



Diagnosis of multi-operational machining processes through variation propagation analysis

Qiang Huang, Shiyu Zhou, Jianjun Shi*

Department of Industrial and Operations Engineering, The University of Michigan, Michigan, USA

Abstract

It is a very challenging task to develop effective process control methodologies for multi-operational manufacturing processes. Although Statistical Process Control (SPC) has been widely used as the primary method in the control of quality, it mainly serves as a change detection tool rather than a method to identify root causes of process changes. This paper proposes a systematic approach to overcome the limitations faced by SPC. In this method, a state space variation propagation model is derived from the product and process design information. The virtual machining concept is applied to isolate faults between operations, and further used in the root cause determination. The detailed methodology is presented, and a case study is conducted to illustrate and verify the developed diagnosis method. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Process diagnosis; Variation propagation; Root cause identification

1. Introduction

In order to improve quality and productivity, it is highly desirable to develop a fast, accurate, and robust fault diagnosis methodology to identify the root causes of quality-related problems for manufacturing processes. Current practice in industries is a product-inspection-oriented measurement strategy. The finished and intermediate products are measured and the measurements are compared with specifications in the product design. If it is within the specification limit, the process is assumed acceptable and it will continue to be run. If the measurement is out of specification limit, an exhaustive search is conducted to find the root causes. This search is usually based on the experience of the operators and sometimes it is very time consuming. Statistical Process Control (SPC) is a quality improvement methodology that involves more statistical analysis than the pure inspection method. However, it largely depends on reliable historical data to generate reference for successful process monitoring, and it cannot determine root causes either. Therefore, a systematic fault diagnosis methodology for quality improvement of manufacturing processes is highly desirable in practice.

A manufacturing system usually involves multiple operations to produce a product. Examples of such multi-operational processes include automotive body assembly, machining, microelectronic manufacturing, and transfer/progressive die processes. In such a process, the part goes through each operation from the beginning to the end of the line. The quality features of the part in different operations are correlated. The final part variation is a stack up of feature variations from all operations. For example, a machining process could have many operations. At each single operation, there are many types of variation sources, such as the geometric and kinematic errors, thermal errors, cutting force induced errors, and fixturing errors. These errors will be accumulated on the product when it passes through the whole process. A huge body of literature can be found on the fault diagnosis and error compensation on a single machining station. A review of these papers can be found in Ramesh et al. [1,2]. However, due to the complicated interactions between different variation sources, very few attempts have been made on the variation propagation analysis and fault diagnosis for the multi-operational machining process.

Mantripragada and Whitney [3] adopted the concept of output controllability from control theory to evaluate and improve the automotive body structure design. Lawless et al. [4] and Agrawal et al. [5] investigated

*Corresponding author. Tel.: +1-734-763-5321; fax: +1-734-764-3451.

E-mail address: shihang@umich.edu (J. Shi).

variation transmission in both assembly and machining process by using an AR(1) model. Their method can be viewed as a data-driven method. Jin and Shi [6] proposed a state space model to depict the variation propagation in a multistage body assembly process. Ding et al. [7] proposed a diagnostic approach for assembly process based on a state space model. However, his approach cannot be directly applied in machining processes.

In this paper, a systematic diagnostic methodology is proposed to monitor process and determine root causes of quality-related problems for multi-operational machining processes. This methodology is based on a state space model proposed by Huang et al. [8].

The paper is divided into five sections. A brief review of deviation propagation model and the variation propagation model are given in Section 2. In Section 3, the process level diagnosis methodology is introduced. In Section 4, a three-operation machining process is used to illustrate the proposed methodology. The conclusions are given in the last section.

2. Variation propagation and observation

2.1. Review of deviation propagation model

In a machining process, the variation in the final product is accumulated as the product moves along the machining line. This accumulating process is shown in Fig. 1.

Up to operation k (oper. k), the part key surfaces are represented by a vector $\mathbf{X}(k)$ and quality characteristics are denoted as $\mathbf{Y}(k)$. $\mathbf{x}(k)$, representing deviation of $\mathbf{X}(k)$, consists of the previous surface deviation $\mathbf{x}(k-1)$ and newly generated surface deviation $\mathbf{x}^u(k)$. The main variation input at oper. k consists of setup error e_k^f (geometric errors of fixture elements) and machining error e_k^m (machine tool geometric and kinematic errors only). $B(k)$ is an indicator matrix which labels all surfaces that are formed at oper. k . To indicate all other surfaces that are not formed at oper. k , $A(k)$ is defined as $A(k) = I - B(k)$, where I is the identity matrix with the same dimension of $A(k)$. $C(k)$ is a mapping matrix that relates $\mathbf{x}(k)$ to the deviation of measurement, i.e. $\mathbf{y}(k)$. $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are the noise terms. A state space form model can be obtained to describe the deviation

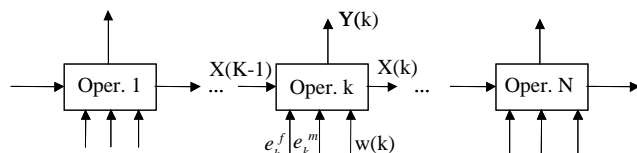


Fig. 1. Error propagation in the multi-operational machining process.

propagation in the process.

$$\mathbf{x}(k) = A(k)\mathbf{x}(k-1) + B(k)\mathbf{x}^u(k) + \mathbf{w}(k),$$

$$\mathbf{y}(k) = C(k)\mathbf{x}(k) + \mathbf{v}(k). \quad (1)$$

The details of the derivation of this model can be found in Huang et al. [8].

2.2. Variation propagation model

At oper. k , the part variation is expressed as the covariance of vector $\mathbf{x}(k)$,

$$K_{\mathbf{x}(k)} = \text{cov}(\mathbf{x}(k)). \quad (2)$$

In the following derivation, $K_{\mathbf{x}}$ is used to represent the covariance of vector \mathbf{x} . Assume $\mathbf{w}(k)$ follows a normal distribution, i.e. $\mathbf{w}(k) \sim N(0, K_{\mathbf{w}(k)})$. By Eq. (1), we have

$$K_{\mathbf{x}(k)} = A(k)K_{\mathbf{x}(k-1)}A^T(k) + B(k)K_{\mathbf{x}^u(k)}B^T(k) + 2A(k)\text{cov}(\mathbf{x}(k-1), \mathbf{x}^u(k))B^T(k) + K_{\mathbf{w}(k)}. \quad (3a)$$

Eq. (3a) indicates that besides the variation of each machined surface, the covariance between finished surfaces and newly generated surfaces also contributes to the total part variation. These correlations, which are primarily caused by datums, are quantified by this equation. The term $B(k)K_{\mathbf{x}^u(k)}B^T(k)$ in Eq. (3a), can be further expressed as

$$B(k)K_{\mathbf{x}^u(k)}B^T(k) = \text{cov}\{R_{GP}(k)B(k)[\mathbf{x}_G^u(k) - T(k)] + \Delta R(k)X^o(k) + R_{GP}(k)B(k)T^o(k)\}, \quad (3b)$$

where R_{GP} is the rotation transformation matrix from the global machine coordinate to the part coordinate, \mathbf{x}_G^u is the deviation of the newly formed surfaces in the global coordinate. $T(k)$ is the translational transformation vector. $X^o(k)$ and $T^o(k)$ are nominal values of the surfaces and translation vector at oper. k . $\Delta R(k) = R_{GP}(k)B(k)R_{PG}^o(k) - B(k)$ is the deviation of the rotation transformation matrix, where $R_{PG}^o(k)$ is the nominal rotation transformation matrix from the part coordinate to the global machine coordinate at oper. k . It can be shown that the terms $R_{GP}(k)$, $\Delta R(k)$ and $T(k)$ are related with setup errors and $\mathbf{x}_G^u(k)$ is caused by machining error only. The details of the definitions of these terms can be found in Huang et al. [8]. To simplify Eq. (3b), we define

$$\Pi_k = \Delta R(k)X^o(k) + R_{GP}(k)B(k)T^o(k) \quad (3c)$$

and $K_k^u = B(k)K_{\mathbf{x}^u(k)}B^T(k)$.

Thus Eq. (3b) is rewritten as

$$K_k^u = K_{R_{GP}(k)B(k)[\mathbf{x}_G^u(k) - T(k)]} + K_{\Pi_k} + 2\text{cov}(R_{GP}(k)B(k)[\mathbf{x}_G^u(k) - T(k)], \Pi_k). \quad (3d)$$

Eq. (3d) shows that the variance of newly machined surfaces K_k^u is consisted of three components: (1)

$K_{R_{GP}(k)B(k)[\mathbf{x}_G^u(k)-T(k)]}$, where $R_{GP}(k)B(k)[\mathbf{x}_G^u(k)-T(k)]$ can be interpreted as machining error expressed in the part coordinate; (2) K_{Π_k} is the variation caused by setup error only; (3) covariance of the first two components. It also shows that without machining error ($\mathbf{x}_G^u(k)=\mathbf{0}$), setup induced error still causes product variation.

2.3. Variation observation

At oper. k , the variation of part characteristics is expressed as the covariance of vector $\mathbf{y}(k)$, i.e., $K_{\mathbf{y}(k)} = \text{cov}(\mathbf{y}(k))$. By Eq. (1), we have

$$K_{\mathbf{y}(k)} = C(k)K_k C^T(k) + K_{\mathbf{v}(k)}. \quad (4)$$

Therefore, variation propagation and observation can be expressed by Eqs. (3a), (3b) and (4).

Suppose that only the end-of-line observation $Y(N)$ is available. By Eq. (1), we have $\mathbf{y}(N) = C(N)\mathbf{x}(N) + \mathbf{v}(N)$. The solution of state space equation is

$$\begin{aligned} \mathbf{x}(N) &= \sum_{i=1}^N \Phi(N, i)\mathbf{B}(i)\mathbf{x}^u(i) + \Phi(N, 0)\mathbf{x}(0) \\ &\quad + \sum_{i=1}^N \Phi(N, i)\mathbf{w}(i), \end{aligned} \quad (5a)$$

$$\begin{aligned} \mathbf{y}(N) &= \sum_{i=1}^N C(N)\Phi(N, i)\mathbf{B}(i)\mathbf{x}^u(i) \\ &\quad + C(N)\Phi(N, 0)\mathbf{x}(0) + \boldsymbol{\varepsilon}, \end{aligned} \quad (5b)$$

where the state transition matrix $\Phi(\cdot, \cdot)$ is defined as $\Phi(N, i) = A(N)A(N-1)\cdots A(i)$ and $\Phi(i, i) = I$. $\mathbf{x}(0)$ represents the raw workpiece surface deviation. $\boldsymbol{\varepsilon}$ is the summation of all modeling uncertainty and sensor noise terms and $\boldsymbol{\varepsilon} = \sum_{i=1}^N C(N)\Phi(N, i)\mathbf{w}(i) + \mathbf{v}(N)$. If we define $\gamma(i)$ as $\gamma(i) = C(N)\Phi(N, i)$, we have

$$\begin{aligned} K_{\mathbf{y}(N)} &= \sum_{i=1}^N \gamma(i)K_i^u \gamma^T(i) + 2 \sum_{j=1}^N \sum_{i=1}^j \gamma(i)\mathbf{B}(i) \\ &\quad \times \text{cov}(\mathbf{x}^u(i), \mathbf{x}^u(j))\mathbf{B}^T(j)\gamma^T(j) \\ &\quad + \gamma(0)K_0\gamma(0)^T + K_{\boldsymbol{\varepsilon}}. \end{aligned} \quad (6)$$

Eq. (6) shows that four components contribute final product variation: machined surface variation, covariance between surfaces machined at different operations, raw workpiece variation and variation from background noise. Once the product design is determined, the number N in the term $\sum_{i=1}^N \gamma(i)\mathbf{x}^u(i)\gamma^T(i)$ is fixed. Only the magnitude of this term can be reduced by process improvement. However, the second term is determined by the process design. If there is no surface in $X^u(i)$ used as datum at operation j , then $\text{cov}(\mathbf{x}^u(i), \mathbf{x}^u(j)) = 0$. Therefore, product variation can be reduced by avoiding correlation among operations.

3. Process diagnosis

The proposed process diagnostics is to systematically identify faulty operations in the multi-operational machining processes. The virtual machining concept is applied to isolate faults between operations, and further used in root cause determination. The input data to this diagnosis method is the part measurement and we assume that the normal condition of the machining process is known.

3.1. Fault isolation by virtual machining

If a surface is machined by using previously machined surfaces as datum, the deviation or variations of this surface is jointly affected by faults in the current and previous operations. The proposed diagnostics is to isolate the faults at the current operation and previous operations.

The virtual oper. k is proposed to distinguish transferred faults and newly generated faults. It is defined as the metal cutting process without e_k^m and e_k^f , while datum error is still the same as the real oper. k , hence the term ‘‘virtual machining’’.

As the output difference between virtual and real operation is only caused by e_k^m and e_k^f , the proposed process level fault isolation methodology is based on the following steps:

1. Collect measurement data and compute statistics. We have $\mathbf{y}(k)$, $\boldsymbol{\mu}_{\mathbf{y}(k)} = E(\mathbf{y}(k))$ and $K_{\mathbf{y}(k)} = \text{cov}(\mathbf{y}(k))$.
2. Virtual machining can be performed based on the following equations.

$$\mathbf{x}'(k) = A(k)\mathbf{x}(k-1) + B(k)\mathbf{x}^u(k) + \mathbf{w}(k),$$

$$\mathbf{y}'(k) = C(k)\mathbf{x}'(k) + \mathbf{v}(k), \quad (7)$$

where \mathbf{x}' , \mathbf{y}' are the part deviation and measurement deviation from virtual machining. $B(k)\mathbf{x}^u(k)$ is simplified as

$$\begin{aligned} B(k)\mathbf{x}^u(k) &= \Delta R(k)X^o(k) \\ &\quad - R_{GP}(k)B(k)\Delta T(k), \end{aligned} \quad (8)$$

where $R_{GP}(k)$, $\Delta R(k)$ and $\Delta T(k)$ are only caused by the datum error at oper. k . Denote $\boldsymbol{\mu}_{\mathbf{y}'(k)} = E(\mathbf{y}'(k))$ and $K_{\mathbf{y}'(k)} = \text{cov}(\mathbf{y}'(k))$. Fault isolation can be performed by comparing $\mathbf{y}(k)$ and $\mathbf{y}'(k)$.

3. If the mean and covariance of $\mathbf{y}(k)$ and $\mathbf{y}'(k)$ are used as the patterns of real and virtual operation k , occurrence of faults can be identified by performing hypothesis testing between patterns of $\mathbf{y}(k)$ and $\mathbf{y}'(k)$.

The above procedure can be repeated for operations $N, N-1, \dots, 2, 1$. As a result, fault isolation between operations can be achieved. Fig. 2 shows the methodology of fault isolation between operations.

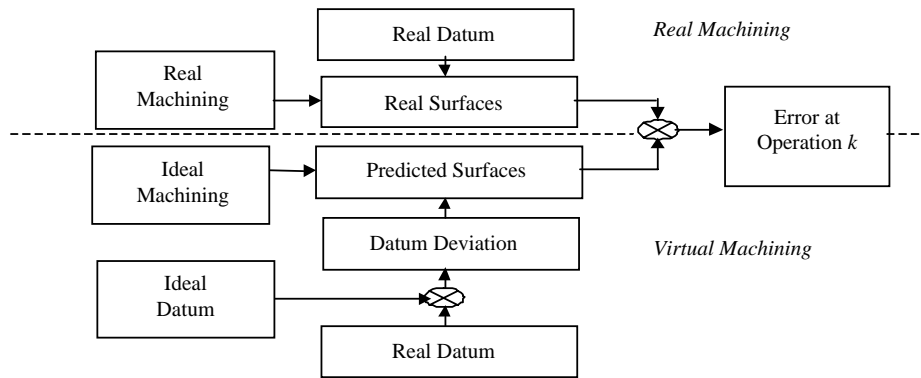


Fig. 2. Fault isolation between operations.

3.2. Faulty operation determination by hypothesis tests

Statistics of $\mathbf{y}(k)$ and $\mathbf{y}'(k)$ are estimated based on the measurement data and the simulation data, respectively. Assume $\mathbf{y}(k)$ and $\mathbf{y}'(k)$ all follow multivariate normal distribution, in which $\mathbf{y}(k)$ and $\mathbf{y}'(k)$ are p -variant, i.e., p characteristics will be inspected.

Suppose we take n_1 random samples of $\mathbf{y}(k)$. Assume the machining datum data is available from $\mathbf{y}(k-1)$. Then $\mathbf{x}^{it}(k)$ is attainable by Eq. (8). Thus, we can have n_2 samples of $\mathbf{y}'(k)$ by simulation and measurement. From these measurements and simulation results, we can easily obtain the sample mean and sample standard deviation. Hence, hypothesis tests can be conducted to determine the faulty operations. The hypothesis test procedure is given below:

1. $H_0 : K_{\mathbf{y}(k)} = K_{\mathbf{y}'(k)}$ vs. $H_1 : K_{\mathbf{y}(k)} \neq K_{\mathbf{y}'(k)}$.

If H_0 hypothesis is rejected at significance level α , faults are assumed to occur at oper. k and we terminate the test. If H_0 hypothesis fails to be rejected, we go on to the second test in step 2. The details of hypothesis tests can be referred to Muirhead [9]. The basic result is as follows. The unbiased likelihood ratio statistic is given as

$$\Lambda_k^* = \frac{(\det((n_1 - 1)S_{\mathbf{y}(k)}))^{(n_1 - 1)/2} (\det((n_2 - 1)S_{\mathbf{y}'(k)}))^{(n_2 - 1)/2}}{[\det((n_1 - 1)S_{\mathbf{y}(k)} + (n_2 - 1)S_{\mathbf{y}'(k)})]^{(n_1 + n_2 - 2)/2}} \times \frac{(n_1 + n_2 - 2)^{(n_1 + n_2 - 2)/2p}}{(n_1 - 1)^{(n_1 - 1)/2p} (n_2 - 1)^{(n_2 - 1)/2p}} \quad (9)$$

Its approximate distribution is

$$P(-2\rho \log \Lambda_k^* \leq x) = P(\chi_f^2 \leq x) + \frac{r}{M^2} [P(\chi_{f+4}^2 \leq x) - P(\chi_f^2 \leq x)] + O(M^{-3}) \quad (10)$$

for large $M = \rho(n_1 + n_2 - 2)$, where

$$\rho = 1 - \frac{2p^2 + 3p - 1}{6(p+1)(n_1 + n_2 - 2)} \left(\frac{n_1 - 1}{n_2 - 1} + \frac{n_2 - 1}{n_1 - 1} + 1 \right),$$

$$f = \frac{p(p+1)}{2},$$

$$r = \frac{p(p+1)}{48} \left\{ (p-1)(p+2) \left(\frac{n_1 - 1}{n_2 - 1} + \frac{n_2 - 1}{n_1 - 1} + 1 \right) - 6[(n_1 + n_2 - 2)(1 - \rho)]^2 \right\}$$

and $P(\chi_f^2 \leq x)$ is Chi-square distribution with f degrees of freedom. We reject H_0 if $-2\rho \log \Lambda_k^* > c_f(\alpha)$, where $c_f(\alpha)$ denotes the upper 100 $\alpha\%$ point of the χ_f^2 distribution.

2. $H_0 : \boldsymbol{\mu}_{\mathbf{y}(k)} = \boldsymbol{\mu}_{\mathbf{y}'(k)}$ vs. $H_1 : \boldsymbol{\mu}_{\mathbf{y}(k)} \neq \boldsymbol{\mu}_{\mathbf{y}'(k)}$ with $\mathbf{K}_{\mathbf{y}(k)} = \mathbf{K}_{\mathbf{y}'(k)} = \text{unknown covariance}$

If H_0 hypothesis is rejected at level α , mean shift are assumed to occur at oper. k . If H_0 hypothesis fails to be rejected, no fault occurs at oper. k . The two-sample T^2 -statistic is given by

$$T_{\alpha,p,n_1+n_2-2}^2 = (\overline{\mathbf{y}(k)} - \overline{\mathbf{y}'(k)})^T \times \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) \frac{(n_1 - 1)S_{\mathbf{y}(k)} + (n_2 - 1)S_{\mathbf{y}'(k)}}{n_1 + n_2 - 2} \right]^{-1} \times (\overline{\mathbf{y}(k)} - \overline{\mathbf{y}'(k)}) \quad (11)$$

with $n_1 + n_2 - 2 > p$. $\overline{\mathbf{y}(k)}$ represents sample mean and $S_{\mathbf{y}(k)}$ represents sample covariance matrix.

Although the dimension of $\mathbf{y}(k)$ is p , the number of varied components in $\mathbf{y}(k)$ is less than p at each operation, because surfaces are not machined only at one operation (for $N > 1$). Therefore, $\mathbf{K}_{\mathbf{y}(k)}$ is always a singular matrix. Since hypothesis tests require covariance matrices to be full rank, instead of $\mathbf{y}(k)$, we test its sub-vector, whose components varies in $\mathbf{y}(k)$. This is shown clearly in the case study.

4. Case study

A case study is performed to show the application of the proposed process level diagnostic methodology. The machining process is assumed to produce the part shown in Fig. 3. Table 1 is the process sheet. According to the design specification of the part in Fig. 3 and the process sheet, we know that planes A–D are design datum, which are also used as machining datum. D–F, are surfaces to be machined. D1 stands for the machined surface D. Thus A–F are the seven surfaces in vector X , where $X^o(3)$, the nominal values of X after the 3rd operation, is shown as an example in Table 2.

The measured characteristics at different operations and their specifications are determined by the part design. $Y^o(3)$ is listed in Table 3 for illustration. Only dimension, location, and orientation related tolerances are considered in this study.

The process is simulated by using the state space equations. The final product characteristics (listed in Table 3) are quality measures. The procedure of the case study is as follows:

- (1) Simulate the machining process under three cases: no faulty operation (machining error and setup error are at the normal level at each operation), one faulty operation (machining error and setup error are increased at the first operation), and two correlated faulty operations (machining error and setup error are increased at the first and the third operation). The parameter chosen for noise and fault distributions are given in Table 4.
- (2) Randomly select 50 samples for each case. The sample data is treated as from real operations, in contrast to virtual operations.
- (3) With sample information, virtual machining is performed to generate 50 virtual parts for each case.
- (4) Set significance level $\alpha = 0.05$, conduct hypothesis test for each case and identify faulty operation(s).

Table 1
Process sheet

Oper.	Before	After	Description	Locating datum (primary + secondary + tertiary datum)
1			Mill face D	A + C + B
2			Drill hole E through	A + C + B
3			Mill face F	D1 + C + B

Table 2
Part representation: sub-vectors of $X^o(3)$

	A	B	C	D	E	F
n_x	0	-1	0	0	0	0.707
n_y	0	0	-1	0	1	0
n_z	-1	0	0	1	0	0.707
p_x	0	0	0	0	80	240
p_y	0	0	0	0	0	0
p_z	0	0	0	120	60	120
d_1	0	0	0	0	60	0

The seven characteristics in Table 3 are of interest. They are divided into three sub-vectors ($y^k(k)$) as shown in Table 5.

The results are shown in Table 6. The hypothesis testing result shows all three operations are statistically in control for case 1, because the test statistics values are less than the threshold, e.g. in oper.1, $1.8548 < 7.8147$ and $4.1931 < 6.6422$. The hypothesis testing result suggests faults happen in the first operation in case 2, since the variance test for oper.1 shows $35.5818 > 7.8147$. Although the first operation and the third operation are correlated in case 3, the variance tests of oper.1 and oper.3 show that $18.8749 > 7$ and

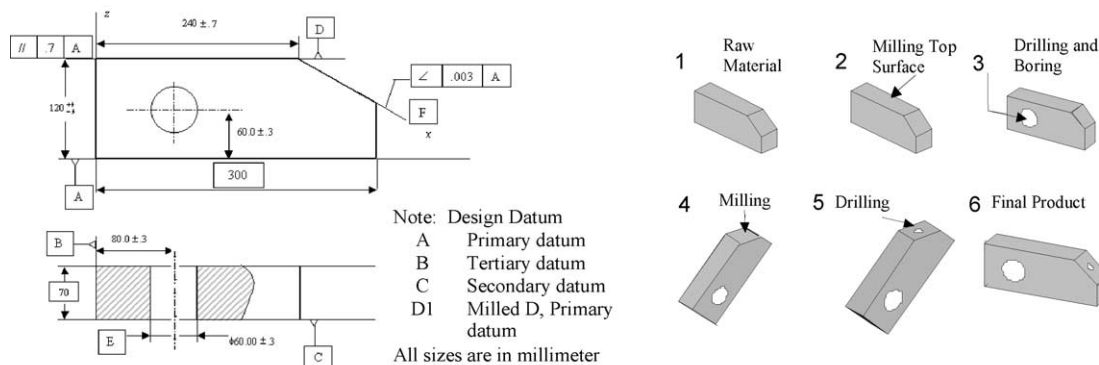


Fig. 3. Part drawing and operation steps for this part.

Table 3
Characteristics $Y^o(3)$

n th elements of $Y^o(3)$	Characteristic	Dimension	Tolerance
1	Height of surface D	120 mm	± 0.4 mm
2	Parallelism between surface D and A	0 mm	0.7 mm
3	Diameter of hole F	60 mm	± 0.3 mm
4	Position of hole F in x direction	80 mm	± 0.3 mm
5	Position of hole F in z direction	60 mm	± 0.3 mm
6	Length of top face	240 mm	± 0.7 mm
7	Angularity of surface F	0.7854 rad	± 0.003 rad

Table 4
Mean and standard deviation (STD) chosen for noise and fault

Component		Rotation	Translation	Size
Noise level	Mean	0	0	0
	STD	0.001 rad	0.01 mm	0.2 mm
Fault	Mean	0.004–0.01 rad	0.04–0.1 mm	0.3–0.6 mm
	STD	0.004–0.01 rad	0.04–0.1 mm	0.3–0.6 mm

Table 5
Sub-vector $y^k(k)$

Operation k	$y^k(k)$	Characteristics of Interest	p
1	$y^1(1)$	Height of surface D Parallelism between surface D and A	2
2	$y^2(2)$	Diameter of hole E Position of hole E in x direction Position of hole E in z direction	3
3	$y^3(3)$	Length of top face Angularity of surface F	2

16.273 > 7.8147, which suggest that both operations have faults.

5. Conclusion

The proposed monitoring and diagnostic methodology are applicable in multi-operational machining

processes. Even in the situations of no historical measurement data, quality improvement can still be achieved by utilizing a process model and prior knowledge about machine tool and fixture, like machine tool repeatability and fixture tooling tolerance. The prior knowledge will be transformed into noise distribution in the process model. By doing all of this, cause–effect relation between the part variations and operation variations can be analyzed through variation propagation model. It provides insight understanding of the process variation, which is helpful for process design, analysis and diagnosis. It is proved successful by the simulation study.

The proposed diagnostic methodology is limited to the process level. Operation level problems, such as fixture or machine tool failures, cannot be distinguished. Further work on operation level diagnostics should be done based on the state space model.

Acknowledgements

The authors gratefully acknowledge the financial support of the NSF Engineering Research Center for Reconfigurable Machining Systems (NSF Grant EEC95-92125) at the University of Michigan and the valuable input from the Center's industrial partners.

References

- [1] Ramesh R, Mannan MA, Poo AN. Error compensation in machine tools—a review part I: geometric, cutting-force induced and fixture dependent errors. *Int J Mach Tools Manuf* 2000;40:1235–56.
- [2] Ramesh R, Mannan MA, Poo AN. Error compensation in machine tools—a review part II: thermal errors. *Int J Mach Tools Manuf* 2000;40:1235–56.
- [3] Mantripragada R, Whitney DE. Modeling and controlling variation propagation in mechanical assemblies using state transition models. *IEEE Trans Robotics Autom* 1999;15(1):124–40.
- [4] Lawless JF, Mackay RJ, Robinson JA. Analysis of variation transmission in manufacturing processes—part I. *J Qual Technol* 1999;31(2):131–42.
- [5] Agrawal R, Lawless JF, Mackay RJ. Analysis of variation transmission in manufacturing processes—part II. *J Qual Technol* 1999;31(2):143–54.

Table 6
Hypothesis testing results

Threshold/test statistics	Operation 1		Operation 2		Operation 3	
	Covariance test	Mean test	Covariance test	Mean test	Covariance test	Mean test
Threshold	7.8147	6.6422	12.5916	9.0055	7.8147	6.6422
Case 1	1.8548	4.1931	6.864	0.2546	2.4967	0.8491
Case 2	35.5818	32.4807	9.7861	0.8924	0.0075	0.11
Case 3	18.8749	18.8148	11.8092	0.3508	16.2737	4.3283

- [6] Jin J, Shi J. State space modeling of sheet metal assembly for dimensional control. *J Manuf Sci Eng* 1999;121L:756–62.
- [7] Ding Y, Ceglarek D, Shi J. Modeling and diagnosis of multistage manufacturing processes: Part I & Part II. JAPAN/USA Symposium on Flexible Automation 2000, Ann Arbor, MI, 2000.
- [8] Huang Q, Zhou N, Shi J. Stream-of-Variation Modeling and Diagnosis of Multi-station Machining Processes. Proceedings of the 2000 ASME International Mechanical Engineering Congress & Exposition, MED-Vol. 11, Orlando, FL, November 5–10, 2000. p. 81–88.
- [9] Muirhead RJ. Aspects of multivariate statistical theory. New York: Wiley, 1982.