MODELING AND DIAGNOSIS OF MULTISTAGE MANUFACTURING PROCESSES: PART I – STATE SPACE MODEL

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ABSTRACT

This two-part paper focuses on the modeling and diagnosis of multistage manufacturing processes. Part I develops a state space model characterizing variation propagation in a multistage process. The state space model describes a discrete-time LTV (Linear Time Varying) stochastic system, which strongly indicates that the existing system and control theory could be used to perform systematic analysis and achieve variation control of the manufacturing process. Moreover, the state space model integrates information of product quality with process information such as tooling status, therefore providing the basis for fault diagnosis. The model is validated through simulation comparison with the widely used software, Variation System Analysis (VSA). The work of fault diagnosis in a multistage manufacturing process is continued in Part II. Despite the research is presented in the specific context of sheet metal assembly process, the methodology is applicable to generic multistage manufacturing processes.

NOMENCLATURE:

J, N	the index and number of all parts in final product
MLP_ξ^J	the ξ^{th} Measurement Location Point at part J
PLP	Primary Locating Point
STD	standard deviation
$\mathbf{U}(i)$	fixturing deviation at station i, defined as $\left[\Delta P_{11}(i) \quad \Delta P_{12}(i) \quad \cdots \quad \Delta P_{n_i 1}(i) \quad \Delta P_{n_i 2}(i)\right]^T$
$\vec{\mathbf{X}}^J(i)$	the state variable that is the deviation vector of part J at station i, defined as $\left[\Delta X^J(i) \quad \Delta Z^J(i) \quad \Delta\alpha^J(i)\right]^T$
X, Z	the global coordinate variables
i, m	the index and number of stations
s, n_i,	the index and number of subassemblies at station i
x, z	the local coordinate variables
$\mathbf{y}, \vec{\mathbf{Y}}$	deviation vector of measurement point
Δ	deviation operator
λ^J	the index of the station on which part J is at its first time welded with other parts or subassemblies in the assembly stream

1. INTRODUCTION

Dimensional quality is one of the deciding factors in today's manufacturing competition. The complexity of a manufacturing process puts high demands on process modeling and design optimization as well as on fault diagnosis to ensure the dimensional integrity of the product. There are two popular directions of research aiming at quality assurance, i.e., using design approach and using diagnostic approach, each of which is reviewed in the following.

1.1 Design-Oriented Quality Improvement

The work of design-oriented quality improvement has been primarily done in *fixture design* and *tolerance design*.

Fixture, which determines the positions of parts during assembling, directly affects the dimensional quality of final products. *Fixture design* attempts to ensure and improve the accuracy, repeatability and reusability of fixture. Asada and By (1985) used a kinematic analysis to comprehend conditions for deterministic positioning, accessibility and detachability. Chou *et al.* (1989) proposed a fixture synthesis method, which automatically generates the locating and clamping points and clamping forces by solving certain constrained linear programming problems. Rong and Bai (1996) evaluated effects of locator errors on workpiece geometrical accuracy and compare different manufacturing paths.

Another design technique to reduce product variation is *tolerance allocation*, which attempts to limit the magnitude of input variations. Tolerance allocation is normally formulated as a constrained optimization problem (Lee and Woo, 1990; Roy and Fang, 1997). A fundamental shortcoming of the existing synthesis methods is that they only include the variation information of product variables while ignoring that caused by process variables. Rong and Bai (1996) and Choudhuri and De Meter (1999) in fact build up a connection between process and product by developing models to describe the impact of fixture error upon product variation. In spite of their contribution in tolerance analysis and accuracy evaluation, they fall short of proposing tolerance allocation scheme.

From the viewpoint of system control, design-oriented quality improvement is equivalent to an open loop feedforward control of manufacturing processes, which is only effective in the absence of noise and disturbance. Due to the complexity and randomness of uncertainties and disturbances in manufacturing processes, a good design cannot guarantee the delivery of desired product quality. Therefore, an effective method for detecting and diagnosing component failures based on in-line measurements would be highly desirable.

1.2 Modeling and Diagnosis of Manufacturing Processes

The modeling and diagnosis of manufacturing processes has become an emerging area on the boundaries of engineering and statistics research and has grown rapidly since its inception in the early 1990s. Earlier research primarily preoccupied with statistical descriptions of variation patterns from in-line measurement data (Hu and Wu,1992) or rule-based fault isolation method (Ceglarek *et al.*, 1994). These diagnostic methodologies are based on heuristic knowledge. Engineering model-based diagnostics of fixture fault in manufacturing process started with Ceglarek and Shi (1996) on the assumptions of single fault, single fixture, and rigid part. From then on, much subsequent research has been finished on more general assumptions, such as multiple faults, compliant components, and multiple fixtures (Apley and Shi, 1998; Shiu *et al*, 1996; Chang and Gossard, 1997; Rong *et al*, 1999). Regrettably, all these works focused only on individual stage/station in the process. An overall process-level model is much more challenging than the earlier stage-level research because the model could be much more complex and data and system information are tremendous. We have to explicitly address the variation propagation issue from the global viewpoint and cover all significant stage-to-stage interactions.

As such, some work has already been initiated to describe variation propagation in multistage manufacturing processes. Mantripragada and Whitney (1999) employ a state transition model to describe the propagation of variation in assembly processes. In their work, the fixture is assumed to be perfect, implying they actually modeled the variation accumulation in assembly caused by part fabrication imperfection. Alternative part-joint design is proposed to improve capability of variation reduction. Lawless *et al.* (Lawless *et al.*, 1999; Agrawal *et al.*, 1999) used an AR(1) model to investigate dimensional variation in assembly process as well as in machining process. They identify interrelations between the statistical model and the physical process by estimating the model parameters from real process measurements. The amount of variation attributable to different stages of a process is then studied to identify opportunities for variation reduction. A physical model that can characterize the stage-to-stage propagation of variation is proposed by Jin and Shi (1999). However, there are some limitations of their work: 1) their model was based on assumptions that only two parts are welded together at each station; 2) their model can only represent sequential process instead of both sequential and parallel processes because the newly added-in part has to be a single part rather than a subassembly; 3) they did not give any complete examples in their paper and no further research on how to apply the state space model in quality control has been conducted.

1.3 Objective and Organization

The objective of this research is aimed at modeling and diagnosing fixture variation in multistage manufacturing processes. A refined state space model is developed to incorporate process information into the representation of product variability. A diagnostic algorithm is then proposed based on the system model. The whole research work is divided into two parts: Part I presents the development of state space model and Part II focuses on the diagnostic method. In order to root our research in engineering realities, the proposed methodology is carried out within the specific context of automotive body assembly processes. Moreover, if there is only one type of operation performed on each station, then every station corresponds to one stage in a manufacturing process. In the following discussions, "station" is used to replace "stage" on many occasions in order to facilitate the visualization of the process.

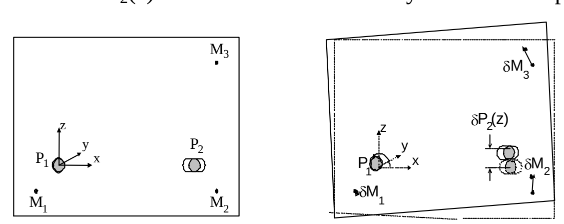
Part I consists of five sections. After the introduction, Section 2 discusses the factors affecting variation propagation in an assembly process. Section 3 derives the state space representation. In section 4, an example from the automotive industry is used to validate the developed model. In section 5 the results are summarized, perspectives of the state space model are addressed, and Part II's diagnostic research is pointed toward.

2. VARIATION FACTORS AND ASSUMPTIONS FOR MODELING

Since the dimensional accuracy is of our basic concern, the factors impacting the dimensional variation propagation in assembly process should be thoroughly understood before modeling. These factors can be divided into variation existing at every single station and variation due to the transference of parts between stations.

2.1 Station-level Variation Factors

At each station, there are two major variation sources, fixturing imperfection and manufacturing imperfection. Fixturing imperfection causes the part supported by this set of fixtures to be away from its nominal position. Figure 1 shows an example of fixture fault manifestation. $\delta P_2(z)$ is a fixture error caused by the failure of pin P_2.



(a) Locators and measurements (b) Manifestation of P_2 failure

Figure 1. An example of fixture fault manifestation

Manufacturing imperfection refers to the deviation of part dimension or feature geometry coming from the sheet metal stamping process and can be considered as initial condition for an assembly process. Whether manufacturing error could affect downstream process depends on what type of part-joint is involved. Ceglarek and Shi (1998) reported that there are two basic types of joint: one is the lap joint (Fig. 2 (a)) and the other is the butt joint (Fig. 2 (b)). The lap joint can absorb variation in the direction of the slip plane while variation will propagate in the defined direction through the mating feature for the butt joint. Figure 2(c) shows a generic joint geometry, which can partially allow motion in the defined direction.

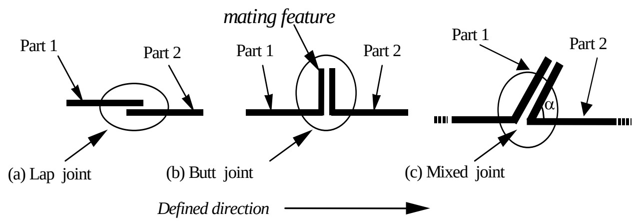


Figure 2. Cross-sectional views of joint geometries

2.2 Reorientation-induced Variation Between Stations

Shiu *et al.* (1996) discovered a phenomenon labeled as reorientation in multi-station process when fixture deviation is present. The reorientation is defined as repositioning due to transferring parts between stations that consist of different datum schemes. For example, in Fig. 3, after part 1 and part 2 were joined together at station i and transferred to station i+1, only pin-holes at part 2 are used to hold the subassembly "1+2". Part 1 deviates from its nominal position even if fixtures at station i+1 are perfect (The dashed line represents the nominal positions of parts while the solid line indicates the actual positions.).

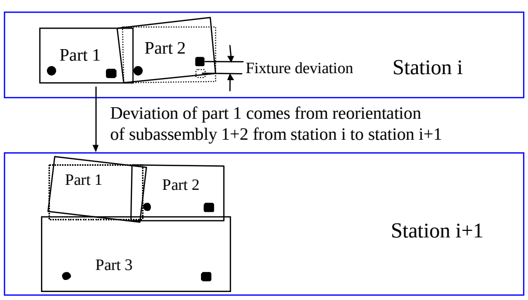


Figure 3. Reorientation-induced deviation

2.3 Summary and Assumptions for Modeling

Besides the variation factors discussed in the previous subsections, it is known that sheet metal part is rather flexible in the direction perpendicular to its panel plane, which will also affect product variation if there is part interference or overconstraint. Thus, the key variation factors can be summarized as:

- Fixture deviations at each assembly station,
- Part-to-part joint types,
- Datum shift scheme between assembly stations,
- Sheet metal compliance.

Considering all the factors during initiation of the problem will result in a model too complex to handle for diagnosis and optimization. Assumptions are made to simplify the problem. In the current research, we only consider a 2-D sheet panel positioned by the 3-2-1 fixture layout (Ceglarek *et al.*, 1994). The model will also apply to n-2-1 nonrigid body fixturing if the fixture faults being considered cause panel motion only in the plane of rigidity. This can cover up to 80% situations in a typical automotive assembly line (Shiu, 1996). Moreover, the part fabrication error is not included so that we can concentrate on modeling and diagnosis of fixture variation. All assumptions for modeling are stated as follows:

(i) 2-D rigid body part;
(ii) 3-2-1 fixture layout for rigid part;
(iii) Lap joint only so that part fabrication error does not affect variation propagation.

3. STATE SPACE MODEL

3.1 Coordinate System

A local coordinate system is fixed on each part or subassembly in order to represent its position and orientation. As shown in Fig. 4, two reference points are assigned to each part and coincide with its PLP holes, denoted as R_1^J and R_2^J, where R_1^J is always assigned to the 4-way pinhole and R_2^J to the 2-way one. A local coordinate system *x-z* is fixed on each part, with the origin at R_1^J, $R_1^J R_2^J$ as *x*-axis and the direction from R_2^J to R_2^J as the positive direction of the *x*-axis. The position and orientation of a local coordinate system in global coordinates are always expressed by position of its origin and angle between the *x*-axis and the X-axis, i.e., X_1^J, Z_1^J and α^J in Fig. 4. The local coordinate system of a subassembly is defined in a similar way by using its two locating points on this subassembly, denoted as P_{sk}, k = 1, 2, where the subscript s refers to the subassembly and the second subscript k indicates a 4-way pinhole (if k=1) or a 2-way pinhole (if k=2). Similarly, a local coordinate system is fixed on subassembly s with the origin at P_{s1}, $P_{s1}P_{s2}$ as *x*-axis and the direction from P_{s1} to P_{s2} as the positive direction of the *x*-axis. X_s, Z_s, α_s are used to represent the corresponding position and orientation of subassembly s.

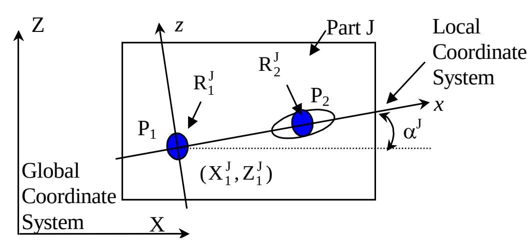


Figure 4. Global and local coordinate systems

3.2 Lemmas and Corollaries

Two lemmas are presented here before setting up the model. These lemmas are used intensively during the future derivations of the state space expression.

Lemma 1. When a rigid body undergoes a translation and a rotation away from its nominal position, if the rotation angle (denoted as $\Delta\beta$) is small, then deviations of any two points on that part (denoted as $[\Delta X_a, \Delta Z_a]^T$ and $[\Delta X_b, \Delta Z_b]^T$) have the relationship,

$$\begin{bmatrix}\Delta X_b\\ \Delta Z_b\\ \Delta\beta\end{bmatrix} = \mathbf{M}(a,b)\begin{bmatrix}\Delta X_a\\ \Delta Z_a\\ \Delta\beta\end{bmatrix} \tag{1}$$

where

$$\mathbf{M}(a,b) = \begin{bmatrix}1 & 0 & -(Z_b - Z_a)\\ 0 & 1 & X_b - X_a\\ 0 & 0 & 1\end{bmatrix}_{3\times 3} \tag{2}$$

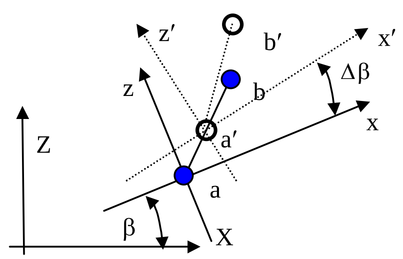


Figure 5. Deviational relationship between two points

Proof. If a local coordinate system is fixed on the rigid body with the origin at point "a" and orientation angle β (Fig. 5), then the global coordinate of point "b" can be written as

$$\begin{bmatrix}X_b\\ Z_b\end{bmatrix} = \mathbf{T}\begin{bmatrix}x_b\\ z_b\end{bmatrix} + \begin{bmatrix}X_a\\ Z_a\end{bmatrix} \tag{3}$$

where **T,** the transformation matrix is expressed as

$$\mathbf{T} = \begin{bmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{bmatrix} \tag{4}$$

Applying the variational operation on both sides of Eq. (3)

$$\delta\begin{bmatrix}X_b\\ Z_b\end{bmatrix} = (\delta\mathbf{T})\begin{bmatrix}x_b\\ z_b\end{bmatrix} + \mathbf{T}\delta\begin{bmatrix}x_b\\ z_b\end{bmatrix} + \delta\begin{bmatrix}X_a\\ Z_a\end{bmatrix} \tag{5}$$

Based on the rigid body assumption, the relative position between "a" and "b" does not change, implying the second term is zero. Also the following relationship holds

$$(\delta\mathbf{T})\begin{bmatrix}x_b\\ z_b\end{bmatrix} = \begin{bmatrix}-\sin\beta & -\cos\beta\\ \cos\beta & -\sin\beta\end{bmatrix}\begin{bmatrix}x_b\\ z_b\end{bmatrix}\delta\beta \tag{6}$$

$$\begin{bmatrix}x_b\\ z_b\end{bmatrix} = \mathbf{T}^{-1}\begin{bmatrix}(X_b - X_a)\\ (Z_b - Z_a)\end{bmatrix}_{2\times 1} \tag{7}$$

Thus, Eq. (5) yields

$$\delta\begin{bmatrix}X_b\\ Z_b\end{bmatrix} = \begin{bmatrix}-(Z_b - Z_a)\\ X_b - X_a\end{bmatrix}\delta\beta + \delta\begin{bmatrix}X_a\\ Z_a\end{bmatrix} \tag{8}$$

which could lead to Eqs. (1) and (2) by rearranging vectors and entities in the matrix. Q.E.D.

Lemma 2. If part J has small deviation at its reference points R_1^J and R_2^J, which is denoted as $\begin{bmatrix}\Delta X_1^J & \Delta Z_1^J & \Delta X_2^J & \Delta Z_2^J\end{bmatrix}^T$ (See Fig. 6), then the deviation of part J, denoted as $\vec{X}^J$, is

$$\vec{\mathbf{X}}^J = \begin{bmatrix}\Delta X^J\\ \Delta Z^J\\ \Delta\alpha^J\end{bmatrix} = \mathbf{Q}^J\begin{bmatrix}\Delta X_1^J\\ \Delta Z_1^J\\ \Delta X_2^J\\ \Delta Z_2^J\end{bmatrix} \tag{9}$$

where

$$\mathbf{Q}^J = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ \frac{\sin\alpha^J}{L^J} & -\frac{\cos\alpha^J}{L^J} & -\frac{\sin\alpha^J}{L^J} & \frac{\cos\alpha^J}{L^J}\end{bmatrix}_{3\times 4} \tag{10}$$

and L^J is the distance between R_1^J and R_2^J.

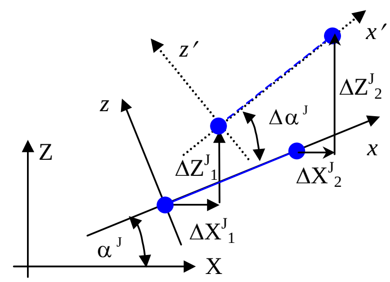


Figure 6. Deviation of the local coordinate system

Proof. The relationship of positional deviation is obvious, so only proof of the angular deviation relationship is needed. In a small deviation case, the angle $\Delta\alpha^J$ is also small and can then be expressed as

$$\Delta\alpha^J = \frac{\Delta z_2^J - \Delta z_1^J}{L^J} \tag{11}$$

Noting that nominal orientation angle of the local coordinate system is α^J, the deviation values in global coordinate system are related with local coordinates as

$$\Delta z_1^J = -\Delta X_1^J \sin\alpha^J + \Delta Z_1^J \cos\alpha^J \tag{12}$$

$$\Delta z_2^J = -\Delta X_2^J \sin\alpha^J + \Delta Z_2^J \cos\alpha^J \tag{13}$$

which leads to the expression of matrix $\mathbf{Q}^J$. Q.E.D.

This conclusion can be extended straightforward to a subassembly as stated in the following two corollaries without proof.

Corollary 1. If there is a subassembly s at station i located by points $P_{s1}(i)$ and $P_{s2}(i)$, then the deviation of this subassembly due to deviations at locating points is

$$\begin{bmatrix}\Delta X_s\\ \Delta Z_s\\ \Delta\alpha_s\end{bmatrix} = \mathbf{Q}_s(i)\begin{bmatrix}\Delta X_{P_{s1}(i)}\\ \Delta Z_{P_{s1}(i)}\\ \Delta X_{P_{s2}(i)}\\ \Delta Z_{P_{s2}(i)}\end{bmatrix} \tag{14}$$

where

$$\mathbf{Q}_s(i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \dfrac{\sin\alpha_s(i)}{L_s(i)} & -\dfrac{\cos\alpha_s(i)}{L_s(i)} & -\dfrac{\sin\alpha_s(i)}{L_s(i)} & \dfrac{\cos\alpha_s(i)}{L_s(i)} \end{bmatrix}_{3\times 4} \quad (15)$$

and $L_s(i)$ is the distance between $P_{s1}(i)$ and $P_{s2}(i)$, $\alpha_s(i)$ is the orientation angle of the subassembly.

The next corollary characterizes the motion of transferring parts between stations. Supposing a subassembly s be supported by $P_{s1}(i)$ and $P_{s2}(i)$ at station i, if there is no fixture deviation at the current station i but there is deviation accumulated in $P_{s1}(i)$ and $P_{s2}(i)$ at the previous station i-1, then the subassembly undergoes rotation and translation when moving from station i-1 to station i.

Corollary 2. The translation and rotation of subassembly s when moving from station i-1 to station i can be expressed as a linear combination of deviation accumulated in its locating points $P_{s1}(i)$ and $P_{s2}(i)$ at the previous station i-1.

$$\begin{bmatrix} \Delta X_s \\ \Delta Z_s \\ \Delta\alpha_s \end{bmatrix} = \mathbf{R}_s(i) \begin{bmatrix} \Delta X_{P_{s1}(i)}(i-1) \\ \Delta Z_{P_{s1}(i)}(i-1) \\ \Delta X_{P_{s2}(i)}(i-1) \\ \Delta Z_{P_{s2}(i)}(i-1) \end{bmatrix} \quad (16)$$

where

$$\mathbf{R}_s(i) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -\dfrac{\sin\alpha_s(i)}{L_s(i)} & \dfrac{\cos\alpha_s(i)}{L_s(i)} & \dfrac{\sin\alpha_s(i)}{L_s(i)} & -\dfrac{\cos\alpha_s(i)}{L_s(i)} \end{bmatrix}_{3\times 4} \quad (17)$$

Here, $\Delta X_{P_{sk}(i)}(i-1)$ and $\Delta Z_{P_{sk}(i)}(i-1)$ (k=1, 2) are deviations of the locating point $P_{sk}(i)$ at the previous station i-1. $L_s(i)$ and $\alpha_s(i)$ are the same as in *Corollary 1*.

3.3 State Space Representation

The deviation accumulated at station i comes from fixture error-induced deviation $\mathbf{E}_1^J$ and reorientation-induced deviation $\mathbf{E}_2^J$, that is,

$$\vec{\mathbf{X}}^J(i) = \vec{\mathbf{X}}^J(i-1) + \mathbf{E}_1^J(i) + \mathbf{E}_2^J(i) \quad (18)$$

Equation (18) is illustrated in Fig. 7.

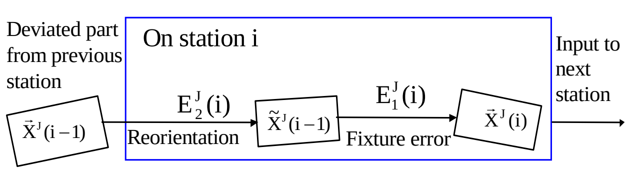


Figure 7. Deviation accumulation of part J on station i

Two deviation terms, $\mathbf{E}_1^J$ and $\mathbf{E}_2^J$ in Eq. (18), are given by the following theorems.

Theorem 1. Suppose part J is on subassembly s at station i as shown in Fig. 8, its fixture error-induced deviation $\mathbf{E}_1^J$ can be expressed as

$$\mathbf{E}_1^J(i) = \mathbf{B}_1^J(i)\mathbf{U}(i) \quad (19)$$

where

$$\mathbf{B}_1^J(i)_{3\times 4n_i} = \begin{cases} \mathbf{M}(P_{s1}(i), R_1^J)\mathbf{Q}_s(i)\mathbf{W}_1(s) & \text{if } \ i \ge \lambda^J \\ \mathbf{0}^{3\times 4n_i} & \text{if } \ i < \lambda^J \end{cases} \quad (20)$$

$$\mathbf{W}_1(s) = \left[\delta_{1s}\mathbf{I}^{4\times 4} \quad \delta_{2s}\mathbf{I}^{4\times 4} \quad \cdots \quad \delta_{ns}\mathbf{I}^{4\times 4}\right]_{4\times 4n(i)} \quad (21)$$

$$\delta_{ks} = \begin{cases} 1 & \text{if } \ k = s \\ 0 & \text{if } \ k \ne s \end{cases} \ \text{is the Kronecker Delta} \quad (22)$$

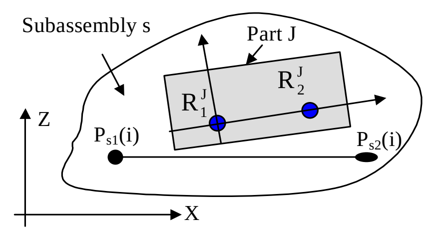


Figure 8. Fixture error-induced deviation

Proof. According to *Corollary 1*, the deviation of subassembly s due to fixture error at $P_{s1}(i)$ and $P_{s2}(i)$ is expressed in Eq. (14). Since part J is on rigid subassembly s, *Lemma 1* can be applied to get the deviation of part J, which is

$$\mathbf{E}_1^J = \mathbf{M}(P_{s1}(i), R_1^J) \begin{bmatrix} \Delta X_s \\ \Delta Z_s \\ \Delta\alpha_s \end{bmatrix} \quad (23)$$

Substituting Eq. (14) into Eq. (23) yields

$$\mathbf{E}_1^J = \mathbf{M}(P_{s1}(i), R_1^J)\mathbf{Q}_s(i) \begin{bmatrix} \Delta P_{s1}(i) \\ \Delta P_{s2}(i) \end{bmatrix} \quad (24)$$

The selecting matrix $\mathbf{W}_1(s)$ defined in Eqs. (21) and (22) is employed to determine which set of fixture contributes to the deviation of part J, i.e.,

$$\begin{bmatrix} \Delta P_{s1}(i) \\ \Delta P_{s2}(i) \end{bmatrix} = \mathbf{W}_1(s)\mathbf{U}(i) \quad (25)$$

Combining Eqs. (24) and (25) will result in the expression of the upper part in Eq. (20). The lower part in Eq. (20) implies that fixture error will not be reflected in the deviation of part J before this part is merged into the assembly stream.

Theorem 2. If part J is on subassembly s at station i, its reorientation-induced deviation $\mathbf{E}_2^J$ can be expressed as

$$\mathbf{E}_2^J(i) = \mathbf{B}_2^J(i)\mathbf{F}_1\vec{\mathbf{X}}^K(i-1) + \mathbf{B}_2^J(i)\mathbf{F}_2\vec{\mathbf{X}}^G(i-1) \quad (26)$$

where

$$\mathbf{B}_2^J(i)_{3\times4} = \begin{cases} \mathbf{M}(P_{s1}(i), R_1^J) R_s(i) & \text{if} \quad i > \lambda^J \\ \mathbf{0}^{3\times4} & \text{if} \quad i \le \lambda^J \end{cases} \tag{27}$$

$$\mathbf{F}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{4\times3} \qquad \mathbf{F}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{M}(R_1^G, R_2^G)_{4\times3} \tag{28}$$

and K and G are the indices of parts that contain locating points $P_{s1}(i)$ and $P_{s2}(i)$, respectively.

Proof. The term $\mathbf{E}_2^J$ at station i is actually caused by the deviation of locating points accumulated up to station i-1, therefore, it reads as

$$\mathbf{E}_2^J(i) = \mathbf{M}(P_{s1}(i), R_1^J)\mathbf{R}_s(i) \begin{bmatrix} \Delta X_{P_{s1}(i)}(i-1) \\ \Delta Z_{P_{s1}(i)}(i-1) \\ \Delta X_{P_{s2}(i)}(i-1) \\ \Delta Z_{P_{s2}(i)}(i-1) \end{bmatrix} \tag{29}$$

Here, $\mathbf{R}_s(i)$ converts fixture deviations at the previous station into subassembly rotation and translation during datum shift according to *Corollary 2*. $\mathbf{M}(P_{s1}(i), R_1^J)$ further translates the deviation of subassembly s to that of part J based on *Lemma 1*. Noticing that $P_{s1}(i)$ is on part K and $P_{s2}(i)$ on part G, respectively, deviations of these two points at the previous station i-1 are actually saved in variables $\vec{\mathbf{X}}^K(i-1)$ and $\vec{\mathbf{X}}^G(i-1)$, namely

$$\begin{bmatrix} \Delta X_{p_{s1}(i)}(i-1) \\ \Delta Z_{p_{s1}(i)}(i-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{X}}^K(i-1) \tag{30}$$

and

$$\begin{bmatrix} \Delta X_{p_{s2}(i)}(i-1) \\ \Delta Z_{p_{s2}(i)}(i-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{M}(R_1^G, R_2^G)\vec{\mathbf{X}}^G(i-1) \tag{31}$$

Putting them together, Eqs. (30) and (31) yield

$$\begin{bmatrix} \Delta X_{P_{s1}(i)}(i-1) \\ \Delta Z_{P_{s1}(i)}(i-1) \\ \Delta X_{P_{s2}(i)}(i-1) \\ \Delta Z_{P_{s2}(i)}(i-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vec{\mathbf{X}}^K(i-1) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{M}(R_1^G, R_2^G)\vec{\mathbf{X}}^G(i-1) \tag{32}$$

Substituting Eq. (32) into Eq. (29), and with definitions in Eqs. (27) – (28), Eq. (26) can be obtained. Again, the lower part of Eq. (27) is **0** because the reorientation-induced deviation only contributes starting from station i+1 if part J is put into the assembly stream on station i.

Q.E.D.

With the help of *Theorems 1* and *2*, Eq. (18) can be written as,

$$\vec{\mathbf{X}}^J(i) = \vec{\mathbf{X}}^J(i-1) + \mathbf{B}_1^J(i)\mathbf{U}(i) + \mathbf{B}_2^J(i)\mathbf{F}_1\vec{\mathbf{X}}^K(i-1) + \mathbf{B}_2^J(i)\mathbf{F}_2\vec{\mathbf{X}}^G(i-1) \tag{33}$$

This implies that deviation error of part J is coupled with deviations of other parts. It is impossible to write a de-coupled deviation propagation equation for each individual part. In order to obtain a standard format of state space equation, all individual deviation vectors are put together to generate a large-dimension vector as

$$\vec{\mathbf{X}}(i) = \begin{bmatrix} \vec{\mathbf{X}}^1(i) \\ \vec{\mathbf{X}}^2(i) \\ \vdots \\ \vec{\mathbf{X}}^N(i) \end{bmatrix}_{3N\times1} \tag{34}$$

Then, $\vec{\mathbf{X}}^K$ and $\vec{\mathbf{X}}^G$ in Eq. (33) can be replaced with the new deviation vector $\vec{\mathbf{X}}$. A selecting matrix $\mathbf{W}_2$ is used to determine reorientation contribution from parts K and G,

$$\mathbf{W}_2(s,i) = \begin{bmatrix} \mathbf{w}_{11}^{3\times3} & \mathbf{w}_{12} & \cdots & \mathbf{w}_{1N} \\ \mathbf{w}_{21} & \mathbf{w}_{22} & \cdots & \mathbf{w}_{2N} \end{bmatrix}_{6\times3N} \tag{35}$$

where

$$\mathbf{w}_{pq}^{3\times3} = \begin{cases} \mathbf{I}^{3\times3} & \text{if } (p, q) = (1, K) \text{ or } (2, G) \\ \mathbf{0}^{3\times3} & \text{otherwise} \end{cases} \tag{36}$$

Defining $\mathbf{H}^J$ as

$$\mathbf{H}^J(i) = \begin{bmatrix} \mathbf{B}_2^J(i)\mathbf{F}_1 & \mathbf{B}_2^J(i)\mathbf{F}_2 \end{bmatrix}_{3\times6} \tag{37}$$

then, Eq. (33) turns out to be

$$\vec{\mathbf{X}}^J(i) = \vec{\mathbf{X}}^J(i-1) + \mathbf{B}_1^J(i)\mathbf{U}(i) + \mathbf{H}^J(i)\mathbf{W}_2(s,i)\vec{\mathbf{X}}(i) \tag{38}$$

Furthermore, defining matrices **B**(i) and **H**(i-1) as

$$\mathbf{B}(i) = \begin{bmatrix} \mathbf{B}_1^1(i) \\ \mathbf{B}_1^2(i) \\ \vdots \\ \mathbf{B}_1^N(i) \end{bmatrix}_{3N\times4n_i} \tag{39}$$

$$\begin{cases} \mathbf{H}(i-1) = \begin{bmatrix} \mathbf{H}^1(i)\mathbf{W}_2(s,i) \\ \mathbf{H}^2(i)\mathbf{W}_2(s,i) \\ \vdots \\ \mathbf{H}^N(i)\mathbf{W}_2(s,i) \end{bmatrix}_{3N\times3N} & \text{if} \quad i = 2, 3, ..., m \\ \mathbf{H}(0) = \mathbf{0}^{3N\times3N} \end{cases} \tag{40}$$

Equation (38) becomes

$$\vec{\mathbf{X}}(i) = (\mathbf{I}^{3N\times3N} + \mathbf{H}(i-1))\,\vec{\mathbf{X}}(i-1) + \mathbf{B}(i)\mathbf{U}(i) \tag{41}$$

In this equation, the dynamic matrix is

$$\mathbf{A}(i-1) = \mathbf{I}^{3N\times3N} + \mathbf{H}(i-1) \tag{42}$$

Equation (41) can be written as

$$\vec{\mathbf{X}}(i) = \mathbf{A}(i-1)\vec{\mathbf{X}}(i-1) + \mathbf{B}(i)\mathbf{U}(i) \tag{43}$$

This is the state space equation governing deviation propagation in an assembly process under assumptions listed in section 2.3.

If sensors are put along the assembly line, the observation equation could be attained. If there are r^J measurement points on part J at station i, then the deviation of each point on part J is

$$\mathbf{y}_\xi^J(i) = \mathbf{M}_o(R_1^J, MLP_\xi^J)\vec{\mathbf{X}}^J(i) \qquad \xi = 1,2,\cdots,r^J \tag{44}$$

where

$$\mathbf{M}_o(\mathrm{R}_1^J, \mathrm{MLP}_\xi^J) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{M}(\mathrm{R}_1^J, \mathrm{MLP}_\xi^J) \tag{45}$$

Thus, the measurement vector of part J is

$$\vec{\mathbf{Y}}^J(i) = \begin{bmatrix} \mathbf{y}_1^J(i) & \mathbf{y}_2^J(i) & \cdots & \mathbf{y}_{r^J}^J(i) \end{bmatrix}^T_{2r^J \times 1} \tag{46}$$

which can be written as

$$\vec{\mathbf{Y}}^J(i) = \mathbf{C}^J(i)\vec{\mathbf{X}}^J(i) \tag{47}$$

where

$$\mathbf{C}^J(i) = \begin{bmatrix} \mathbf{M}_o(\mathrm{R}_1^J, \mathrm{MLP}_1^J) \\ \mathbf{M}_o(\mathrm{R}_1^J, \mathrm{MLP}_2^J) \\ \vdots \\ \mathbf{M}_o(\mathrm{R}_1^J, \mathrm{MLP}_{r^J}^J) \end{bmatrix}_{2r^J \times 3} \tag{48}$$

Then, the deviation vector of all the measurement points is

$$\vec{\mathbf{Y}}(i) = \begin{bmatrix} \vec{\mathbf{Y}}^1(i) \\ \vec{\mathbf{Y}}^2(i) \\ \vdots \\ \vec{\mathbf{Y}}^N(i) \end{bmatrix}_{(2\sum_{J=1}^{N} r^J) \times 1} = \mathbf{C}(i)\vec{\mathbf{X}}(i) \tag{49}$$

where

$$\mathbf{C}(i) = \begin{bmatrix} \mathbf{C}^1(i) & \mathbf{0}^{2r^2 \times 3} & \cdots & \mathbf{0}^{2r^N \times 3} \\ \mathbf{0}^{2r^1 \times 3} & \mathbf{C}^2(i) & \cdots & \mathbf{0}^{2r^N \times 3} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}^{2r^1 \times 3} & \mathbf{0}^{2r^2 \times 3} & \cdots & \mathbf{C}^N(i) \end{bmatrix}_{(2\sum_{J=1}^{N} r^J) \times 3N} \tag{50}$$

If process disturbance and measurement noise are considered together with Eqs. (43) and (49), then a complete set of deviation propagation model and its observation is

$$\vec{\mathbf{X}}(i) = \mathbf{A}(i-1)\vec{\mathbf{X}}(i-1) + \mathbf{B}(i)\mathbf{U}(i) + \boldsymbol{\eta}(i) \tag{51}$$

$$\vec{\mathbf{Y}}(i) = \mathbf{C}(i)\vec{\mathbf{X}}(i) + \boldsymbol{\zeta}(i) \qquad i = 1,2, \ldots, m, \tag{52}$$

where $\boldsymbol{\eta}$ and ζ are process disturbance and measurement noise vectors, respectively.

4. MODEL VALIDATION

In order to verify the model, an example is generated from the assembly process of an autobody side aperture. The side aperture consists of three main parts: rear quarter, A-B ring, and fender side (Fig. 9(a)). The shape of each part is simplified to be rectangular and coordinate values are rounded off as integers from the real CAD model (Fig. 9 (b)). These three parts are referred as part 1, part 2, and part 3, respectively, with the number marked on each part. The coordinates of locating points P_{1-6} and measurement points $A_{1\text{-}3}$ are marked on Fig. 9 (b) as well. All units are in millimeter.

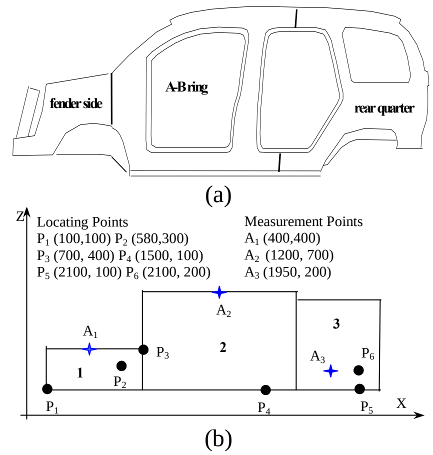


Figure 9. Geometry of the assembly

The assembly sequence and datum shift scheme are shown in Fig. 10. There are three stations in this example. Part 1 and Part 2 are joined together at station I and Part 3 is assembled with subassembly "1+2" at station II. Finally, assembly "1+2+3" is put on station III to measure deviations of three points A_1, A_2 and A_3.

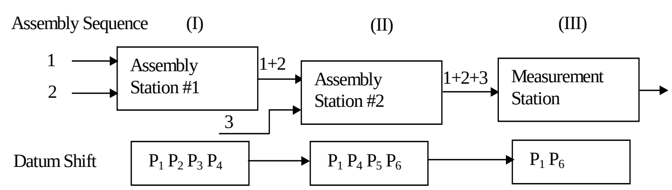


Figure 10. Assembly sequence and datum shift

Suppose there are tolerances of ±2.0 mm associated with all the locating points at both assembly stations. Fixture deviations at the measurement station are substantially smaller to be neglected. Also, it is assumed that deviations at the locating points are normally distributed and all tolerance limits correspond to their 6σ values. Monte Carlo simulation can compute the simulated variation of three measurement points.

The results are compared with those from the widely used commercial software Variation System Analysis (VSA, 1998). The standard deviation of all measurement points from both models are compared in Table 1. The difference between two simulations of this specific example is at the level of 0.015% that is negligible. Thus, we conclude that the state space model can simulate variation propagation correctly.

Table 1. Comparison between state space model and VSA

	VSA		State Space		Discrepancy of STD (%)
	Mean	STD	Mean	STD	
X_{A1}	400.0064	0.5121	400.0074	0.5121	0.0000
Z_{A1}	399.9928	0.5121	399.9926	0.5121	0.0000
X_{A2}	1199.9907	1.0694	1199.9922	1.0694	0.0000
Z_{A2}	699.9857	0.6967	699.9836	0.6968	0.0144
X_{A3}	1950.0014	0.9555	1949.9949	0.9556	0.0105
Z_{A3}	200.0058	1.4440	200.0064	1.4442	0.0139

5. CONCLUSIONS AND PERSPECTIVES

In this paper, a state space model has been developed to provide a mathematical representation of multistage assembly processes. If the station index i is considered as analogous to the "time index" in a dynamic system, the state space model actually describes a discrete time LTV (Linear Time Varying) stochastic process. This analogy implies that the existing optimal control theory could be utilized to conduct systematic analysis and to control the variation in an assembly process.

A straightforward application of the state space model is variation simulation. However, the application of the model goes beyond variation simulation analysis. Perspectives of the proposed model can be viewed through the following three steps: (1) process analysis, (2) process optimization, and (3) process control.

1) *Process Analysis*: Diagnosability and sensitivity will be investigated. The fault domain and diagnostic model need to be developed. Assembly process information is connected to fault pattern in such a way that questions of under what circumstance the fixture faults are diagnosable and how sensitive they are could be answered.

2) *Process Optimization*: Since the fixture layout and assembly sequence will affect variation of assembly product, optimization of system parameters will lead to a robust design of PLP layout and assembly process planning. Furthermore, maximum diagnosability can also be ensured under optimizations of sensor distribution strategy. Success in this task will achieve real integration of design and manufacturing.

3) *Process Feedback Control*: A good process design cannot completely prevent defects from occurring in manufacturing processes. The ultimate solution for variation control is a closed-loop feedback controller in the presence of noise and disturbance. Adaptive tooling system as actuator is technically available (Lizuka *et al.*, 1992). The LQG control algorithm (Stengel, 1986) can be used to solve a terminal control problem.

The proposed methodology will be applicable to generic manufacturing processes such as machining processes although the work is carried out within the context of assembly processes. A diagnostic method, which falls in the category of process analysis work, will be presented in Part II of this paper.

REFERENCES:

1. Agrawal, R., Lawless, J.F. and Mackay, R.J., 1999, Analysis of Variation Transmission in Manufacturing Processes – Part II, *Journal of Quality Technology*, Vol. 31, No. 2, pp. 143 – 154.
2. Apley, D.W. and Shi, J., 1998, Diagnosis of Multiple Fixture Faults in Panel Assembly, *ASME Transactions, Journal of Manufacturing Science & Engineering*, Vol. 120, pp. 793 – 801.
3. Asada, H., By, B., 1985, Kinematic Analysis of Workpart Fixturing for Flexible Assembly with Automatically Reconfigurable Fixtures, *IEEE Journal of Robotics and Automation*, Vol. RA-1, No. 2, pp. 86 – 94.
4. Ceglarek, D. and Shi, J., 1996, Fixture Failure Diagnosis for Auto Body Assembly Using Patter Recognition, *ASME Transactions, Journal of Engineering for Industry*, Vol. 118, pp. 55 – 65.
5. Ceglarek, D. and Shi, J., 1998 Design Evaluation of Sheet Metal Joints for Dimensional Integrity, *ASME Transactions, Journal of Manufacturing Science & Engineering*, Vol. 120, pp. 452 – 460.
6. Ceglarek, D., Shi, J. and Wu, S. M., 1994, A Knowledge-based Diagnosis Approach for the Launch of the Autobody Assembly Process, *ASME Transactions, Journal of Engineering for Industry*, Vol. 116, pp. 491 – 499.
7. Chang, M., Gossard, D.C., 1997, Modeling the assembly of compliant, no-ideal parts, *Computer Aided Design*, Vol. 29, No. 10, pp. 701-708.
8. Chou, Y-C., Chandru, V. and Barash, M. M., 1989, A Mathematical Approach to Automatic Configuration of Machining Fixtures: Analysis and Synthesis, *ASME Transactions, Journal of Engineering for Industry*, Vol. 111, pp. 299 – 306.
9. Choudhuri, S.A. and De Meter, E.C., 1999, Tolerance Analysis of Machining Fixture Locators, *ASME Transactions, Journal of Manufacturing Science & Engineering*, Vol. 121, pp. 273 – 281.
10. Hu, S.J. and Wu, S.M., 1992, Identifying Root Cause of Variation in Automobile Body Assembly Using Principal Component Analysis, *Transactions of NAMRI*, Vol. XX, pp. 311 –316.
11. Jin, J. and Shi, J., 1999, State Space Modeling of Sheet Metal Assembly for Dimensional Control, *ASME Transactions, Journal of Manufacturing Science & Engineering*, Vol. 121, no. 4, pp. 756 - 762.
12. Lawless, J.F., Mackay, R. J. and Robinson, J.A., 1999, Analysis of Variation Transmission in Manufacturing Processes – Part I, *Journal of Quality Technology*, Vol. 31, No. 2, pp. 131 – 142.
13. Lee, W-J, Woo, T.C., 1990, Tolerances: Their Analysis and Synthesis, *ASME Transactions, Journal of Engineering for Industry*, Vol. 112, pp. 113 – 121.
14. Lizuka, K., Nakao, H., Okuyama, K., Honda, S. and Noumaru, M., 1992, The Development of Intelligent Body Assembly System, *JAPAN/USA Symposium on Flexible Automation - Volume 2*, pp. 1539 – 1542.
15. Mantripragada, R. and Whitney, D. E., 1999, Modeling and Controlling Variation Propagation in Mechanical Assemblies Using State Transition Models, *IEEE Trans. On Robotics and Automation*, Vol. 15, No. 1, pp.124 - 140.
16. Rong, Q., Ceglarek, D. and Shi, J., 1999 Dimensional Fault Diagnosis for Compliant Beam Structure Assemblies, Accepted to *ASME Transactions, Journal of Manufacturing Science & Engineering*.
17. Rong, Y., Bai, Y., 1996, Machining Accuracy Analysis for Computer-aided Fixture Design Verification, *ASME Transactions, Journal of Manufacturing Science and Engineering*, Vol. 118, pp. 289 – 299.
18. Roy, U. Fang, Y. C., 1997, Optimal tolerance re-allocation for generative process sequence, *IIE Transactions*, vol. 29, pp. 37-44.
19. Shiu, B., 1996, *Modeling of An Automotive Body Assembly System for Dimentional Control*, Ph.D. Dissertation, The University of Michigan, Ann Arbor MI.
20. Shiu, B., Ceglarek, D., and Shi, J., 1996, Multi-Station Sheet Metal Assembly Modeling and Diagnostics, *NAMRI/SME Transactions*, Vol XXIV. , pp. 199 – 204.
21. Stengel, R.F., 1986, *Stochastic Optimal Control, Theory and Application*, John Wiley & Sons, New York.
22. VSA, 1998, *VSA-3D Release 12.5 User Manual*, Variation System Analysis, Inc., 300 Maple Park Boulevard, St. Clair Shores, MI 48081.