturing process. The cost is associated with the tolerances assigned to both *process variables* and *product variables*(Fig. 4). Variations of process/product variables determined by their tolerances will affect the quality of final product. The variation of *process variables* contributing at different manufacturing stages is the focus of this paper. The variation of *product variables*, i.e., variation of each component from precedent process is treated as initial variation condition to the current fabrication process. Later on, with the example from autobody assembly process, we will simplify the problem by setting the initial condition to zero, namely we are only concerned with process variables in the current development.

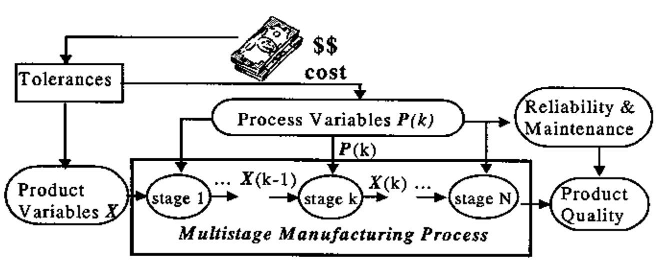


Figure 4. Overview of process-oriented tolerancing

Differentiated from product variables, process variable carries the dynamic process information such as tooling degradation and thus they are strongly related to process reliability and the corresponding maintenance policies. If the tolerances are allocated without considering tooling degradation, product quality can only be guaranteed at the very initial stages of production. However, the quality criteria should be satisfied not only during initial stages of production but during the whole life cycle of the production system. Currently, for many real production systems, maintenance service is conducted following fixed time schedule, e.g., all locating pins at assembly stations are replaced every half a year. In this case, the initial tolerances need to be tighter to accommodate tooling degradation between conservative maintenance schedule to avoid out-of-specification products. Mathematically, the optimal tolerance $\boldsymbol{T}^*$ can be formulated as the following constrained optimization.

$$
\begin{aligned}
&\boldsymbol{T}^* = \min_{\boldsymbol{T}} \mathrm{C_T}(\boldsymbol{T}) \\
&\text{subject to } \Im_{\mathrm{Q}}(\boldsymbol{T}, \mathrm{t}) < \mathrm{C} \text{ for } \{\mathrm{t} : 0 < \mathrm{t} < \mathrm{t_m}\}
\end{aligned} \tag{1}
$$

where C_T represents the cost function of tolerancing, $\boldsymbol{T}$ is the tolerance vector of selected key process variables, $\Im_Q(\cdot,\cdot)$ is a given measure or index of product quality, C is the threshold of specified product quality, t is time, and t_m represents the maintenance time period.

The cost function is determined by the tolerances assigned to process variables. Generally speaking, the tighter the tolerance, the higher the cost of satisfying it. The choice of cost function of process variables strongly depends on physics of the variables. The reciprocal function and negative exponential function are widely used as cost functions. The second question is how to relate the tolerances to the product quality index, which actually is the constraint function. The traditional WC/RSS models are no longer applicable in this situation since the relationship between tolerance of process variable and product quality depends on the physics of manufacturing process. In fact, the construction of the constraint function relating the tolerance to quality needs development of several essential models shown in Fig. 5 with both forward (prediction) and backward (allocation) propagation.

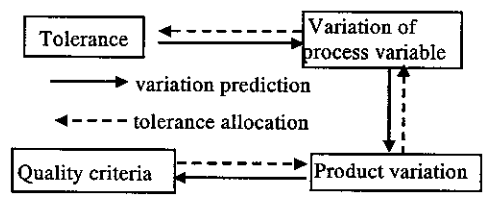


Figure 5. Relationship between tolerance and quality

From the above diagram, it can be seen that the tolerance is first related to variation of process variables. Product variation propagates along the production line with contribution and accumulation from process variables at each stage. Eventually, some proper measure is exerted to compare the variation of final product with the specified quality index. Therefore, there are four key elements to realize the above optimization formulation:

- Variation propagation model,
- Tolerance-variation relationship,
- Process degradation model,
- Cost function.

2.2 State Space Model of Variation Propagation

A multistage assembly process such as automotive body assembly is described in detail in Ceglarek *et al.* (1994). Fig. 6 demonstrates a process with N assembly stations. A state space model has been developed by the authors (Jin and Shi, 1999; Ding *et al.*, 2000) to describe the variation propagation in a multi-station assembly process as

$$
\boldsymbol{X}(\mathrm{k}) = \boldsymbol{A}(\mathrm{k}-1)\boldsymbol{X}(\mathrm{k}-1) + \boldsymbol{B}(\mathrm{k})\boldsymbol{P}(\mathrm{k}) + \boldsymbol{W}(\mathrm{k}) \tag{2}
$$

$$
\boldsymbol{Y}(\mathrm{k}) = \boldsymbol{C}(\mathrm{k})\boldsymbol{X}(\mathrm{k}) + \boldsymbol{V}(\mathrm{k}) \tag{3}
$$

where X(k) is the part deviation measured in selected points after operation at station k, P(k) is the process variation contributed at station k, Y(k) is the measurement vector, and W(k) and V(k) are process disturbance and sensor noise, respectively. A, B, and C are system matrices encoding process configuration information such as fixture layout and sensor location. Detailed expression regarding matrices A, B, and C can be found in (Ding *et al.*, 2000).

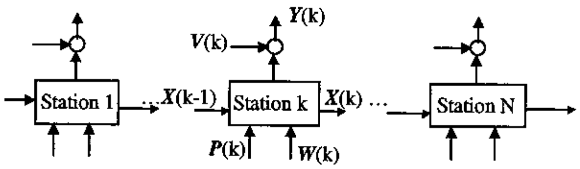


Figure 6. An assembly process with N stations

Suppose there is only end-of-line observation, that is, k = N. Eq. (3) can be transformed to

$$Y(\mathrm{N}) = \sum_{\mathrm{k=1}}^{\mathrm{N}} C(\mathrm{N})\Phi(\mathrm{N,k})B(\mathrm{k})P(\mathrm{k}) + C(\mathrm{N})\Phi(\mathrm{N},0)X(0) + \varepsilon \quad (4)$$

where the state transition matrix is defined as

$$\Phi(\mathrm{N,k}) = A(\mathrm{N}-1)A(\mathrm{N}-2)\cdots A(\mathrm{k}) \quad \text{and} \quad \Phi(\mathrm{k,k}) = I \quad (5)$$

Here, X(0) corresponds to the initial conditions, coming from manufacturing imperfection of stamped parts, and ε is the summation of all modeling uncertainty and sensor noise terms. Moreover, it was assumed that this process involves sheet metal assembly with only lap-lap joint (Ding *et al.*, 2000) and thus stamping imperfection of part dimensions will not affect propagation of variations. Then, we can set initial conditions to zero. The uncertainty term ε can be neglected in the design stage or estimated based on the historical data. The variation propagation can then be approximated as a pure linear relationship as

$$K_{\mathrm{Y}} = \sum_{\mathrm{k=1}}^{\mathrm{N}} \gamma(\mathrm{k}) K_{\mathrm{P}}(\mathrm{k}) \gamma^{\mathrm{T}}(\mathrm{k}) \quad (6)$$

where $K_{(\cdot)}$ is a covariance matrix and $\gamma(\mathrm{k}) = C(\mathrm{N})\Phi(\mathrm{N,k})B(\mathrm{k})$. Thus, the product quality is affected by K_P(k) that is the covariance of *process variables*. Based on the engineering knowledge, it is known that process variable in this problem is related to fixturing error at every assembly station, which is most often caused by clearance of locating pin-hole pair. More discussion on the locating pairs is presented in Ceglarek *et al.* (1994).

2.3 Relationship between Tolerance and Variation

There are two major types of pin-hole locating pair commonly used in the automotive body assembly industry: (1) 4-way pin-hole and (2) two-way pin-hole locating pair, shown in Fig. 7, where d_{pin} or d_{hole} is the diameter of a pin or a hole and T_i is the specified tolerance of the clearance, that is, the upper limit of the clearance. The 4-way pin-hole locating pair includes a homogeneous circular hole and controls motion in both X and Z directions (Fig. 7(a)). The 2-way pin-hole locating pair consists of a slot and a circular pin and thus only controls the motion perpendicular to the long axis of the slot, i.e., Z direction in Fig. 7(b). These two types of locating pairs are used together to position a part during assembly. Due to the free motion along the x axis of 2-way pin-hole locating pair, the part is not overconstrained in fixturing.

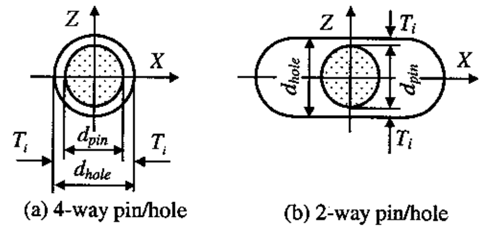


Figure 7. Diagram of pin/hole locating pairs

Our primary interest is the variation associated with a pin-hole locating pair caused by its clearance. The clearance-induced deviation is shown in Fig. 8. The geometrical relationship is obtained by Jin *et al.* (1999). The deviation of a 4-way locating pair is exemplified in Fig. 8 (a), in which the deviations of P_1' (center of the pin-hole) from P_1 (center of the pin) in both X and Z directions are

$$\Delta X = \delta \cos\theta \quad (7)$$

$$\Delta Z = \delta \sin\theta \quad (8)$$

where δ is the distance between P_1' and P_1 and θ is the contact orientation.

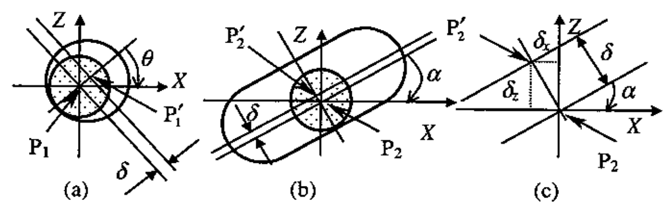


Figure 8. Clearance-induced deviation

If the clearance tolerance is T_i, δ is the random variable representing the actual clearance in one setup. Although δ is in fact bounded by [0, T_i], it can be reasonably approximated by a normal distribution $N(\frac{T_i}{2}, (\frac{T_i}{6})^2)$. The clearance of a 4-way locating pair is considered as homogenous in all directions and thus the orientation angle θ is of the uniform distribution between 0 and 2π, i.e., $\theta \sim U(0, 2\pi)$. Given that the two random variables δ and θ are independent with each other, the statistics regarding ΔX and ΔZ are shown as follows.

$$E[\Delta X] = E[\delta\cos\theta] = E[\delta]\cdot E[\cos\theta] = 0 \quad (9)$$

$$E[\Delta Z] = E[\delta\sin\theta] = E[\delta]\cdot E[\sin\theta] = 0 \quad (10)$$

$$\sigma^2_{X,4\text{-way}} = E[\Delta X^2] = E[\delta^2]\cdot E[\cos^2\theta] = \frac{5T_i^2}{36} \quad (11)$$

$$\sigma^2_{Z,4\text{-way}} = E[\Delta Z^2] = E[\delta^2]\cdot E[\sin^2\theta] = \frac{5T_i^2}{36} \quad (12)$$

$$\mathrm{Cov}(\Delta X, \Delta Z) = E[\Delta X \Delta Z] = E[\delta^2]\cdot E[\sin\theta\cos\theta] = 0 \quad (13)$$

where $E[\cdot]$ is the expectation and $\mathrm{Cov}(\cdot,\cdot)$ represents covariance of two random variables. These equations imply that deviations of P_1' in both directions have zero mean and the same variances. They are also uncorrelated according to Eq. (13).

The geometrical relationship of a 2-way locating pair with orientation angle α shown in Fig. 8 (b) and (c) reads

$$\delta_X = \delta \sin\alpha \cdot \kappa \text{ and } \delta_Z = -\delta\cos\alpha\cdot\kappa, \quad (14)$$

where δ is defined in the same way as before and κ is a binary random variables with value either 1 or -1. We postulate that if the pin touches the top (or left if α approaches 90°) edge of pin-hole, then κ is 1; if the pin touches the bottom (or right if α approaches 90°) edge of pin-hole, then κ is -1. Also, κ is independent of δ. Hence, the variation associated with a 2-way locating pair can then be expressed as

$$E[\delta_X] = E[\delta_Z] = 0, \quad (15)$$

$$\sigma^2_{X,2\text{-way}} = E[\delta^2 \sin^2\alpha\cdot\kappa^2] = \frac{5T_i^2}{18}\cdot\sin^2\alpha, \quad (16)$$

$$\sigma^2_{Z,2\text{-way}} = E[\delta^2 \cos^2\alpha\cdot\kappa^2] = \frac{5T_i^2}{18}\cdot\cos^2\alpha, \quad (17)$$

$$\mathrm{Cov}(\delta_X,\delta_Z) = E[\delta^2 \cos\alpha\sin\alpha\cdot\kappa^2] = \frac{5T_i^2}{18}\cos\alpha\sin\alpha. \quad (18)$$

Eq. (18) implies that the deviations of a 2-way locating pair in arbitrary orientation angle α are correlated. In order to eliminate this correlation, the angle α is usually made to be around 0° (horizontal) or 90° (vertical). In light of this argument, it can be concluded that the matrices $\boldsymbol{K}_P(k)$ are diagonal for all k according to Eqs. (13) and (18). Eqs. (11), (12), (16), and (17) will be iteratively applied to every pin-hole locating pair on each station so that $\boldsymbol{K}_P(k)$ can be expressed in terms of corresponding tolerances.

2.4 Process Degradation Model

A stochastic degradation model of a locating pin is described in Jin *et al.* (1999). The locating pin aggregates wear with the increase in the number of operations. The aggregated wear $\Delta_d(t)$ at operation t is expressed as

$$\Delta_d(t) = \Delta_d(t-1) + \Delta_r(t) \quad (19)$$

where $\Delta_r(t)$ is the incremental wear due to operation t. According to Jin *et al.* (1999), $\Delta_r(t)$ is of lognormal distribution, i.e., $\Delta_r(t) \sim \mathrm{LOGNORM}(\mu_\Delta(t), \sigma^2_\Delta(t))$. The mean of wear-out rate μ_Δ consists of two components, a constant wear-out rate plus a higher initial wear-out rate that decreases exponentially. The mean of wear out rate at operation t is assumed to be

$$\mu_\Delta(t) = \mu_0 + \mu_1 e^{-\beta t} \quad (20)$$

where $\mu_0+\mu_1$ is the initial wear out rate, μ_0 is the constant rate and β determines how fast the wear-out will reach its steady state. The change of clearance of pin-hole locating pair can be computed by

$$d(t) = \delta + \Delta_d(t) \quad (21)$$

where d(t) is the clearance after operation t and the δ is the initial clearance same as that in Eqs. (7) and (8). This implies that the clearance increases after pin wears out and thus the locating variation increases as well. We should substitute the enlarged clearance at time t_m into Eqs. (7), (8), and (14) and recalculate the locating variation. In the following derivations, we make the following assumptions: 1) the initial clearance δ, orientation variables θ and κ, and aggregated wear $\Delta_d(t)$ are assumed to be independent to each other, 2) the variance of wear out rate σ^2_Δ is assumed to be the same for all operations, 3) according to the Central Limit Theorem, the aggregated wear $\Delta_d(t)$ will be close to normal distribution after large enough number of operations. Based on these properties and assumptions, the following relationships can be obtained by substituting Eq. (21) into Eqs. (11), (12), (16) and (17), respectively.

$$\sigma^2_{X,4\text{-way}}(t_m) = E[(\delta+\Delta_d(t_m))^2\cdot\cos^2\theta] = \frac{1}{2}E[(\delta+\Delta_d(t_m))^2] \quad (22)$$

$$\sigma^2_{Z,4\text{-way}}(t_m) = E[(\delta+\Delta_d(t_m))^2\cdot\sin^2\theta] = \frac{1}{2}E[(\delta+\Delta_d(t_m))^2] \quad (23)$$

$$\sigma^2_{X,2\text{-way}}(t_m) = E[(\delta+\Delta_d(t_m))^2\sin^2\alpha\cdot\kappa^2] = \sin^2\alpha\cdot E[(\delta+\Delta_d(t_m))^2] \quad (24)$$

$$\sigma^2_{Z,2\text{-way}}(t_m) = E[(\delta+\Delta_d(t_m))^2\cos^2\alpha\cdot\kappa^2] = \cos^2\alpha\cdot E[(\delta+\Delta_d(t_m))^2] \quad (25)$$

where

$$\begin{aligned} E[(\delta+\Delta_d(t_m))^2] &= E[\delta^2] + 2\cdot E[\delta]\cdot\bar{d}(t_m) + \mathrm{Var}(\Delta_d(t_m)) + \bar{d}(t_m)^2 \\ &= \frac{5}{18}\cdot(T_i + \frac{9}{5}\bar{d}(t_m))^2 + t_m\cdot\sigma^2_\Delta + \frac{1}{10}\cdot\bar{d}(t_m)^2 \end{aligned} \quad (26)$$

and $\bar{d}(t_m) = E[\,\Delta_d(t_m)\,]$ is the average aggregated wear.

2.5 Cost Function

Many different cost function of tolerances have been proposed for different tolerance allocation schemes (Wu *et al.*, 1988). As for a dimensional tolerance made from a fabrication process, the reciprocal function and negative exponential function are most often used representations. In this problem, the cost function is chosen to be reciprocal function represented as

$$C_T = \sum_{i=1,\ldots n_T} \frac{w_i}{T_i}, \quad (27)$$

where T_i is the i^{th} tolerance, i = 1, 2, ..., n_T, w_i is the weighting coefficient associated with T_i.

2.6 Optimization Formulation and Optimality

As long as these essential process models have been made available, a constrained optimization problem is formulated for the multistage assembly process as

$$\begin{aligned} &\boldsymbol{T}^* = \min_{\boldsymbol{T}}\{C_T(\boldsymbol{T})\}\text{ subject to} \\ &\Im_Q(\boldsymbol{T},t) = \sigma_s^2 - \|\mathrm{diag}(\boldsymbol{K}_Y)\|_\infty \ge 0 \text{ for all } 0<t<t_m \text{ and } T_i>0\ \forall i \end{aligned} \quad (28)$$

where $\mathrm{diag}(\boldsymbol{K}_Y)$ extracts the diagonal elements of $\boldsymbol{K}_Y$ (see Eq. (6)), i.e., $\mathrm{diag}(\boldsymbol{K}_Y)$ includes variances of those selected measurement points on final product, which are Key Product Characteristics (KPCs). The current choice of constraint function requires that variations of all KPCs on the final product must be less than the given upper variation limit. This constraint function is only one of the choices, corresponding to the criteria used in industry.

It can be shown that this formulation will achieve the global optimality by investigating the properties of objective function and constraint function through the following *Lemmas*.

Lemma 1. The cost function C_T is a convex function.

This is obvious since the reciprocal function is convex and the summation of convex functions is also convex.

Definition. (Zangwill, 1967, pp. 34) A functional h is called *quasi-concave* if given x_1 and x_2, then

$$h((1-\varphi)x_1+\varphi x_2)\geq \min[h(x_1),h(x_2)] \qquad \forall\, \varphi\in[0,1] \tag{29}$$

Also according to Zangwill (1967), if a function h is quasi-concave, the set it forms by $H = \{\, x \mid h(x) \geq 0 \,\}$ is convex. This implies that the convex cost function converges to its global minimum within the bound formed by constraints if constraint function is quasi-concave.

Lemma 2. The constraint function in Eq. (28) is quasi-concave.

Proof. Given $\boldsymbol{K}_p(k)$ as diagonal matrix, KPC variance vector $\mathrm{diag}(\boldsymbol{K}_Y)$ can be expressed as

$$\mathrm{diag}(\boldsymbol{K}_Y)=\sum_{k=1}^{N}[\gamma(k)^2]\cdot \mathrm{diag}(\boldsymbol{K}_p(k)) \tag{30}$$

where $[\gamma(k)^2]$ represents a matrix whose entity is the square of the corresponding entity in matrix $\gamma(k)$. Moreover, the j^{th} element in vector $\mathrm{diag}(\boldsymbol{K}_P(k))$ is the variance of either a 4-way locating pair or a 2-way locating pair. Based on Eqs. (22) – (25), the j^{th} element can be written in a general format as

$$\mathrm{diag}(\boldsymbol{K}_P(k))_j=c_{jk}\cdot\{\frac{5}{18}(T_{jk}+a)^2+b\,\} \tag{31}$$

where

$$\begin{aligned} c_{jk} &= \frac{1}{2} && \text{for 4-way pin}\\ &= \sin^2\alpha && \text{for 2-way pin in X direction}\\ &= \cos^2\alpha && \text{for 2-way pin in Z direction}\end{aligned} \tag{32}$$

and

$$a=\frac{9}{5}\bar{d}(t_m),\qquad b=t_m\cdot\sigma_\Delta^2+\frac{1}{10}\bar{d}(t_m)^2 \tag{33}$$

If no degradation is considered, term a and b are zero and T_{jk} is corresponding to the initial limit of clearance. Substituting Eqs. (30) and (31) into Eq. (28), the constraint function turns out to be

$$\Im_Q(\boldsymbol{T})=\sigma_s^2-\left\|\sum_{k=1}^{N}\left[\gamma(k)^2\right]\begin{bmatrix}\vdots\\ c_{jk}\{\frac{5}{18}(T_{jk}+a)^2+b\}\\ \vdots\end{bmatrix}\right\|_\infty \tag{34}$$

Considering the tolerance range $[\boldsymbol{T}_1, \boldsymbol{T}_2]$ and for arbitrary $\varphi\in[0,1]$,

$$\Im_Q((1-\varphi)\boldsymbol{T}_1+\varphi\boldsymbol{T}_2)=\sigma_s^2-\left\|\sum_{k=1}^{N}\left[\gamma(k)^2\right]\begin{bmatrix}\vdots\\ c_{jk}\{\frac{5}{18}((1-\varphi)T_{jk,1}+\varphi T_{jk,2}+a)^2+b\}\\ \vdots\end{bmatrix}\right\|_\infty \tag{35}$$

If $T_{jk,1} < T_{jk,2}$, then

$$c_{jk}\cdot\{\frac{5}{18}(T_{jk,1}+a)^2+b\}<c_{jk}\cdot\{\frac{5}{18}((1-\varphi)T_{jk,1}+\varphi T_{jk,2}+a)^2+b\}$$

$<c_{jk}\cdot\{\frac{5}{18}(T_{jk,2}+a)^2+b\}$ for any $\varphi\in[0,1]$ and all j, k,

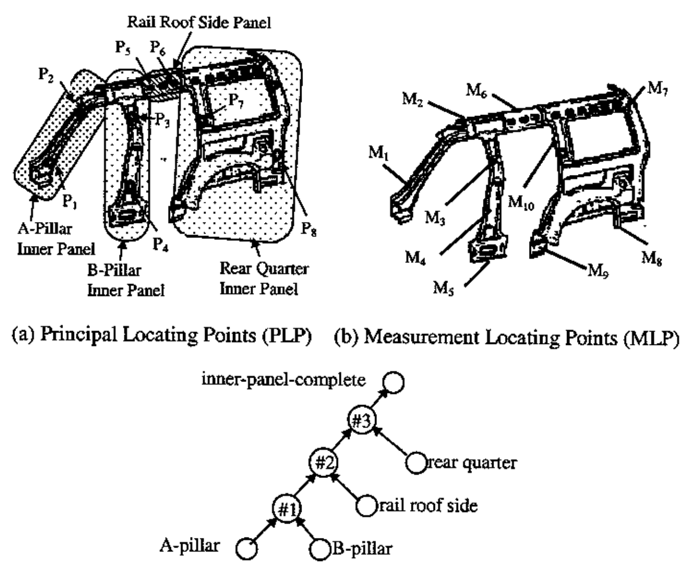
$\therefore\ \Im_Q(\boldsymbol{T}_1)>\Im_Q((1-\varphi)\boldsymbol{T}_1+\varphi\boldsymbol{T}_2)>\Im_Q(\boldsymbol{T}_2)$, implying that $\Im_Q(\boldsymbol{T})$ is quasi-concave.

Theorem. The nonlinear optimization problem (NLP) stated in Eq. (28) converges to a global minimum $\boldsymbol{T}^*$.

Proof. If there is a local optimum $\boldsymbol{T}^*$ satisfying Kuhn-Tucker conditions, it is the global minimum according to Theorem 2.15 in Zangwill (1967).

3. EXAMPLE

The assembly process of side aperture inner panel is used to illustrate the tolerancing of multi-station assembly processes. The inner panel consists of four parts: A-pillar, B-pillar, rail roof side panel, and rear quarter inner panel. Each part contains two Principal Locating Points (PLPs) and several Measurement Location Points (MLPs) that are the KPCs with specified upper limit on their variation levels. The inner panel and the corresponding PLPs and MLPs are shown in Fig. 9 (a) and (b). There are three assembly stations involved in this process. At the first station, A-pillar and B-pillar are welded together and then the rail roof side panel is added at the second station. Finally, the subassembly from the second station consisting of A-pillar, B-pillar, and rail roof side panel is assembled with the rear quarter to yield the inner-panel-complete. The dimensional data are attained at the end-of-line measurement station. The assembly sequence is shown in Fig. 9 (c). The proposed approach is applied to this process first without considering tooling degradation and then with degradation considered.



(a) Principal Locating Points (PLP) (b) Measurement Locating Points (MLP)

(c) Assembly sequence

Figure 9. Side aperture inner panel assembly

3.1 Tolerance allocation when tooling degradation is not considered

There are twelve tolerance variables of clearance $T_1 \sim T_{12}$ to be allocated in this three-station process (each station has four pin-hole locating pairs). It is assumed that all process variables are subject to the same manufacturing cost, that is, $w_i = 1$ for i = 1, 2, ..., 12 in Eq. (27). The designer requires that the final product (the inner-panel-complete) must have Six-Sigma value less than 1.5 mm at all KPCs, namely $\sigma_s^2 = (\frac{1.5}{6})^2$ in Eq. (28). From the industrial practice, it is known that the tolerance of a clearance is usually above 0.01mm. Thus, the initial tolerance is then picked up from the interval [0.01, 2] mm. The procedure for tolerance allocation is shown in the following flow chart (Fig. 10).

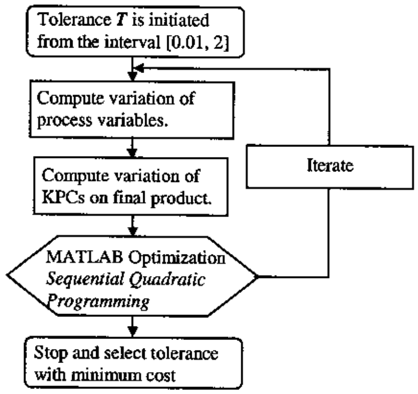


Figure 10. Tolerance allocation w/o degradation model

The optimization problem is solved by using MATLAB function *fmincon* that uses a Sequential Quadratic Programming (SQP) method (MATLAB, 1999). Because the availability of analytical models developed in Section 2, the time-consuming Monte Carlo simulation can be avoided in order to obtain the variation of process variables. The program converges and yields the optimal tolerance after 290 times of iterations in minutes. The optimally allocated tolerances for these process variables are listed in Table 1.

Table 1. Tolerances without tooling degradation. (mm)

T_1	T_2	T_3	T_4	T_5	T_6
0.21	0.36	0.19	0.31	0.30	0.42
T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
0.63	0.36	0.34	0.34	0.34	0.32

Compared with the current industry practice, where the tolerance of locating clearance is allocated empirically as uniform for all locating pairs, the proposed approach no longer allocates tolerances uniformly. This non-uniformity is consistent with the process sensitivity, that is, the more variation the process variable contributes to the final product, the tighter the corresponding tolerance should be. It is too hard for the empirical approach to determine which tolerance should be tight and which one should be loose. As a result, either the cost is higher or the variation of final product is above the threshold. In another word, optimality is difficult to achieve for empirical method.

3.2 Tolerance allocation with consideration of tooling degradation

Under this circumstance, the tolerances are allocated at the beginning of production while the quality criteria are checked for all products produced by the degraded process. The procedure for tolerance allocation with consideration of tooling degradation model is shown in Fig. 11.

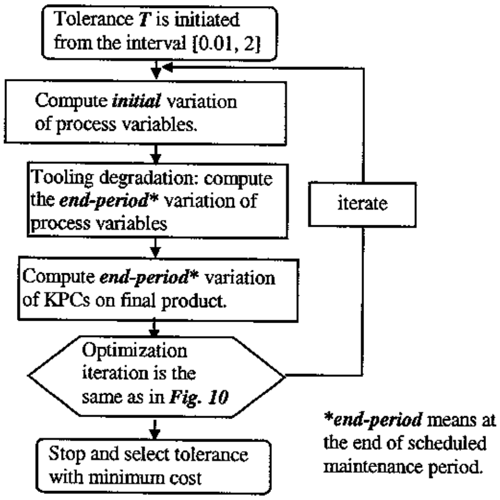


Figure 11. Tolerance allocation with degradation model

The optimization is still solved by using MATLAB function *fmincon* but with the tooling degradation model implemented. Based on the industry experience, parameters needed in the degradation model such as operation rate, maintenance period, and pin wear-out rate are listed in Table 2. The program converges and yields the optimal tolerance after almost the same number of iterations as in Section 3.1. The new tolerances become tighter and are shown in Table 3.

Table 2. Parameters in degradation model

μ_0 (mm)	μ_1 (mm)	β	σ_Δ (mm)	t_m	operations/day
5×10^{-7}	1×10^{-6}	1×10^{-3}	5×10^{-5}	6 months	500

Table 3. Tolerances with tooling degradation. (Unit : mm)

T_1	T_2	T_3	T_4	T_5	T_6
0.16	0.31	0.14	0.25	0.23	0.34
T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
0.58	0.26	0.30	0.27	0.28	0.26

3.3 Comparison and Discussion

In the current automotive industry, the tolerances are uniformly set to be 0.25 mm for all clearances. Substituting these tolerances into the system model described in Sections 2, the maximum Six-Sigma values of KPCs at the beginning of production and after the half year production are listed in Table 4. Although the assigned tolerance can produce qualified products at the beginning of production period, many out-of-specification products will be fabricated after tooling elements degraded.

Table 4. Maximum 6σ of KPCs for 0.25 mm tolerance

Beginning	Half Year	Specified
$6\sigma = 1.44$ mm	$6\sigma = 1.77$ mm	$6\sigma = 1.50$ mm

Furthermore, the manufacturing cost of different cases, represented by the summation of the reciprocal of all the tolerances

(Eq. 27) are compared in Table 5. When degradation is not considered, the tolerance is allocated non-uniformly and result in the manufacturing cost decreasing 20.6% compared to uniform 0.25 mm tolerance. When process degradation is considered, product quality is ensured throughout the production without increasing manufacturing cost from that of the uniform 0.25mm tolerance. Since defective product will be unavoidably produced under the scheme of uniform 0.25mm tolerance, the actual cost is even higher for the empirical method when the quality-loss related costs such as rework, labor, and material waste are counted. It can be concluded that process-oriented tolerance allocation can deliver high quality product in comparably lower cost and thus it is advantageous over the empirical approach.

Table 5. Comparison of cost of different scenarios

Scenario	w/o degradation	with degradation	uniform 0.25 mm
Cost	38.1	47.9	48

4. CONCLUSIONS

The framework of *process-oriented tolerance synthesis* is proposed and demonstrated in this paper with the application to a multistage assembly process. The tolerance of process variable is optimally allocated by solving a nonlinear constrained optimization problem. Four kinds of relationship or models are essential in the process-oriented tolerance synthesis: cost functions, tolerance-variation model, variation propagation model, and process degradation model. Recent progress in modeling and analysis of multistage manufacturing process presents inherent understanding of the process and facilitates the development of process-oriented tolerancing technique.

The advantage of the new approach over the traditional product-oriented tolerancing is that process-oriented tolerancing is no longer concerned only with information of product design. A much broader category of information regarding process design and quality requirements is included. The process-oriented approach really integrates the design and manufacturing and can allocate the tolerance of process variables throughout the entire system with remarkably low manufacturing cost. Furthermore, the approach can guarantee the delivery of desired quality during process life-time service without raising manufacturing cost. This kind of dynamic process information is hard to be included in the traditional product-oriented method. Therefore, the switch to process-oriented technology is a critical technological trend as being pointed out by Thurow (1992) *"In the future sustainable competitive advantage will depend more on new process technologies and less on new product technologies."*

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