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Optimal fixture locator adjustment strategies for multi-station assembly processes

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Fixture locating errors directly impact the dimensional quality of products in assembly processes. During a production run, fixture locators may deviate from their designed positions and this can possibly lead to defects and quality loss in the final assembled products. Mass production in multi-station assembly processes involves multiple fixtures/stations, which leads to extreme complexity in dimensional control through locator position adjustment. This research aims to develop a systematic methodology for fixture locator adjustment to minimize total production costs in multi-station assembly processes. In this paper, a linear model is derived to describe the complex propagation effect of fixture adjustments throughout all stations in an assembly process. Bayesian estimation with iterative algorithms is used to adaptively estimate the unknown parameters of locator deviation errors during production. An optimal fixture locator adjustment strategy is obtained through dynamic programming based on the given process and product design scheme. A case study is provided to illustrate the implementation procedures and the significance of the proposed methodology.

[Supplementary materials are available for this article. Go to the publisher's online edition of *IIE Transactions* for the following free supplemental resource: Appendix]

Keywords: Bayesian estimation, dynamic programming, fixture locator adjustment, multi-station assembly process, state space model

1. Introduction

The calibration of part locating fixtures has significant effects on the dimensional quality of final products in a multi-station assembly process. Here, "multi-station" refers to the multiple stations or operations that are involved in producing a product within a complex assembly system. Examples of multi-station assembly processes include automotive body assembly, aircraft assembly and home appliances assembly.

Earlier research has indicated that 72% of all dimensional faults in automotive body assembly is due to fixture locator errors (Ceglarek and Shi, 1995). These fixture locators refer to the locating pins, or NC blocks, that directly determine the position and orientation of a part. In a 3-2-1 locating mechanism, the six degrees of freedom need to be controlled to position a rigid part. In a typical automobile body assembly process, there are more than 150 sheet metal parts that are assembled by more than 100 assembly stations using more than 1000 locators. During mass production, fixture locators may deviate from their designed positions which can lead to defects and quality loss in the final products. As a result, effective identification and control of those fixture locator errors are important, yet challenging, tasks due to the complexity of variation and propagation in a multi-station assembly process. The existing quality assurance techniques are mainly based on Statistical Quality Control (SQC), which focuses on on-line monitoring of quality measurements during production. In general, SQC techniques are effective in detecting quality changes, but they may not always provide a systematic means for adjusting fixture locators to eliminate the root causes of quality changes, especially for complex multi-station assembly processes. As a result, it is desirable to develop an effective approach to optimally adjust fixture locators based on in-process quality measurement with consideration of the overall cost of quality defects and the cost of adjustments throughout a whole production run. In this paper, the production run is defined as the total number of products produced by the existing production setup.

The proposed fixture locator adjustment strategy is based on the integration of Statistical Process Control (SPC) with Automatic Process Control (APC). SPC is used to monitor

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process changes while APC utilizes feedback or feedforward control to compensate the process change. Box and Kramer (1992) provided a detail discussion on SPC and APC integration. Grubbs (1983) first studied the adjustment problem by setting machine or process parameters to produce the desired outputs. His rule is to adjust a process by an entire observed deviation after the first item is produced. Then, a half deviation is adjusted after producing the second item and so on. More investigations have been done to extend Grubbs' work (Trietsch, 1998, 2000; Del Castillo *et al.*, 2003). Triethsch (1998, 2000) referred to Grubbs' adjustment rule as the harmonic rule. In contrast to Grubbs, he allowed some adjustments to be skipped (Trietsch, 1998) and also took into consideration measurement and adjustment cost (Trietsch, 2000). Del Castillo *et al.* (2003) proposed a general formulation for the setup adjustment problems, including the Grubbs harmonic rule. The Bayesian method based on a Kalman filter is used in their formulation.

One major objective of the process adjustment is to reduce manufacturing cost, including the loss due to off-target quality and process adjustment cost. Various efforts (Crowder, 1992; Luceño, 2003; Lian and Del Castillo, 2006) have been made to control a manufacturing process to achieve those objectives with great success. However, most of those research efforts focus on a single variable or a single-station process. Some recent research provided the analysis of the adjustment problem in the case of multiple inputs in the model (Sachs *et al.*, 1995). Del Castillo and Rajagopal (2002) applied the double EWMA control scheme to a multiple-input mutiple-output case. Although there are some studies that discuss multiple inputs and outputs in the process adjustment, the multi-station assembly process has not been fully investigated due to the complexity of the variation propagation and interactions among different stations. As a result, the fixture adjustment based on a single station alone (e.g., current, intermediate station) may not be able to fully ensure the optimal solution in terms of the total manufacturing cost in a multi-station assembly process. In the case of making adjustments in a multiple station process, Wang and Huang (2007) developed an automatic process adjustment method to compensate the mean shift of machining processes by adjusting fixture locators based on the equivalent fixture error (EFE) concept. A minimum-mean-square-error controller is designed based on the dynamic EFE model. However, this method cannot be directly applied to multi-station assembly process control. Therefore, this paper aims to develop an optimal fixture adjustment strategy by considering the variation propagation and interaction among different stations in a multi-station assembly process.

The proposed fixture locator adjustment methodology was inspired by the work by Lian and Del Castillo (2006), in which they proposed an optimal machine setup adjustment strategy for a single machine. However, their method is not directly applicable to the adjustment in multi-station assembly processes due to the inability to model the complex interrelations among stations. The complexity of variation propagation in a multi-station assembly process can be shown in two aspects: (i) fixture errors and adjustments on the upstream stations' effect on the performance of the downstream stations; and (ii) a part being transferred through different fixtures at different stations may generate propagated variations that combine all fixture errors among stations. Therefore, this paper aims to extend the existing adjustment strategy from single station control to multi-station assembly process control by using a dynamic programming approach.

The objective of this research is to develop a systematic methodology to determine the optimal fixture adjustment strategy for a prespecified control interval using the quadratic off-target cost function and the constant adjustment cost in a multi-station assembly process. In this study, it is assumed that the fixture position can be accurately adjusted or the variance of the adjustment errors can be obtained either from the tooling specifications or through off-line tooling calibration tests. The initial fixture errors are random variables that follow an unknown multivariate normal distribution. In order to adjust a multi-station assembly process, a state space model proposed by Jin and Shi (1999) and Shi (2006) is adopted to capture the fixture error propagation through all stations and its effect on product quality. This model describes the complex interrelation of variation propagation among stations and therefore provides an opportunity to further develop an adjustment strategy in a multi-station assembly process.

The remainder of this paper is organized as follows. Section 2 provides an overview of the proposed methodology and introduces the state space model representation for multi-station assembly processes. An optimal fixture adjustment strategy is developed in Section 3 by using dynamic programming with the integration of Bayesian estimation of fixture errors. Section 4 provides a case study on a multi-station assembly process to illustrate the proposed methodology. Finally, the conclusion and future work are given in Section 5.

2. Overview of proposed methodology and process modeling

This section gives an overview of the proposed methodology and introduces the process model to be used in the methodology development.

2.1. *Overview of the proposed methodology*

The proposed methodology addresses the fixture adjustment problem for multi-station assembly processes with a prespecified control interval. The development of the proposed methodology consists of four essential components, as illustrated in Fig. 1. In the proposed approach, a model

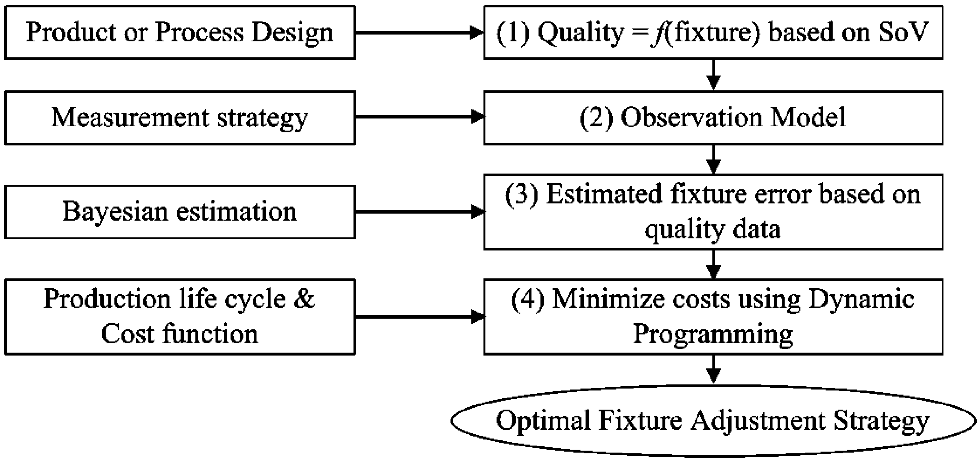

Fig. 1. General framework of the proposed methodology.

is constructed to describe the relationships between quality measurements and fixture position deviations from their nominal positions. Typically, a state space model is constructed off-line by using the product and process design data (Jin and Shi, 1999; Zhou, Huang and Shi, 2003). An observation model is further derived to describe the relationships among the quality measurements and fixture errors or adjustment effects. With the help of the state space model and the observation model, Bayesian estimation is adopted to estimate the unknown fixture errors on-line based on the measured quality data. Finally, an optimization problem is formulated to minimize the overall production run cost due to both the product quality loss and the fixture adjustment cost. Considering the nature of variation propagation in a multi-station assembly process, dynamic programming is used to solve this optimization problem to obtain an optimal adjustment strategy for quality improvement. The details of each step in this framework are discussed in the following sections.

2.2. *Review of state space model for multi-station manufacturing processes*

An N-stage manufacturing process is shown in Fig. 2. The notations to be used in the model are defined as follows:

k = the index of a manufacturing station, where $k = 1, 2, \ldots, N$;

$\mathbf{x}_k$ = the product feature vector defined as the deviations of the key features from their nominal values at station k, which is the resultant deviation due to the accumulated effects of all previous stages $k = 1, 2, \ldots, k-1$, as well as the effect contributed at stage k;

$\mathbf{u}_k$ = the fixture position offset at stage k, which reflects the combined effect of fixture locator errors and all fixture adjustments up to stage k (the elements of $\mathbf{u}_k$ are assumed to be independent of each other);

$\mathbf{w}_k$ = process disturbance or uncertainties (it is assumed that $\mathbf{w}_k$ has a zero mean and the elements of $\mathbf{w}_k$ are independent);

$\mathbf{y}_k$ = observed Key Product Characteristics (KPCs) at stage k, which is the measurement vector of product quality (dimensional deviations) at stage k;

$\mathbf{v}_k$ = measurement noise (it is assumed that $\mathbf{v}_k$ has a zero mean and the elements of $\mathbf{v}_k$ are independent).

Also, suppose p_k, q_k and m_k are the dimension of vectors $\mathbf{u}_k$, $\mathbf{w}_k$ and $\mathbf{y}_k$, respectively. For a process with N stations, P, Q and M are the following:

P : $\sum_{k=1}^{N} p_k$, total number of fixture errors or process faults;

Q : $\sum_{k=1}^{N} q_k$, total number of process noises;

M : $\sum_{k=1}^{N} m_k$, total number of measurements (KPCs).

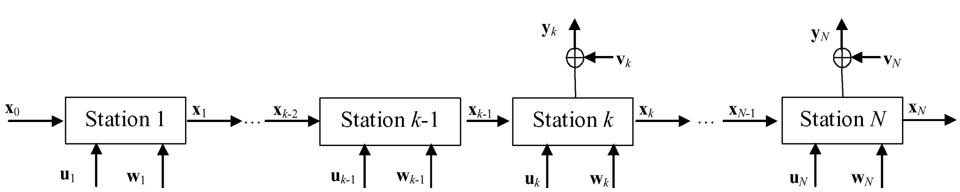

Fig. 2. Diagram of a multi-station manufacturing process.

A state space model describing the relationship between product quality and fixture errors is (Jin and Shi, 1999):

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k,$$
$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k, \quad k = 1, 2, \ldots, N, \tag{1}$$

where $\mathbf{A}_{k-1}$ represents the variation propagating effect due to the process datum transform from station $k-1$ to station k, $\mathbf{B}_k$ represents a contribution of the fixture offset $\mathbf{u}_k$ at the current station k on the product characteristics $\mathbf{x}_k$ and $\mathbf{C}_k$ is the measurement matrix that represents the relationship between product key features $\mathbf{x}_k$ and measured product quality data $\mathbf{y}_k$. Equation (1) can be re-written as a linear input–output model:

$$\mathbf{y}_k = \sum_{i=1}^{k} \mathbf{C}_k\mathbf{\Phi}_{k,i}\mathbf{B}_i\mathbf{u}_i + \mathbf{C}_k\mathbf{\Phi}_{k,0}\mathbf{x}_0 + \sum_{i=1}^{k} \mathbf{C}_k\mathbf{\Phi}_{k,i}\mathbf{w}_i + \mathbf{v}_k, \tag{2}$$

where $\mathbf{\Phi}_{(.,.)}$ is called the state transition matrix; that is, $\mathbf{\Phi}_{k,i} = \mathbf{A}_{k-1}\mathbf{A}_{k-2}\cdots\mathbf{A}_i$ for $k > i$ and $\mathbf{\Phi}_{k,k} = \mathbf{I}$ and $\mathbf{x}_0$ is the initial deviation of the part features before the part enters the assembly process. Without loss of generality, $\mathbf{x}_0$ is assumed to be a zero vector. Equation (2) can be simplified as

$$\mathbf{y} = \mathbf{\Gamma} \times \mathbf{u} + \mathbf{\Psi} \times \mathbf{w} + \mathbf{v}, \tag{3}$$

where $\mathbf{y} = [\mathbf{y}_1^{\mathrm{T}} \cdots \mathbf{y}_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{M\times 1}$, $\mathbf{u} = [\mathbf{u}_1^{\mathrm{T}} \cdots \mathbf{u}_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{P\times 1}$, $\mathbf{w} = [\mathbf{w}_1^{\mathrm{T}} \cdots \mathbf{w}_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{Q\times 1}$, $\mathbf{v} = [\mathbf{v}_1^{\mathrm{T}} \cdots \mathbf{v}_N^{\mathrm{T}}]^{\mathrm{T}} \in R^{M\times 1}$, and

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{C}_1\mathbf{B}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{C}_2\mathbf{\Phi}_{2,1}\mathbf{B}_1 & \mathbf{C}_2\mathbf{B}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_N\mathbf{\Phi}_{N,1}\mathbf{B}_1 & \mathbf{C}_N\mathbf{\Phi}_{N,2}\mathbf{B}_2 & \ldots & \mathbf{C}_N\mathbf{B}_N \end{bmatrix} \in R^{M\times P},$$

and

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{C}_2\mathbf{\Phi}_{2,1} & \mathbf{C}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_N\mathbf{\Phi}_{N,1} & \mathbf{C}_N\mathbf{\Phi}_{N,2} & \ldots & \mathbf{C}_N \end{bmatrix} \in R^{M\times Q}.$$

Suppose $\{\sigma_{w_i}^2\}_{i=1\ldots Q}$ are the variance components of process noises $\mathbf{w}$ and σ_v^2 is the identical variance of sensor noises $\mathbf{v}$. A linear input–output model for product t is

$$[\mathbf{y}]_t = \mathbf{\Gamma} \times [\mathbf{u}]_t + \mathbf{\Psi} \times [\mathbf{w}]_t + [\mathbf{v}]_t = \mathbf{\Gamma} \times [\mathbf{u}]_{\mathrm{t}} + [\boldsymbol{\varepsilon}]_t, \tag{4}$$

where $[\boldsymbol{\varepsilon}]_t = \mathbf{\Psi} \times [\mathbf{w}]_t + [\mathbf{v}]_t$. Assume that $[\mathbf{w}]_t \sim N(0, \mathrm{diag}\{\sigma_{w_l}^2\}_{l=1,\ldots,Q})$ and $[\mathbf{v}]_t \sim N(0, \mathrm{diag}\{\sigma_v^2\})$, then the variance of independent and identically distributed (i.i.d) $\boldsymbol{\varepsilon}$ is

$$\mathbf{\Sigma}_{\boldsymbol{\varepsilon}} = \mathbf{\Psi} \times \mathbf{\Sigma}_{\mathbf{w}} \times \mathbf{\Psi}^{\mathrm{T}} + \mathbf{\Sigma}_{\mathbf{v}}. \tag{5}$$

The linear input–output model (4) serves as the basis for formulating an optimization problem on fixture adjustment.

3. Dynamic programming formulation

This section discusses the formulation of the fixture locator adjustment problem into a dynamic programming problem. Section 3.1 describes the problem setup for dynamic programming, which provides the definitions for the stages of dynamic programming, the state variables and the decision variables. Sections 3.2 and 3.3 describe the observation model and Bayesian estimation of fixture locating errors respectively. Afterwards, a cost function is defined in Section 3.4 and the proposed optimal adjustment solutions are discussed in Section 3.5.

3.1. *Problem setup for dynamic programming*

In this paper, the stage defined in the dynamic programming formulation corresponds to the control action index. The total number of stages is equal to the total number of products in a production run divided by the number of products produced within each control interval prespecified by the users. The decision variables are the adjustments of the fixture locations.

Let n denote the total number of control intervals in one production run; and b denote the number of products produced during each control interval. For example, if a production run consists of 10 000 products and a control interval of 500 products is prespecified, then there is a total of 20 control intervals. In this case, $n = 20$ and $b = 500$. It is reasonable to assume that the product quality measurements within each control interval are i.i.d. distributed because the fixture adjustment can only be made at each prespecified control time. Therefore, the process status at stage i is represented by the average of fixture location offsets ($\bar{\mathbf{u}}^i$) and the average of product quality measurements ($\bar{\mathbf{y}}^i$) at control interval i (right after taking control action $i-1$ and right before taking control action i).

In this paper, $\bar{\mathbf{u}}^i$ is assumed to follow a multivariate normal probability distribution. Since $\bar{\mathbf{u}}^i$ is not directly measurable, a posterior estimation of its mean $\boldsymbol{\mu}^i$ is needed, which can be obtained by $\hat{\boldsymbol{\mu}}^i|\bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)} \sim N(\boldsymbol{\mu}^i, \mathbf{\Lambda}^i)$ to be discussed in Section 3.2. Therefore, the state variables at stage i are defined to include $\boldsymbol{\mu}^i$, $\mathbf{\Lambda}^i$ and $\bar{\mathbf{y}}^i$, which can uniquely represent the process status at stage i for determining control action i. The decision of fixture adjustments will be optimally determined by using a dynamic programming approach to be discussed in Section 3.5 based on the predefined cost function in Section 3.4. The state transition is affected by the fixture design scheme and product quality measurements in a multi-station assembly process, which will be discussed in Sections 3.2 and 3.3.

3.2. *Observation model*

The observation model represents the relationship among product quality measurements, fixture errors and fixture adjustments. Suppose $\mathbf{y}^i$ and $\mathbf{u}^i$ are the deviations from their nominal values right before control action i ($i \geq 1$). The initial fixture offset vector ($\mathbf{u}^1$) represents the fixture offset errors right before the first control action is applied to the system. Let $[\mathbf{a}^i]_j$ be the amount of the fixture adjustment made at control action i for fixture component $j, j = 1, \ldots, P$. Since the product quality within each control interval is i.i.d. distributed, the average observation vector ($\bar{\mathbf{y}}^i$) and the average observation noise ($\bar{\boldsymbol{\varepsilon}}^i$) within a control interval i are used in the observation model:

$$\bar{\mathbf{y}}^i = \boldsymbol{\Gamma}\bar{\mathbf{u}}^i + \bar{\boldsymbol{\varepsilon}}^i \quad \text{for } i = 1, \ldots, n, \tag{6}$$

$$\bar{\mathbf{u}}^i = \boldsymbol{\mu}^i + \bar{\boldsymbol{\eta}}^i, \tag{7}$$

$$\boldsymbol{\mu}^i = \boldsymbol{\mu}^{i-1} + \mathbf{a}^{i-1} = \boldsymbol{\mu}^1 + \sum_{k=1}^{i-1} \mathbf{a}^k \quad \text{for } i = 2, \ldots, n, \tag{8}$$

where $\bar{\boldsymbol{\varepsilon}}^i \sim N(\mathbf{0}, (1/b)\Sigma_\varepsilon)$, and $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$ is a diagonal matrix and assumed to be known. Also, $\bar{\mathbf{u}}^i \sim N(\boldsymbol{\mu}^i, \boldsymbol{\Sigma}_{\boldsymbol{\eta}^i})$ and $\bar{\boldsymbol{\eta}}^i \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\eta}^i})$, where $\boldsymbol{\Sigma}_{\boldsymbol{\eta}^i}$ is also a diagonal matrix used to represent the variance induced by the natural fixture locating errors due to the design tolerances of locators and assembled parts, as well as tooling adjustment errors. By considering the cumulative effect of tooling adjustment errors after the $(i-1)$th adjustment, $\boldsymbol{\Sigma}_{\boldsymbol{\eta}^i}$ can be calculated by $\boldsymbol{\Sigma}_{\boldsymbol{\eta}^i} = \boldsymbol{\Sigma}_{\boldsymbol{\eta}} + (i-1)\boldsymbol{\Sigma}_{\mathbf{a}}$, where $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ represents the inherent variance of locating errors induced by the design tolerances of locators and assembled parts, and $\boldsymbol{\Sigma}_{\mathbf{a}}$ represents the additional variance due to each time of tooling adjustment action. $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ and $\boldsymbol{\Sigma}_{\mathbf{a}}$ are assumed to be preknown, which can be obtained through tooling/product tolerance design, and tooling specifications or off-line calibration tests respectively. In practice, in order to ensure an effective tooling adjustment, it is usually required to have an accurate tooling adjustment, i.e., the diagonal elements of $\boldsymbol{\Sigma}_{\mathbf{a}}$ due to adjustment errors are relatively small when compared with the corresponding element of $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ due to the design tolerances of locators. Therefore, without loss of generality, it is reasonable to use $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ instead of $\boldsymbol{\Sigma}_{\boldsymbol{\eta}^i}$ in the following analyses. Online Appendix A shows the derivation of the observation model (6).

3.3. *Bayesian estimation of fixture locating errors*

Before the first control adjustment, the initial fixture offset error of $\boldsymbol{\mu}^1$ is unknown. This section discusses how to estimate this unknown initial offset by using the Bayesian estimation method. DeGroot (1969) provides a Bayesian estimation for a multivariate normal distribution with known variances. In this paper, $\boldsymbol{\Sigma}_\varepsilon$ and $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}$ are assumed to be known or estimated through model calibrations and/or historical data. From Equations (6) to (8), after taking control action i, the predicted probability distribution function right before control action $i+1$ is

$$\bar{\mathbf{y}}^{i+1}\big|\boldsymbol{\mu}^i, \mathbf{a}^i \sim N(\boldsymbol{\Gamma}\boldsymbol{\mu}^i + \boldsymbol{\Gamma}\mathbf{a}^i, \boldsymbol{\Sigma}_\varepsilon/b + \boldsymbol{\Gamma}\boldsymbol{\Sigma}_{\boldsymbol{\eta}}\boldsymbol{\Gamma}^{\mathrm{T}}). \tag{9}$$

In practice, the fixture offset $\boldsymbol{\mu}^i = \boldsymbol{\mu}^1 + \sum_{k=1}^{i-1} \mathbf{a}^k$ is unknown due to an unknown $\boldsymbol{\mu}^1$. However, $\boldsymbol{\mu}^i$ can be estimated based on all observed product quality measurements $\bar{\mathbf{y}}^{(i)}$ with the known control action of $\mathbf{a}^{(i)}$, where $\bar{\mathbf{y}}^{(i)}$ and $\mathbf{a}^{(i)}$ correspond to the whole set of the average product quality measurements and the adjustments from control action 1 up to control action i, respectively. The posterior distribution of the estimator $\hat{\boldsymbol{\mu}}^i$ is

$$\hat{\boldsymbol{\mu}}^i|\bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)} \sim N(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i), \tag{10}$$

and $\boldsymbol{\Lambda}^i$ is the variance of the posterior distribution of the estimator $\hat{\boldsymbol{\mu}}^i$. The posterior predictive density function of $f(\bar{\mathbf{y}}^{i+1} \mid \bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)})$ follows a multivariate normal distribution (DeGroot, 1969), which can be obtained by

$$f(\bar{\mathbf{y}}^{i+1} \mid \bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)}) = \int f(\bar{\mathbf{y}}^{i+1} \mid \hat{\boldsymbol{\mu}}^i) f(\hat{\boldsymbol{\mu}} \mid \bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)}) \mathrm{d}\hat{\boldsymbol{\mu}}^i, \tag{11}$$

where $\int$ in this equation denotes a multi-dimensional integral. It can be shown that the posterior predictive density is a multivariate normal distribution and expressed as

$$f(\bar{\mathbf{y}}^{i+1} \mid \bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)}) \sim N(\boldsymbol{\Gamma}\boldsymbol{\mu}^i + \boldsymbol{\Gamma}\mathbf{a}^i, \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}/b + \boldsymbol{\Gamma}\boldsymbol{\Sigma}_{\boldsymbol{\eta}}\boldsymbol{\Gamma}^{\mathrm{T}} + \boldsymbol{\Gamma}(\boldsymbol{\Sigma}_{\boldsymbol{\eta}} + \boldsymbol{\Lambda}^i)\boldsymbol{\Gamma}^{\mathrm{T}}). \tag{12}$$

Also, the recursive updates of $\hat{\boldsymbol{\mu}}^i$ and $\boldsymbol{\Lambda}^i$ can be obtained (Meinhold and Singpurwalla, 1983) by

$$\hat{\boldsymbol{\mu}}^i = (\hat{\boldsymbol{\mu}}^{i-1} + \mathbf{a}^{i-1}) + (\boldsymbol{\Lambda}^{i-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}})\boldsymbol{\Gamma}^{\mathrm{T}} \times (\boldsymbol{\Sigma}_\varepsilon/b + \boldsymbol{\Gamma}(\boldsymbol{\Lambda}^{i-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}})\boldsymbol{\Gamma}^{\mathrm{T}})^{-1}(\bar{\mathbf{y}}^i - \boldsymbol{\Gamma}\hat{\boldsymbol{\mu}}^{i-1} - \boldsymbol{\Gamma}\mathbf{a}^{i-1}), \tag{13}$$

$$\boldsymbol{\Lambda}^i = (\boldsymbol{\Lambda}^{i-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}}) - (\boldsymbol{\Lambda}^{i-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}})\boldsymbol{\Gamma}^{\mathrm{T}} \times \left(\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}/b + \boldsymbol{\Gamma}\left(\boldsymbol{\Lambda}^{i-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}}\right)\boldsymbol{\Gamma}^{\mathrm{T}}\right)^{-1} \boldsymbol{\Gamma}(\boldsymbol{\Lambda}^{i-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}}). \tag{14}$$

The estimations obtained from Equations (13) and (14) provide the basis for the optimal fixture adjustment decision. The optimization algorithm, based on dynamic programming, is discussed in Section 3.5.

3.4. *Cost function*

A cost function needs to be defined for an objective function in order to obtain an optimal fixture adjustment decision. There are two cost components that need to be considered in the control objective function: the fixture adjustment cost and product quality loss cost. In this paper, the fixture adjustment cost is fixed and independent of the magnitude of adjustment. Therefore, whenever the adjustment vector of $\mathbf{a}^i$ is non-zero, a fixed adjustment cost is incurred. The quality loss cost is represented by a quadratic cost function of deviation $\mathbf{y}^i$ from its target value, and is commonly used in the Taguchi method (Kackar, 1985; Nair, 1992).

Assume c is the ratio of the adjustment cost to the quality loss as defined in Crowder (1992). The expectation of the total cost is expressed as

$$L(n) = E\left\{\sum_{i=1}^{n}\left(b \times (\bar{\mathbf{y}}^i)^{\mathrm{T}}(\bar{\mathbf{y}}^i) + c\sum_{j=1}^{P}\delta([\mathbf{a}^{i-1}]_j)\right)\right\}, \quad (15)$$

where

$$\delta(a) = \begin{cases} 0 & \text{if } a = 0 \\ 1 & \text{if } a \neq 0. \end{cases}$$

Since

$$E[\mathbf{y}^{\mathrm{T}}\mathbf{y}] = \mathrm{trace}[\Sigma_{\mathbf{y}}] + E[\mathbf{y}]^{\mathrm{T}}E[\mathbf{y}], \quad (16)$$

the expected total cost becomes:

$$L(n) = \sum_{i=1}^{n}\left(b \times \left(\mathrm{trace}[\boldsymbol{\Sigma}_{\bar{\mathbf{y}}^i}] + E[\bar{\mathbf{y}}^i]^{\mathrm{T}}E[\bar{\mathbf{y}}^i]\right) + c\sum_{j=1}^{P}\delta\left(\left[\mathbf{a}^{i-1}\right]_j\right)\right). \quad (17)$$

The optimal fixture adjustment decision is determined by minimizing this expected total cost function.

3.5. *Optimal fixture adjustment strategy using dynamic programming*

The determination of an optimal fixture adjustment amount is a sequential decision problem. In this paper, dynamic programming is adopted to determine such an optimal strategy. The fixture position offset vector $\mathbf{u}^i$ at stage i is modeled with a normal probability distribution using mean $\boldsymbol{\mu}^i$ and variance $\boldsymbol{\Lambda}^i$, which are updated with Equations (13) and (14) after observing control action $i-1$ and right before making control action i. Let $V_i(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i)$ be the minimum expected total cost from future control action i to control action $n-1$, given $\boldsymbol{\mu}^i$ and $\boldsymbol{\Lambda}^i$ after the current control action $i-1$. The optimal fixture adjustment strategy is determined recursively backward from the terminal condition (control action $n-1$) to the beginning point (control action 1). The expected total cost at the terminal condition or at control interval $n-1$ is determined first. The density of the average observation vector at the terminal condition is

$$\bar{\mathbf{y}}^n \mid \bar{\mathbf{y}}^{n-1}, \mathbf{a}^{n-1} \sim N\left(\boldsymbol{\Gamma}\boldsymbol{\mu}^{n-1} + \boldsymbol{\Gamma}\mathbf{a}^{n-1}, \boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}/b + \boldsymbol{\Gamma}\Sigma_{\boldsymbol{\eta}}\boldsymbol{\Gamma}^{\mathrm{T}} + \boldsymbol{\Gamma}(\boldsymbol{\Lambda}^{n-1} + \boldsymbol{\Sigma}_{\boldsymbol{\eta}})\boldsymbol{\Gamma}^{\mathrm{T}}\right).$$

Therefore,

$$\begin{aligned} V_{n-1}(\boldsymbol{\mu}^{n-1}, \boldsymbol{\Lambda}^{n-1}) &= \min_{\mathbf{a}^{n-1}} E\left\{b \times \left(\bar{\mathbf{y}}^n\right)^{\mathrm{T}}\left(\bar{\mathbf{y}}^n\right) + c \times \sum_{j=1}^{P}\delta\left(\left[\mathbf{a}^{n-1}\right]_j\right)\middle|\bar{\mathbf{y}}^{n-1}\right\} \\ &= \min_{\mathbf{a}^{n-1}}\left\{b \times \left[\mathrm{tr}\left(\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}/b + \boldsymbol{\Gamma}\Sigma_{\boldsymbol{\eta}}\boldsymbol{\Gamma}^{\mathrm{T}} + \boldsymbol{\Gamma}\left(\boldsymbol{\Sigma}_{\boldsymbol{\eta}} + \boldsymbol{\Lambda}^{n-1}\right)\boldsymbol{\Gamma}^{\mathrm{T}}\right) + (\boldsymbol{\Gamma}\boldsymbol{\mu}^{n-1} + \boldsymbol{\Gamma}\mathbf{a}^{n-1})^{\mathrm{T}} \times \left(\boldsymbol{\Gamma}\boldsymbol{\mu}^{n-1} + \boldsymbol{\Gamma}\mathbf{a}^{n-1}\right)\right] + c \times \sum_{j=1}^{P}\delta\left(\left[\mathbf{a}^{n-1}\right]_j\right)\right\}. \end{aligned} \quad (18)$$

For stage $i < n-1$, the minimum expected total cost at each stage i is defined recursively as

$$\begin{aligned} V_i\left(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i\right) = \min_{\mathbf{a}^i} \Big\{ & b \times \left[\mathrm{tr}\left(\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}/b + \boldsymbol{\Gamma}\Sigma_{\boldsymbol{\eta}}\boldsymbol{\Gamma}^{\mathrm{T}} + \boldsymbol{\Gamma}\left(\boldsymbol{\Lambda}^i + \boldsymbol{\Sigma}_{\boldsymbol{\eta}}\right)\boldsymbol{\Gamma}^{\mathrm{T}}\right) + \left(\boldsymbol{\Gamma}\boldsymbol{\mu}^i + \boldsymbol{\Gamma}\mathbf{a}^i\right)^{\mathrm{T}} \times \left(\boldsymbol{\Gamma}\boldsymbol{\mu}^i + \boldsymbol{\Gamma}\mathbf{a}^i\right)\right] + c \times \sum_{j=1}^{P}\delta\left(\left[\mathbf{a}^i\right]_j\right) \\ & + E\left\{V_{i+1}\left(\boldsymbol{\mu}^{i+1}, \boldsymbol{\Lambda}^{i+1}\right)\middle|\mathbf{a}^i\right\}\Big\}, \end{aligned} \quad (19)$$

where

$$E\left\{V_{i+1}\left(\boldsymbol{\mu}^{i+1}, \boldsymbol{\Lambda}^{i+1}\right)\middle|\mathbf{a}^i\right\} = \int V_{i+1}\left(\boldsymbol{\mu}^{i+1}, \boldsymbol{\Lambda}^{i+1}\right) \times f\left(\bar{\mathbf{y}}^{i+1}\middle|\bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)}\right)\mathrm{d}\bar{\mathbf{y}}^{i+1} \quad (20)$$

and $f(\bar{\mathbf{y}}^{i+1} \mid \bar{\mathbf{y}}^{(i)}, \mathbf{a}^{(i)})$ is given in Equation (12). The estimators of $\hat{\boldsymbol{\mu}}^{i+1}$ and $\boldsymbol{\Lambda}^{i+1}$ are iteratively obtained based on Equations (13) and (14) respectively, and they are substituted into Equation (20) for $\boldsymbol{\mu}^{i+1}$ and $\boldsymbol{\Lambda}^{i+1}$.

A backward induction method is used to solve this dynamic programming problem. The expected cost in Equation (20) can be calculated by using a Monte Carlo simulation to solve the integration. The accuracy of the solutions depends on the sampling size. However, the larger the sample size, the more time-consuming the computation. The function of Equation (19) is convex because all the components in Equation (19) are either positive definite or semi-positive definite.

In this paper, the modified standard Lipschitizian optimization algorithm (Jones *et al.*, 1993) is used to solve Equation (19), which finds the global minimum with a fast convergence. The Matlab platform provides this optimization function as "`gblSolve.m`". The implementation of the proposed methodology is summarized in two steps: (i) off-line construction of the adjustment or control tables using dynamic programming; and (ii) on-line estimation of the errors using the Bayesian estimation method and performing the adjustment on the process according to the adjustment strategy.

3.6. *Construction of control tables for optimal fixture adjustment*

This section discusses the generation of off-line control tables through state space discretization for the optimal fixture adjustment decision. It is important to notice that the parameters of $(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i)$ are in $R^P \oplus R^{P\times P}$, meaning they are situated in continuous and unbounded space.

Therefore, a discretization step is needed to map a practical, bounded discrete space with finite elements. If this bounded discrete space is denoted as F, i.e., $(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i) \in F$ with $F = F_{\boldsymbol{\mu}} \oplus F_{\boldsymbol{\Lambda}}$, then an approximate dynamic programming with backward induction can be used to determine $V_i(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i)$. For each stage i, $\boldsymbol{\Lambda}^i$ can be predetermined given $\boldsymbol{\Lambda}^1$, b, $\boldsymbol{\Gamma}$, $\boldsymbol{\Sigma}_\eta$ and $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$ (refer to Equation (14)); therefore, $F_{\boldsymbol{\Lambda}}$ is the set consisting of the predetermined $\boldsymbol{\Lambda}^i$ for each stage i. To find $F_{\boldsymbol{\mu}}$, the possible set of values of the elements of $\boldsymbol{\mu}^i$ is determined as $f_\mu^i = \{\mu^i | \mu^i = j \times \Delta_\mu, j = -b_\mu, -b_\mu + 1, \ldots, b_\mu + 1, b_\mu\}$ where b_μ is a positive integer, and Δ_μ is a small increment. $b_\mu \times \Delta_\mu$ is selected to cover the possible values of each element in $\boldsymbol{\mu}^i$. Therefore, $F_{\boldsymbol{\mu}}$ is a set of the vectors containing all possible combinations of the values from the set of f_μ^i. The parameter of $(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i)$ is then mapped to the discrete space F to the closest values of each element of $\boldsymbol{\mu}^i$ and $\boldsymbol{\Lambda}^i$. The following steps are used to construct the adjustment tables.

Step 1. Set the initial values used for a particular production or process. These values are $\boldsymbol{\mu}^1$, $\boldsymbol{\Lambda}^1$, b, $\boldsymbol{\Gamma}$, $\boldsymbol{\Sigma}_\eta$, $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$, c and n.

Step 2. Generate the discrete space F. The bounded value (b_μ) and increment (Δ_μ) are set based on the accuracy and computation cost.

Step 3. For each stage i, determine $V_i(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i)$ for $(\boldsymbol{\mu}^i, \boldsymbol{\Lambda}^i) \in F$ using Equations (18) to (20).

An example of a multi-station assembly process is used to illustrate the proposed methodology in the next section.

4. Case study on a multi-station assembly processes

The developed method was applied to a three-station assembly process ($N = 3$) (Ding *et al.*, 2002) as shown in Fig. 3. A two-dimensional case was considered, meaning each measurement m_i provides data on the deviations in the x and z directions. There are three fixture systems consisting of 18 potential locator position deviations formed by $\mathbf{u}_i$ at each station $i (i = 1, 2, 3)$ as $\mathbf{u}_1 = [\delta p_1 \cdots \delta p_6]^{\mathrm{T}}$, $\mathbf{u}_2 = [\delta p_7 \cdots \delta p_{15}]^{\mathrm{T}}$, $\mathbf{u}_3 = [\delta p_{16}\ \delta p_{17}\ \delta p_{18}]^{\mathrm{T}}$, where δp_i is the deviation associated with fault i.

Table 1. The coordinates of the nine sensors in Fig. 3

Sensor points	*Coordinates (x, z)*
m_1 (on station I)	(0, 350)
m_2 (on station I)	(1630, 1100)
m_3 (on station II)	(0, 900)
m_4 (on station II)	(1630, 1000)
m_5 (on station II)	(950, 0)
m_6 (on station III)	(950, 900)
m_7 (on station III)	(1630, 1100)
m_8 (on station III)	(2280, 1000)
m_9 (on station III)	(2280, 150)

There are nine sensors used to measure part dimension deviations. These sensors were located based on the diagnosability study of Zhou, Ding, Chen and Shi (2003) to ensure a fully diagnosable system, i.e., all means and variances of the fixture errors can be estimated. Observation vector $\mathbf{y}$ is the deviation from the target value and is expressed in each station as

$$\mathbf{y}_1 = [\delta m_1(x)\quad \delta m_1(z)\quad \delta m_2(x)\quad \delta m_2(z)]^{\mathrm{T}},$$
$$\mathbf{y}_2 = [\delta m_3(x)\quad \delta m_3(z)\quad \delta m_4(x)\quad \delta m_4(z)\ \delta m_5(x)\ \delta m_5(z)]^{\mathrm{T}},$$
$$\mathbf{y}_3 = [\delta m_6(x)\quad \delta m_6(z)\quad \delta m_7(x)\quad \delta m_7(z)\quad \delta m_8(x)\quad \delta m_8(z)\quad \delta m_9(x)\quad \delta m_9(z)]^{\mathrm{T}}.$$

The locations, or coordinates, of these nine sensors in Fig. 3 are provided in Table 1. For these 18 locator error components, this case study assumed that all fixture positions were initially calibrated without error ($\delta p = 0$), except for the following four components: (i) pin P_1 at station I in the z-direction (δp_2); (ii) pin P_4 at station I in the z-direction (δp_6); (iii) pin P_7 at station II in the x-direction (δp_{13}); and (iv) pin P_8 at station III in the z-direction (δp_{18}). In addition, only these four locator components are adjustable at the prespecified control times. In this case, the dimension of

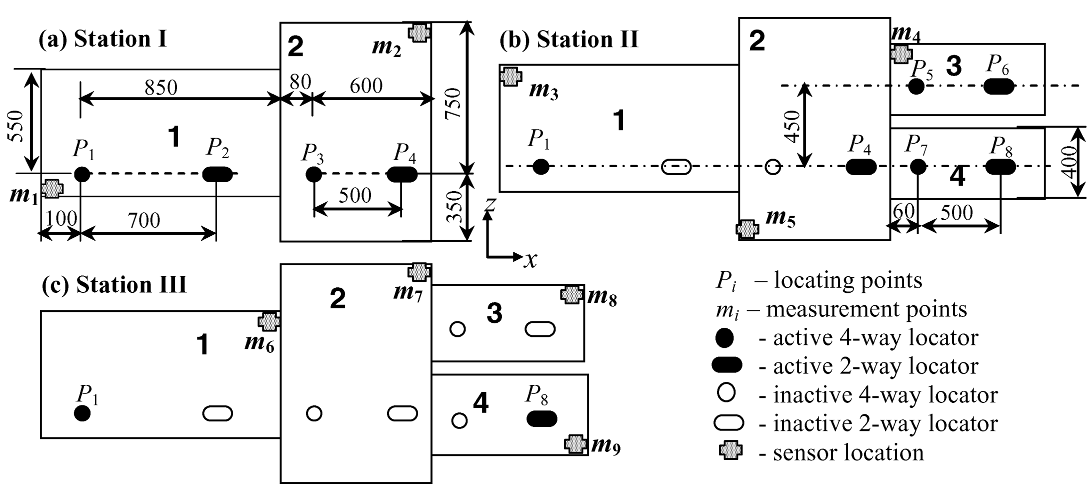

Fig. 3. Three-station assembly process (Ding *et al.*, 2002).

$\mathbf{u}^1$ is changed from a 18×1 vector to a 4×1 vector, and it is expressed as $\mathbf{u}^1 = [\delta p_2, \delta p_6, \delta p_{13}, \delta p_{18}]^{\mathrm{T}}$. As an example, the initial setup deviations of the fixture $\mathbf{u}^1 = [-1.5\ 2\ 1\ 0.5]^{\mathrm{T}}$ are used in the following simulation analysis.

The components of $\mathbf{a}^i$ were associated with these four components of the locator errors. With the assembly scheme in Fig. 3, matrices $\mathbf{A}_k$, $\mathbf{B}_k$, $\mathbf{C}_k$ and $\mathbf{\Gamma}$ for $k = 1, 2, 3$ were determined numerically (Jin and Shi, 1999) as shown in online Appendix B. The covariance matrices of sensor noise ($\mathbf{\Sigma_v}$), natural noise of the locators ($\mathbf{\Sigma_\eta}$) and process noise ($\mathbf{\Sigma_w}$) are given as follows:

$$\mathbf{\Sigma_v} = 0.01 \times \mathbf{I}_{18\times18}, \quad \mathbf{\Sigma_\eta} = 0.005 \times \mathbf{I}_{18\times18},$$
$$\mathbf{\Sigma_w} = 0.000\,01 \times \mathbf{I}_{36\times36},$$

where $\mathbf{I}_{r\times r}$ is an $r \times r$ identity matrix. Based on Equation (5), the covariance matrix of $\mathbf{\Sigma}_\varepsilon$ can be calculated as shown in online Appendix B. Assume an *a priori* distribution of $\mathbf{u}^1$ of:

$$\mathbf{u}^1 = \mathbf{O}_{4\times1} \quad \text{and} \quad \mathbf{\Lambda}^1 = 0.01 \times \mathbf{I}_{4\times4},$$

where $\mathbf{O}_{r\times s}$ is an $r \times s$ zero matrix. After all the initial values have been set up, the control tables are generated using Matlab and stored in Matlab cell arrays.

4.1. *Optimal adjustment and cost performance comparison*

The total cost using the proposed methodology was compared to the total cost when no adjustment policy exists. Suppose that the length of the production run has 20 control intervals ($n = 20$) and each interval produces five products ($b = 5$). A fixed adjustment cost of 25 ($c = 25$) is incurred whenever there is an adjustment regardless of the magnitude of the adjustment. With this set of parameters, the fixture adjustment methodology is applied. A program was developed with Matlab to solve the optimal fixture adjustment problem using the proposed method. The obtained optimal fixture adjustment strategy is

$$\mathbf{a}^1 = [1.4\ \ 0\ \ -0.9\ \ 0]^{\mathrm{T}}, \quad \mathbf{a}^2 = [0\ \ -0.8\ \ 0\ \ 0]^{\mathrm{T}},$$
$$\mathbf{a}^3 = [0\ \ -0.3\ \ 0\ \ 0]^{\mathrm{T}}, \quad \mathbf{a}^4 = [0\ \ -0.3\ \ 0\ \ 0]^{\mathrm{T}},$$
$$\mathbf{a}^5 = [0\ \ 0\ \ 0\ \ -0.3]^{\mathrm{T}}, \quad \mathbf{a}^6 = [0\ \ -0.2\ \ 0\ \ 0]^{\mathrm{T}},$$
$$\mathbf{a}^{14} = [0\ \ -0.4\ \ 0\ \ 0]^{\mathrm{T}}.$$

According to this strategy, there are seven adjustments for this production run. The first adjustment is performed on pin P_1 at station I in the z-direction and on pin P_7 at station II in the x-direction after control interval 1. The second, third and fourth adjustments are conducted on pin P_4 at station I in the z-direction after control intervals 2, 3 and 4, respectively. The fifth adjustment is made on pin P_8 at station III in the z-direction after control interval 5. Finally, pin P_4 at station I in the z-direction is adjusted after control intervals 6 and 14. There are no control adjustments on the other control intervals $i = 7, 8, \ldots, 13$, and after control interval 15. The total costs with and without the implementation of the fixture adjustment strategy is compared as shown in Fig. 4.

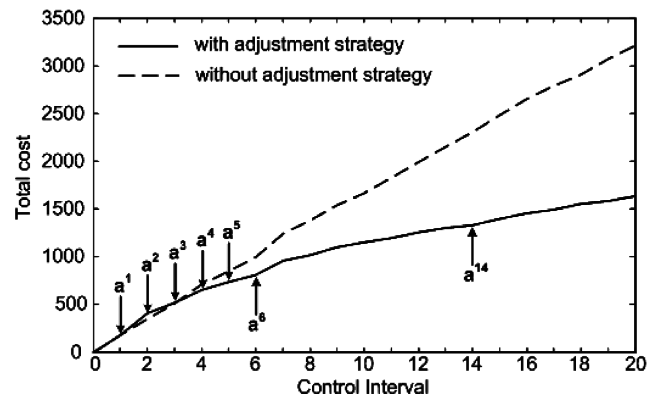

Fig. 4. Cost comparison: with and without adjustment strategy.

Figure 4 shows that a lower cost is obtained by implementing the proposed adjustment strategy compared with the case without any adjustment. In this case study, about 49.10% of the total cost can be saved by using the proposed optimal adjustment strategy, thus highlighting its significance in the total cost reduction.

An intuitive control strategy is that the adjustments to compensate for the initial locator errors can be completed in the early stage of the production run to reduce quality loss. As shown in Fig. 4, the adjustments of the fixture locators are made at every step in the first six control times. After that, no further adjustment is made until the 14th control time. The reason for this later adjustment is that the Bayesian estimation algorithm requires more samples to converge. Consequently, the estimation at steps 7 to 13 is not accurate enough to ensure a cost-effective adjustment by trading off the cost of quality saving and the adjustment cost, especially when the locator deviation magnitude is small after the six steps of adjustments.

Remark 1. The control actions may be taken at different control times. This is because not all locator deviations are adjusted at the beginning of the production due to the convergence of the estimation. The speed of convergence depends on the observation model, measurement noise, etc. Thus, the deviation estimation of some locators may converge faster than others. As an example in Fig. 4, there are some adjustments at the beginning of production (control intervals 1, 2, ..., 6) and later adjustment at control interval 14.

Remark 2. Some fixture locators still deviate from their nominal positions after all adjustments are performed. This is because their impact on quality loss is smaller than the adjustment cost for the remaining life of the production.

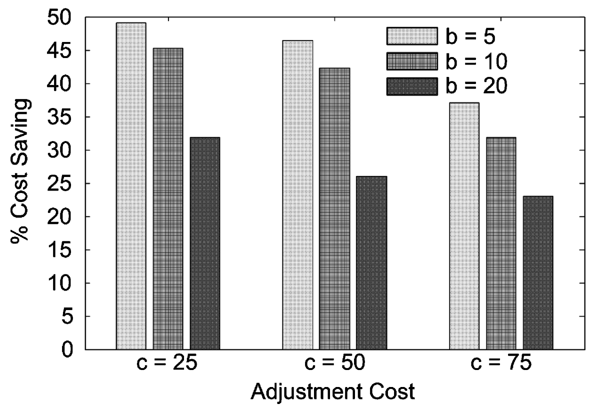

Fig. 5. Percentage of cost savings for different control intervals and adjustment costs.

4.2. *Cost performance comparison under different parameter settings*

This section aims to investigate how the cost of adjustment, as well as the control interval, impacts the effectiveness of the proposed methodology. A set of simulation studies was conducted to investigate the cost performance under different adjustment costs and control intervals.

It is given that 100 parts are to be produced for each production run and the initial process setup conditions are the same. Suppose three different control intervals are considered: $b = 5, 10$ and 20. Therefore, the n value corresponding to each control interval is 20, 10 and 5, respectively. Figure 5 shows the percentage of cost savings for different control intervals and adjustment costs after applying the optimal adjustment strategy.

In Fig. 5, it can be seen that when the control interval is increased from 5 to 20, the percentage of cost savings decreases regardless of the adjustment cost. That is, a smaller control interval will provide a higher cost benefit for all adjustment costs. Also, as the adjustment cost increases, the cost benefit from the adjustment strategy decreases. This conclusion is valid although the specific value of the cost benefit may be different depending on changes in the costs of adjustment and quality, as well as production run. From this case study, Fig. 5 provides some interpretation of a situation when the cost of adjustment is so high compared to the cost due to quality loss that it might not be worthwhile to adjust any locators. Therefore, the most cost-effective results can be expected only if the adjustment cost is low and the control interval is small. This conclusion is consistent with our intuition that small control intervals can provide more opportunities to conduct timely adjustments if the adjustment cost is not relatively high compared with the quality loss cost.

5. Summary and future work

In this paper, a new general methodology was developed under a dynamic programming framework to determine an optimal fixture adjustment strategy in a multi-station assembly process. In this study, a state space model was used to capture the variation propagation over multiple stations between the product quality and the fixture locator errors. The stochastic dynamic programming was formulated to determine the optimal fixture adjustment strategy by integrating Bayesian estimation of unknown fixture offset errors. The cost function was defined to include both quality loss and adjustment cost. In order to demonstrate the effectiveness of the proposed optimal control strategy, a state space discretation was implemented to obtain an approximate solution for the dynamic programming problem. Case studies were conducted to demonstrate that the proposed strategy can significantly reduce the total production cost, in which the control interval had an inverse relationship with the percentage of cost saving.

It should be noted that, due to the nature of dynamic programming, the proposed method might require intensive computations to determine an optimal adjustment strategy, which has the computation complexity of $O(2^n)$. However, this limitation can be relaxed considering the industrial practice of adjusting locator positions that are typically made at early control intervals in a production run. As a result, the number of parts in a production run may be truncated for a smaller n, which make it feasible to solve this dynamic programming problem.

Other future extensions of this research will be to relax some assumptions. For example, instead of assuming a perfect adjustment, one can deal with some adjustment errors that reflect the usual case in real practice. Some degradation of tools or processes may be included in the model as well, such that it becomes a maintenance problem. In addition, there exists some other approaches used for solving dynamic programming problems with an infinite horizon and finite state space, such as reinforcement learning and neurodynamic programming (Si *et al.*, 2004), which may provide less intensive computation. However, further investigation is needed to justify whether those methods can be applied or modified for future research.

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References

Åström, K.J. (1970) *Introduction to Stochastic Control Theory*, Academic Press, New York, NY.

Box, G.E.P. and Kramer, T. (1992) Statistical process monitoring and feedback adjustment—a discussion. *Technometrics*, **34**(3), 251–267.

Ceglarek, D. and Shi, J. (1995) Dimensional variation reduction for automotive body assembly. *Manufacturing Review*, **8**, 139–154.

Crowder, S.V. (1992) An SPC model for short production runs: minimizing expected cost. *Technometrics*, **34**(1), 64–73.

DeGroot, M.H. (1969) *Optimal Statistical Decisions*, McGraw-Hill, New York, NY.

Del Castillo, E., Pan, R. and Colosimo, B.M. (2003) A unifying view of some process adjustment methods. *Journal of Quality Technology*, **35**(3), 286–293.

Del Castillo, E. and Rajagopal, R. (2002) A multivariate double EWMA process adjustment scheme for drifting processes. *IIE Transactions*, **34**, 1055–1068.

Ding, Y., Shi, J. and Ceglarek, D. (2002) Diagnosability analysis of multistation manufacturing processes. *ASME Transactions, Journal of Dynamic Systems, Measurement, and Control*, **124**, 1–13.

Grubbs, F.E. (1983) An optimum procedure for setting machines or adjusting processes. *Journal of Quality Technology*, **15**, 186–189.

Jin, J. and Shi, J. (1999) State space modeling of sheet metal assembly for dimensional control. *ASME Transactions, Journal of Manufacturing Science and Technology*, **121**(4), 756–762.

Jones, D.R., Perttunen, C.D. and Stuckman, B.E. (1993) Lipschitzian optimization without the Lipschitz constant. *Journal of Optimization Theory and Applications*, **79**(1), 157–181.

Kackar, R.N. (1985) Off-line quality control, parameter design and the Taguchi method (with discussions). *Journal of Quality Technology*, **17**, 175–209.

Lian, Z. and Del Castillo, E. (2006) Setup adjustment under unknown process parameters and fixed adjustment cost. *Journal of Statistical Planning and Inference*, **136**(3), 1039–1060.

Luceño, A. (2003) Dead band adjustment charts with asymmetric off-target costs, deterministic process drift and delayed dynamics. *The Statistician*, **52**(4), 501–514.

Meinhold, R.J. and Singpurwalla, N.D. (1983) Understanding the Kalman filter. *The American Statistician*, **37**(2), 123–127.

Nair, V.N. (1992) Taguchi's parameter design: a panel discussion. *Technometrics*, **34**, 127–161.

Sachs, E., Hu, A. and Ingolfsson, A. (1995) Run by run process control: combining SPC and feedback control. *IEEE Transactions on Semiconductor Manufacturing*, **8**(1), 26–43.

Shi, J. (2006) *Stream of Variation Modeling and Analysis for Multistage Manufacturing Processes*, CRC Press, Boca Raton, FL.

Si, J., Barto, A.G., Powell, W.B. and Wunsch, D. (2004) *Handbook of Learning and Approximate Dynamic Programming*, Wiley, New York, NY.

Trietsch, T. (1998) The harmonic rule for process setup adjustment with quadratic loss. *Journal of Quality Technology*, **30**(1), 75–84.

Trietsch, T. (2000) Process setup adjustment with quadratic loss. *IIE Transactions*, **32**, 299–307.

Wang, H. and Huang, Q. (2007) Using error equivalence concept to automatically adjust discrete manufacturing processes for dimensional variation control. *ASME Transactions, Journal of Manufacturing Science and Technology*, **129**(2), 644–652.

Zhou, S., Ding, Y., Chen, Y. and Shi, J. (2003) Diagnosability study of multistage manufacturing process based on linear mixed-effects models. *Technometrics*, **45**(4), 312–325.

Zhou, S., Huang, Q. and Shi, J. (2003) State space modeling of dimensional variation propagation in multistage machining process using differential motion vector. *IEEE Transactions on Robotics and Automation*, **19**(2), 296–309.

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