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Optimization of Coding Gain for Full-Response CPM Space-Time Codes

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***Abstract*— Conditions are derived under which M-ary full-response CPM space-time codes will attain full spatial diversity and optimal coding gain. General code construction rules are desirable due to the nonlinearity and inherent memory of CPM signals which make manual design or computer search difficult. Optimization of the coding gain for CPM space-time codes is shown to depend on the CPM frequency/phase shaping pulse, modulation index, and codewords. The modulation indices and phase shaping functions that optimize the coding gain are specified. Finally, optimization of ST-CPM codewords is discussed.**

I. Introduction

Space-time (ST) coding transmits coded waveforms from multiple antennas to maximize link performance. Tarokh *et al.* [1] devised rank and determinant criteria for spatial diversity that optimizes the worst case pair-wise error probability (PWEP) and presented some simple design rules that guarantee full diversity for linear modulation schemes. In [2], the determinant criterion is strengthened by showing that in order to optimize the product distance one must optimize the Euclidean distance. Furthermore, in [3] is shown that different design criteria apply depending on the diversity order. When a reasonably large diversity order ($L_t \times L_r \geq 4$) is provided, the code performance is determined by the minimum squared Euclidean distance of the code. When the diversity order is small, the rank and determinant criteria will determine the code performance.

Continuous phase modulation (CPM) is a nonlinear modulation scheme with high bandwidth efficiency. Space-time CPM (ST-CPM) code design is more difficult than that for linear modulation due to the nonlinearity of the CPM modulator and its more complex performance matrices. General code construction rules are desirable due to the nonlinearity and inherent memory of CPM signals which make manual design or computer search difficult. Recently, Zhang and Fitz [4] derived a design criteria for ST-CPM and identified the rank criterion for particular CPM schemes. In [5], more general conditions are derived under which M-ary full-response CPM space-time codes attain full diversity. Some attempts to optimize coding gain of CPM space-time codes are made in [6] - [8]. In [6], a ST-CPM scheme for two transmit antennas is proposed, where different mapping rules are used over different antennas to achieve full diversity and optimal coding gain. An orthogonal ST-CPM system with two transmit antennas is proposed in [7]. In [8], the burst-based orthogonal ST-CPM system is proposed. However, a general determinant criterion for ST-CPM space-time codes has yet to be proposed.

The goal of this paper is to provide a general framework for ST-CPM coding gain optimization. Although a linear decomposition of CPM signals is used to identify a determinant criterion for M-ary full-response CPM signals, conclusions from this paper can be applied to any other ST-CPM configuration. It is shown that optimization of the coding gain for ST-CPM depends not only on the codewords as in linear modulation, but also on the frequency/phase shaping function. The modulation indices and frequency shaping functions that optimize the coding gain are specified. Finally, optimization of ST-CPM codewords is discussed.

The remainder of the paper is as follows. Section II describes the coded ST-CPM system on a quasi-static fading channel and reviews a linear decomposition for full-response CPM signals [9]. Section III presents a determinant criterion for M-ary full-response CPM signals. Section IV specifies frequency shaping pulses that optimize the ST-CPM coding gain. Section V discusses optimization of full-response ST-CPM codewords. Section VI presents simulation results which show that full spatial diversity and optimal coding gain are achieved for full-response ST-CPM systems that meet the rank and determinant criterion. Finally. Section VII concludes the paper.

II. ST-CPM System Model and Review of CPM Signals

This paper considers a ST-CPM system with L_t transmit antennas and L_r receive antennas. As shown in Fig. 1, K_b information symbols are input to a space-time (ST) encoder. The ST encoder uses the error control code $\mathcal{C}$ to encode information symbols into codeword vectors $\hat{\mathbf{c}} \in \mathcal{C}$ of length $N = N_c L_t$, and then maps these vectors onto an $L_t \times N_c$ matrix $\mathbf{C}$ in the following manner: codeword

$$\hat{\mathbf{c}} = \left(c_1^{(1)}, \ldots, c_{N_c}^{(1)}, \ldots c_1^{(L_t)}, \ldots, c_{N_c}^{(L_t)}\right) \quad (1)$$

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is mapped onto the $L_t \times N_c$ matrix

$$\mathbf{C} = \begin{bmatrix} c_1^{(1)} & \cdots & c_{N_c}^{(1)} \\ \vdots & \ddots & \vdots \\ c_1^{(L_t)} & \cdots & c_{N_c}^{(L_t)} \end{bmatrix}, \tag{2}$$

where $c_k^{(i)}$ is the code symbol assigned to i-th transmit antenna at time epoch k.

The outputs of the space-time encoder are L_t streams of symbols. Each stream of symbols after modulation mapping is input to a CPM modulator. The CPM modulated signals are simultaneously transmitted from L_t transmit antennas.

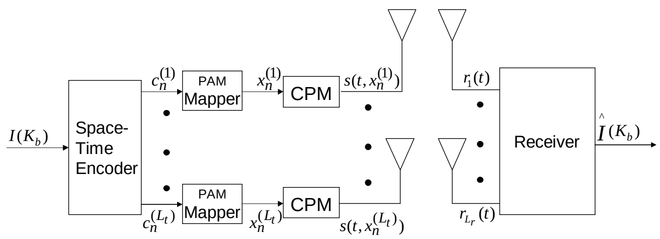

Fig. 1. Space-time coded CPM system.

The received signal at each receive antenna is a noisy superposition of the L_t transmitted signals, each affected by quasi-static flat Rayleigh fading, and independent zero-mean complex additive white Gaussian noise (AWGN). With these assumptions, the received signal can be represented as

$$\mathbf{r}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t), \tag{3}$$

where $\mathbf{s}(t) = [s_1(t), \ldots, s_{L_t}(t)]^\top$ is the vector of transmitted signals, $\mathbf{r}(t) = [r_1(t), \ldots, r_{L_r}(t)]^\top$ is the vector of received signals, $\mathbf{n}(t) = [n_1(t), \ldots, n_{L_r}(t)]^\top$ is the noise vector that contains independent zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension, and $\mathbf{H}(t) = [h_{ij}(t)]_{L_r \times L_t}$ is the matrix of complex channel fading gains.

The performance of a ST-CPM system has a direct analogy to the performance of ST coded linear modulation [4]. Consequently, the rank and the determinant criterion for space-time linear modulation are directly applicable to ST-CPM, the only difference being the "signal" matrix $\mathbf{C}_s$, i.e.,

$$\begin{bmatrix} \int_0^{N_cT_c} |\Delta s_1(t)|^2 \mathrm{d}t & \cdots & \int_0^{N_cT_c} \Delta s_1(t)\Delta s_{L_t}^*(t)\mathrm{d}t \\ \vdots & \vdots & \vdots \\ \int_0^{N_cT_c} \Delta s_{L_t}(t)\Delta s_1^*(t)\mathrm{d}t & \cdots & \int_0^{N_cT_c} |\Delta s_{L_t}(t)|^2 \mathrm{d}t \end{bmatrix}, \tag{4}$$

where $\Delta\mathbf{s}(t) = \mathbf{s}(t) - \hat{\mathbf{s}}(t)$ is the difference between CPM signals $\mathbf{s}(t)$ and $\hat{\mathbf{s}}(t)$, each corresponding to codeword $\mathbf{C}$ and $\hat{\mathbf{C}}$, respectively.

The CPM complex envelope with normalized amplitude can be represented as $s(t) = \exp(j\phi(t;\mathbf{x}))$, where $\phi(t;\mathbf{x}) = 2\pi h \sum_{k=0}^{N_c-1} x_{k+1}\beta(t - kT_c)$ is the excess phase, h is the modulation index, $\mathbf{x} = (x_0, \cdots, x_{N_c-1})$ is the information sequence with elements chosen from the M-ary alphabet $\{\pm1, \pm3, \cdots, \pm(M-1)\}$, and T_c is the symbol duration. The function $\beta(t)$ is the phase shaping pulse defined by $\beta(t) = \int_0^t q(\tau)d\tau$, and $q(t)$ is the frequency shaping pulse of length LT_c, such that $\beta(LT_c) = 1/2$. The CPM waveform has full-response when $L = 1$ and partial-response when $L > 1$.

Mengali and Morelli [9] showed that M-ary full-response CPM signals can be represented as

$$s(t;\mathbf{x}) = \sum_{k=0}^{R-1}\sum_{n=0}^{N_c-1} B_{k,n} g_k(t - nT_c), \tag{5}$$

where $R = M - 1 = 2^F - 1$. Functions $g_k(t)$ are equal to

$$g_k(t) = \prod_{l=0}^{F-1} s^{(l)}(t + e_{l,k}T_c), \tag{6}$$

where the $s^{(l)}(t)$ are defined as

$$s^{(l)}(t) = \begin{cases} \dfrac{\sin\left(2\pi h 2^l \beta(t)\right)}{\sin(\pi h 2^l)} & , \quad 0 \le t < T_c \\ s^{(l)}(2T_c - t) & , \quad T_c \le t \le 2T_c \\ 0 & , \quad \text{otherwise} \end{cases}. \tag{7}$$

Symbols $B_{k,n}$ are defined as

$$B_{k,n} = \exp\left(j2\pi h \sum_{l=0}^{F-1} 2^l A_{l,n-e_{l,k}} - j\pi h d_{k,n} \right), \tag{8}$$

where $A_{l,n-e_{l,k}} = \sum_{r=0}^{n-e_{l,k}} \gamma_{l,r}$, $d_{k,n} = \sum_{l=0}^{F-1} 2^l(n - e_{l,k} + 1)$, and $\gamma_{l,n} \in \{0,1\}$. are coefficients in the binary representation of the code symbol $c_n = (x_n - M + 1)/2$. Integers $e_{l,k}$ used in (6)-(8), are chosen to satisfy $e_{l,k} \in \{0,1\}$ and $\prod_{l=0}^{F-1} e_{l,k} = 0$.

III. Optimization of Coding Gain for ST-CPM

The inputs to the CPM modulators in Fig. 1 are elements from the $L_t \times N_c$ matrix $\mathbf{X} = [x_k^{(i)}]$, where $x_k^{(i)}$ is the PAM mapped symbol assigned to i-th transmit antenna at time epoch k. The outputs from the CPM modulators are signals $s(t;\mathbf{x}^{(i)})$ as defined in (5). Assume that $h = K/P$, where K and P are relatively prime integers. We define $v_{k,n}^{(i)}$ as

$$v_{k,n}^{(i)} = \left[\sum_{r=0}^{n-e_{l,k}} c_r^{(i)} \right]_{\text{mod P}}, \tag{9}$$

where $c_r^{(i)} = (x_r^{(i)} - M + 1)/2$ and $x_r^{(i)}$ is element from the matrix $\mathbf{X}$.

We also define the matrix of accumulative values as $\mathbf{V}_k \triangleq [v_{k,n}^{(i)}]$, denote $e^{\mathbf{V}_k} = [e^{j2\pi h v_k^{(i)}}]$, and denote $\Delta\mathbf{V}_k = e^{\mathbf{V}_k} - e^{\hat{\mathbf{V}}_k}$. Also, we define the diagonal matrix $\mathbf{D}_k \triangleq [e^{-j\pi h d_{k,n}}]$ with elements $d_{k,n} = \sum_{l=0}^{F-1} 2^l(n - e_{l,k} + 1)$. Finally, we define the vector $\mathbf{g}_k(t) = [g_{k,0}(t), g_{k,1}(t), \cdots, g_{k,N_c-1}(t)]^T$, where $g_{k,i}(t) \triangleq g_k(t - iT_c)$. From (6)-(9), the CPM difference signal matrix for two space-time codewords, $\mathbf{C}$ and $\hat{\mathbf{C}}$, can be written as

$$\Delta\mathbf{s}(t) = \sum_{k=0}^{R-1} \Delta\mathbf{V}_k \; \mathbf{D}_k \; \mathbf{g}_k(t). \tag{10}$$

In [4] is shown that the rank and determinant criteria are applicable to ST-CPM signal matrix $\mathbf{C}_s$. Once full diversity is guaranteed, the next objective is to maximize the coding gain, $\xi(\Delta\mathbf{s}(t))$, over all pairs of distinct codewords $\mathbf{C}$ and $\hat{\mathbf{C}}$. The coding gain is defined by geometric-mean of the nonzero eigenvalues of the matrix $\mathbf{C}_s$, i.e., $\xi(\Delta\mathbf{s}(t)) = (\prod_{i=1}^{L_t}\lambda_i)^{1/L_t} = |(\mathbf{C}_s)|^{1/L_t}$, where $|\cdot|$ denotes the determinant operation. Using (4) and (10), matrix $\mathbf{C}_s$ can be written as

$$\sum_{k=0}^{R-1}\sum_{l=0}^{R-1}\Delta\mathbf{V}_k\mathbf{D}_k\int_0^{N_cT_c}\mathbf{g}_k(t)\mathbf{g}_l^H(t)\ \mathrm{d}t\ \mathbf{D}_l^H\Delta{\mathbf{V}_l}^H, \quad (11)$$

where $(\cdot)^H$ denotes the Hermitian operation. Equation (11) shows that maximization of the coding gain for CPM modulated space-time codes is more difficult than for linearly modulated space-time codes, because coding gain is not only a function of codewords $\mathbf{C}$ and $\hat{\mathbf{C}}$, but also depends on selection of the functions $g_{k,i}(t)$ in vectors $\mathbf{g}_k(t)$. To further simplify (11) we collect all functions $g_{k,i}(t)$ in a vector $\mathbf{g}(t) = [g_{0,0}(t),\cdots,g_{R-1,N_c-1}(t)]^T$. Since components in the vector $\mathbf{g}(t)$ are linearly independent functions, we can apply Gram-Schmidt orthonormalization on the vector $\mathbf{g}(t)$ to obtain the orthonormal bases $\Psi(t) = [\psi_{0,0}(t),\cdots,\psi_{R-1,N_c-1}(t)]^T$. Then, each vector $\mathbf{g}_k(t) = [g_{k,0}(t),\cdots,g_{k,N_c-1}(t)]^T$ can be represented as $\mathbf{g}_k(t) = \mathbf{T}_k\Psi_k(t)$, where elements in the matrix $\mathbf{T}_k$ are $t_{k,i,j} =< g_{k,j}(t),\psi_{k,i}(t) >$ and $<\cdot>$ denotes the inner product operation. Functions in the vector $\Psi_k(t) = [\psi_{k,0}(t),\cdots,\psi_{k,N_c-1}(t)]^T$ are selected from the vector $\Psi(t)$ to belong to only one of the vectors $\Psi_k(t)$ for $0 \le k \le R-1$. Since all elements in $\Psi_k(t)$ are orthonormal to the elements in $\Psi_{l\neq k}(t)$, (11) can be simplified to $\mathbf{C}_s = \sum_{k=0}^{R-1}\Delta\mathbf{V}_k\mathbf{D}_k\mathbf{G}_k\mathbf{D}_k^H\Delta{\mathbf{V}_k}^H$, where $\mathbf{G}_k \triangleq \int_0^{N_cT_c}\mathbf{g}_k(t)\mathbf{g}_k^H(t)\ \mathrm{d}t$.

Theorem 1: (**A Determinant Criterion for ST-CPM**) Suppose matrices $\mathbf{G}_k$ are designed to be semi-identity matrices, i.e., $\mathbf{G}_k = Q_k\mathbf{I}_{N_c}$, where $\mathbf{I}_{N_c}$ is the $N_c\times N_c$ identity matrix, N_c is the rank of the matrices $\mathbf{G}_k$, and Q_k is some constant. Then the coding gain $\xi(\Delta\mathbf{s}(t))$ is maximized if the matrices $\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H$ are designed to be semi-identity matrices, i.e., $\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H = H_k\mathbf{I}_{L_t}$, with maximized constants $H_k = \mathrm{tr}(\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H)/L_t$, where $\mathrm{tr}(\cdot)$ denotes the trace operation.

Proof: In [2] is shown that the determinant of a positive definite Hermitian matrix is maximized if the matrix is designed as a semi-identity matrix with maximized trace. Since $\mathbf{C}_s$ is a positive definite Hermitian matrix, this theorem applies. When matrices $\mathbf{G}_k$ are designed as semi-identity matrices, the influence of the phase shaping functions on the coding gain is optimized and $|(\mathbf{C}_s)|$ can be written as

$$|\mathbf{C}_s| = \left|\sum_{k=0}^{R-1}Q_k\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H\right|. \quad (12)$$

Using Hadamard's inequality [10], (12) becomes

$$|\mathbf{C}_s| = \left|\sum_{k=0}^{R-1}\mathbf{A}_k\right| \le \prod_{i=1}^{L_t}\sum_{k=0}^{R-1}a_{iik}, \quad (13)$$

where $\mathbf{A}_k = Q_k\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H$ and a_{iik} are diagonal elements in $\mathbf{A}_k$. The equality holds only if matrices $\mathbf{A}_k$ are diagonal. Further maximization of $|\mathbf{C}_s|$ is possible by choosing elements a_{iik} to maximize $\prod_{i=1}^{L_t}\sum_{k=0}^{R-1}a_{iik}$. Using the arithmetic-geometric mean inequality, (13) becomes

$$|\mathbf{C}_s|_{max}^{1/L_t} = \left(\prod_{i=1}^{L_t}\sum_{k=0}^{R-1}a_{iik}\right)^{1/L_t} \le \frac{1}{L_t}\sum_{k=0}^{R-1}\mathrm{tr}(\mathbf{A}_k). \quad (14)$$

From (13) and (14), it follows that the determinant $|\mathbf{C}_s|$ is maximized if the matrices $\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H$ are semi-identity i.e., $H_k\mathbf{I}_{L_t}$ and constants $H_k = \mathrm{tr}(\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H)/L_t$ are maximized, what was our claim. ⊡

In the following sections, we will investigate conditions under which the matrices $\mathbf{G}_k$ and $\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H$ can be constructed as semi-identity matrices.

IV. Optimization of Coding Gain Through Phase Shaping Pulses

Theorem 1 shows that the influence of the phase shaping functions and the codewords on the coding gain can be analyzed separately if the matrices $\mathbf{G}_k$ are constructed as semi-identity matrices. Here, we investigate when the semi-identity condition is satisfied for full-response CPM signals.

The matrix $\mathbf{G}_k$ is equal to

$$\mathbf{G}_k = \quad (15)$$
$$\begin{bmatrix} \int_0^{2T_c}|g_{k,0}(t)|^2\mathrm{d}t & \cdots & 0 \\ \int_{T_c}^{2T_c}g_{k,1}(t)g_{k,0}^*(t)\mathrm{d}t & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \int_{(N_c-1)T_c}^{(N_c+1)T_c}|g_{k,N_c-1}(t)|^2\mathrm{d}t \end{bmatrix},$$

where $(\cdot)^*$ denotes the complex conjugate operation and functions $g_{k,i}(t)$ are defined as in (6). First, we observe that all diagonal elements are always equal, i.e. $\int_0^{2T_c}|g_k(t)|^2\mathrm{d}t = \int_{T_c}^{3T_c}|g_k(t-T_c)|^2\mathrm{d}t = \cdots = \int_{(N_c-1)T_c}^{(N_c+1)T_c}|g_k(t-(N_c-1)T_c)|^2\mathrm{d}t$. Hence, the matrix $\mathbf{G}_k$ will be semi-identity if all integrals $I_{i,i+1} = \int_{(i+1)T_c}^{(i+2)T_c}g_{k,i}(t)g_{k,i+1}^*(t)\mathrm{d}t$ and $I_{i+1,i} = \int_{(i+1)T_c}^{(i+2)T_c}g_{k,i+1}(t)g_{k,i}^*(t)\mathrm{d}t$ are equal to zero. We start evaluation of these integrals, by evaluating the products

$$g_{k,i}(t)g_{k,i+1}^*(t) = \prod_{l=0}^{F-1}s^{(l)}(t-(i-e_{l,k})T_c)\, s^{(l)}(t-(i+1-e_{l,k})T_c)^* \quad (16)$$

for $k\in\{1,\ldots,R-1\}$. Since integers $e_{l,k}$ are chosen to satisfy $e_{l,k}\in\{0,1\}$ and $\prod_{l=0}^{F-1}e_{l,k} = 0$, when $k\in\{1,\ldots,R-1\}$, at least one integer $e_{l,k}$ is different form zero, and at least one integer $e_{m\neq l,k}$ is equal to zero. Without loss of generality, we can assume that $e_{0,k} = 0$ for $l = 0$. Then, equation (16) can be modified to

$$g_{k,i}(t)g_{k,i+1}^*(t) = s^{(0)}(t-iT_c)s^{(0)}(t-(i+1)T_c)^* \quad (17)$$
$$\prod_{l=1}^{F-1}s^{(l)}(t-(i-e_{l,k})T_c)\, s^{(l)}(t-(i+1-e_{l,k})T_c)^*.$$

Note that the function $s^{(0)}(t-(i+1)T_c)$ and at least one of the functions $s^{(l)}(t-(i-e_{l\neq 0,k})T_c)$ are different from zero in the time intervals $(i+1)T_c \le t \le (i+3)T_c$ and $(i-e_{l,k})T_c \le t \le (i+2-e_{l,k})T_c$, respectively. Since at least one $e_{l\neq 0,k} > 0$, these two functions cannot be different from zero at the same time. Hence, we can conclude that the product $g_{k,i}(t)g^*_{k,i+1}(t)$ is always equal to zero. A similar argument can be used to show that $g^*_{k,i}(t)g_{k,i+1}(t) = 0$. Hence, for an arbitrary phase shaping function, matrices $\mathbf{G}_k$, $k \in \{1,\ldots,R-1\}$, are semi-identity matrices.

Previous reasoning cannot be applied to matrix $\mathbf{G}_0$, because all integers $e_{l,k}$ are equal to zero for $k=0$. Hence, we need to evaluate integrals $I_{i,i+1} = \int_{(i+1)T_c}^{(i+2)T_c} g_{0,i}(t)g^*_{0,i+1}(t)dt$ for particular phase shaping functions. The following lemma shows that for commonly used phase shaping functions, integrals $I_{i,i+1}$ and $I_{i+1,i}$ can be equal to zero.

Lemma 2: For the phase shaping functions [11]

$$\beta(t) = \begin{cases} \frac{bt}{2aT_c} & , \quad 0 \le t < aT_c \\ \frac{ta(1-2b)}{2T_c(1-2a)} + \frac{b}{2} & , \quad aT_c \le t \le (1-a)T_c \\ \frac{b}{2a}\left(\frac{t}{T_c}-1\right)+\frac{1}{2} & , \quad (1-a)T_c \le t \le T_c \\ \frac{1}{2} & , \quad T_c \le t \end{cases} , \quad (18)$$

with adequate selection of parameters a and b, and for the raised cosine (1RC) function with the modulation indices $h = 1/2^n$, where $1 \le n \le F-1$, the integrals $I_{i,i+1} \approx 0$ and $I_{i+1,i} \approx 0$.

The proof of this lemma is omitted for brevity.

Remark 1: If the rectangular pulse (1REC) is selected as the frequency shaping function $q(t)$ (i.e., $\beta(t)$ has $a=1$ and $b=1$) the integrals $I_{i,i+1}$ and $I_{i+1,i}$ are not always zero. However, for minimum-shift keying (MSK) modulation (1REC with $h=1/2$), $I_{i,i+1} = I_{i+1,i} = 0$, because MSK has orthogonal carriers with minimum frequency separation.

V. Optimization of Coding Gain Through Codewords

Theorem 1 shows that if the $\mathbf{G}_k$ have semi-identity form, then the ST-CPM coding gain simplifies to

$$\xi(\Delta\mathbf{s}(t)) = |\mathbf{C}_s|^{1/L_t} = \left|\sum_{k=0}^{R-1} Q_k \Delta\mathbf{V}_k \Delta\mathbf{V}_k{}^H\right|^{1/L_t}, \quad (19)$$

where $Q_k = \int_0^{2T_c} |g_k(t)|^2 \mathrm{d}t$. While it is difficult in general to maximize the exact coding gain $\xi(\Delta\mathbf{s}(t))$, it is possible to maximize the trace upper bound $|\mathbf{C}_s|^{1/L_t} \le \mathrm{tr}(\mathbf{C}_s)/L_t$ on the coding gain. Furthermore, in [3] is shown that for systems with $L_tL_r \ge 4$, maximization of trace $\mathrm{tr}(\mathbf{C}_s)$ is a sufficient condition for coding gain optimization. The trace $\mathrm{tr}(\mathbf{C}_s)$ is equal to

$$\mathrm{tr}(\mathbf{C}_s) = \sum_{k=0}^{R-1} Q_k d^2_{E_k}\left(e^{f_{k,P}(\mathbf{C})}, e^{f_{k,P}(\hat{\mathbf{C}})}\right) = \quad (20)$$
$$\sum_{k=0}^{R-1} Q_k \sum_{i=1}^{L_t}\sum_{n=0}^{N_c-1} 4\sin^2\left(\pi h\left(f_{k,P}\left(c_n^{(i)}\right) - f_{k,P}\left(\hat{c}_{n,}^{(i)}\right)\right)\right),$$

where $c_n^{(i)}$ are code symbols, $d^2_{E_k}(e^{\mathbf{V}_k}, e^{\hat{\mathbf{V}}_k})$ is the squared Euclidean distance between the codewords $\mathbf{C}$ and $\hat{\mathbf{C}}$, $h = K/P$, $P = 2^H$ for $1 \le H \le F-1$, and functions $f_{k,P}(\ \cdot\ )$ are defined as

$$f_{k,P}(\mathbf{C}) = \mathbf{V}_k = \left[f_{k,P}\left(c_n^{(i)}\right)\right] = \left[\left(\sum_{r=0}^{n-e_{j,l}^{(k)}} c_r^{(i)}\right)_{\mathrm{mod\,P}}\right]. \quad (21)$$

From (20) follows that the trace of the matrix $\mathbf{C}_s$ over all pairs of codewords $\mathbf{C} \ne \hat{\mathbf{C}} \in \mathcal{C}$ will be maximized if the squared minimum Euclidean distances $d^2_{E_k\min} = \min\{d^2_{E_k}(e^{f_{k,P}(\mathbf{C})}, e^{f_{k,P}(\hat{\mathbf{C}})}) : \mathbf{C} \ne \hat{\mathbf{C}} \in \mathcal{C}\}$ for $k \in \{0,\ldots,R-1\}$ are maximized.

VI. Simulation Results

In this section we present some simulation results to verify proposed coding gain design criterion. All simulations use a frame length of 130 with $L_r = 1$ and a full-response raised cosine (1RC) frequency shaping function. Each spatial channel is modelled as being independently Rayleigh faded.

The first example uses the space-time codewords from a (4×4) 8-ary CPM space-time code $\mathcal{C}$. Following the rank design criterion in [5], the codewords $\mathbf{C} \in \mathcal{C}$ are constructed as $\mathbf{C} = f^{-1}_{0,8}(\sum_{l=0}^{2} 2^l \mathbf{C}_l)$, where $f^{-1}_{0,8}(\ \cdot\ )$ denotes the inverse of the function $f_{0,8}(\ \cdot\ )$ defined in (21) and $\mathbf{C}_l$ are binary codewords from linear (4×4) space-time codes $\mathcal{C}_l$ described in [12] or [13]. All codewords $\mathbf{C}_0 \in \mathcal{C}_0$ have full rank over $\mathbb{F}$ and all codewords $\mathbf{C}_0$ satisfy $\mathbf{C}_0 \ne \hat{\mathbf{C}}_0$. When the binary space-time code in [12] with the generator matrix $G = [1 \quad \alpha \quad \alpha^2 \quad \alpha^3]$ is used, where α is a zero of the primitive polynomial $f(x) = x^4+x+1$ over $\mathbb{F}$, the minimum trace upper bound of ST-CPM code is $\sum_{k=0}^{6}(Q_k/4)d^2_{E_k\min} \approx 3.66$. On the other hand, when the binary space-time code in [13] is used, the minimum trace upper bound of ST-CPM code is $\sum_{k=0}^{6}(Q_k/4)d^2_{E_k\min} \approx 7.31$.

Fig. 2 compares the performance curves obtained for full-response 8-ary CPM signals with 1RC frequency shaping function, $h = 1/4$ and $L_t = \{1,2,3,4\}$ transmit antennas. The codewords used to obtain the second, third, and fourth curve are constructed using the binary space-time code in [12]. The fifth curve is obtained using the the binary space-time code proposed in [13]. Fig. 2 shows that full diversity and coding gain improvement are achieved when the space-time codes meet both the rank and coding gain design criteria.

The second example uses the space-time codewords from a (2×2) 8-ary CPM space-time code $\mathcal{C}$, where codewords $\mathbf{C} \in \mathcal{C}$ are constructed as $\mathbf{U} = f^{-1}_{0,8}(\sum_{l=0}^{2} 2^l \mathbf{C}_l)$, where $f^{-1}_{0,8}(\ \cdot\ )$ is defined as above and $\mathbf{C}_l$ are binary codewords from linear (2×2) space-time codes $\mathcal{C}_l$ described in [12] with the generator matrix $G = [1 \quad \alpha]$, where α is a zero of the primitive polynomial $f(x) = x^2+x+1$ over $\mathbb{F}$. All codewords $\mathbf{C}_0 \in \mathcal{C}_0$ have full rank over $\mathbb{F}$ and all codewords $\mathbf{C}_0$ satisfy $\mathbf{C}_0 \ne \hat{\mathbf{C}}_0$. The performance curves for this 8-ary, 1RC, $h = 1/4$, CPM space-time code with $L_t = \{1,2\}$ transmit antennas are plotted in Fig. 3. Results shows that full diversity is obtained using this space-time code. The minimum trace upper bound of this ST-CPM code is $\sum_{k=0}^{6}(Q_k/2)d^2_{E_k\min} \approx 3.66$. The coding gain can be improved if the codewords are constructed using the Alamouti's code [14]. Code elements $v^{(1)}_{1,k}$ and $v^{(1)}_{2,k}$ should be selected to maximize $\left|v^{(1)}_{1,k}\right|^2 + \left|v^{(1)}_{2,k}\right|^2$. The other two code elements $v^{(2)}_{1,k}$ and $v^{(2)}_{2,k}$ should be chosen to satisfy $v^{(2)}_{1,k} = (1/(2h) - v^{(1)}_{2,k})_{\mathrm{mod\,P}}$ and $v^{(2)}_{2,k} = (1/h - v^{(1)}_{1,k})_{\mathrm{mod\,P}}$, where the modulation index is $h = K/P$. The minimum trace upper bound of this ST-CPM code is $\sum_{k=0}^{6}(Q_k/2)d^2_{E_k\min} \approx 7.31$. We refer to this design as orthogonal space-time full-response CPM (OST-FCPM) design. Further enhancement of the coding gain can be obtained if the codewords are constructed using the super-orthogonal 2-state trellis space-time code in [15]. The minimum trace upper bound of this ST-CPM code is $\sum_{k=0}^{6}(Q_k/2)d^2_{E_k\min} \approx 9.78$. We refer to this design as super-orthogonal space-time full-response CPM (SOST-FCPM) design. Fig. 3 compares our OST-FCPM and SOST-FCPM designs with OST-FCPM designs [7], [8] and ST-FCPM design with mapping scheme [6], respectively. Results show that our SOST-FCPM design has similar performance as ST-FCPM with mapping scheme [6] and performs better than the OST-FCPM designs [7], [8] and our OST-FCPM design.

VII. Conclusion

By using a linear decomposition of CPM signals, we have identified the determinant criterion for M-ary full-response CPM signals. Optimization of the coding gain for CPM space-time codes is shown to depend on the CPM frequency/phase shaping pulse, modulation index, and codewords. The modulation indices and phase shaping functions that optimize the coding gain are specified and optimization of ST-CPM codewords is discussed. Simulation results show that full spatial diversity and optimal coding gain are achieved for ST-CPM systems that meet the rank and determinant criteria.

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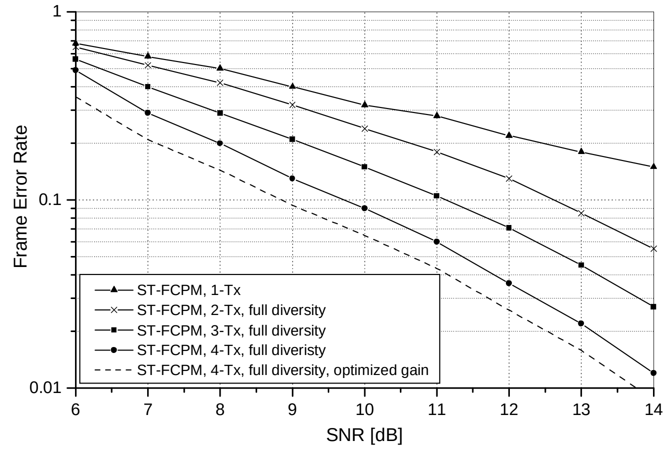

Fig. 2. Frame-error rate of 8-ary ST-CPM with 1RC and $h = 1/4$ in quasi-static fading.

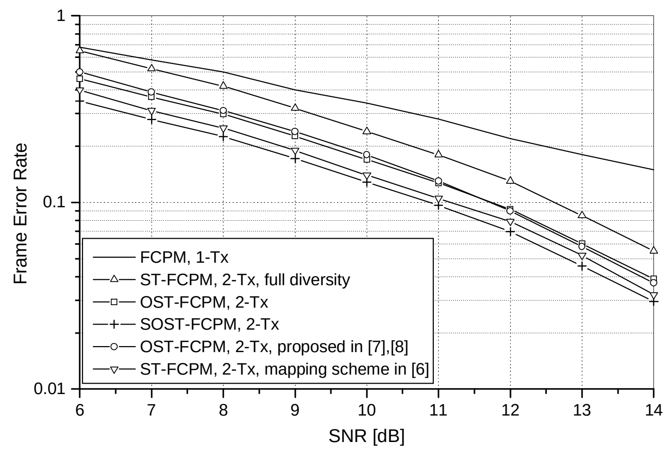

Fig. 3. Frame-error rate of (2×2) 8-ary ST-CPM with 1RC and $h = 1/4$ in quasi-static fading.

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