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A Space-Time Code Design for Partial-Response CPM: Diversity Order and Coding Gain

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***Abstract*— Using a linear decomposition of continuous phase modulated (CPM) signals with tilted-phase, sufficient conditions are derived under which M-ary partial-response CPM space-time codes will attain both full spatial diversity and optimal coding gain. A rank criterion for M-ary partial-response CPM that specifies the set of allowable modulation indices is identified. Furthermore, optimization of the coding gain for CPM space-time codes is shown to depend on the CPM frequency/phase shaping pulse, modulation index, and codewords. The modulation indices and phase shaping functions that optimize the coding gain are specified. Finally, optimization of CPM space-time codewords is discussed.**

I. Introduction

Space-time (ST) coding transmits coded waveforms from multiple antennas to maximize link performance. *Full spatial diversity* is one design objective for ST codes, being upper bounded by the product $L_t \times L_r$, where L_t and L_r are the number of transmit and receive antennas, respectively. *Coding gain* optimization is another design objective for space-time codes. Tarokh *et al.* [1] devised the rank and determinant criteria for spatial diversity that optimizes the worst case pair-wise error probability (PWEP) and presented some simple design rules that guarantee full spatial diversity for linear modulation schemes. In [2], the determinant criterion is strengthened by showing that the Euclidean distance must be optimized in order to optimize the product distance. Finally, in [3] is shown that different design criteria apply depending on the diversity order. For sufficient degrees of freedom ($L_t \times L_r \geq 4$), the code performance is governed by the minimum squared Euclidean distance of the code. For small $L_t \times L_r$, the rank and determinant criteria will govern the code performance.

One of main difficulties encountered when deriving general design rules for linearly modulated ST codes is that the diversity and coding design criteria apply to the complex domain of baseband-modulated waveforms, whereas block code designs are usually carried out over finite fields. Space-time continuous phase modulated (ST-CPM) code design is even more difficult due to the nonlinearity of the CPM modulator and its more complex performance matrices. Zhang and Fitz [4] derived design criteria for ST-CPM on quasi-static fading channels and identified a rank criterion for particular CPM schemes. In [5], more general rank criterion for M-ary full-response ST-CPM codes is proposed.

Some attempts to optimize coding gain of ST-CPM have been made in [6]–[8]. In [6], a ST-CPM scheme is proposed with $L_t = 2$ that uses different mapping rules on the two antennas to achieve full diversity and maximal coding gain. An orthogonal and a burst-based orthogonal ST-CPM system are proposed in [7] and [8], respectively. General conditions for coding gain optimization of M-ary full-response ST-CPM are derived in [9]. However, a general framework for partial-response ST-CPM is still lacking.

This paper derives sufficient conditions under which M-ary partial-response ST-CPM codes attain both full spatial diversity and optimal coding gain. Paralleling the work of Mengali and Morelli [10], we first derive a linear decomposition of CPM signals with tilted-phase. The tilted-phase is time-invariant and significantly simplify the receiver processing [11]. However, we must stress that this paper is not about achieving the lowest complexity, but rather to provide a general framework for ST-CPM. Also, we would like to point out that although a linear decomposition of CPM signals is used to identify rank and determinant criteria for M-ary partial-response CPM signals, conclusions from this paper can be applied to any other ST-CPM configuration. Based on our linear decomposition, we propose a rank criterion for M-ary partial-response CPM that defines a set of optimal modulation indices for partial-response CPM schemas. Then, we show that optimization of the coding gain for ST-CPM depends not only on the codewords, as in linear modulation, but also on the frequency shaping function and modulation index. Finally, the modulation indices and phase shaping functions that optimize the coding gain are specified and the optimization of ST-CPM codewords is discussed.

The remainder of the paper is as follows. Section II describes ST-CPM on a quasi-static fading channel. Section III derives the linear decomposition of CPM signals with tilted-phase. Section IV presents our design criteria for M-ary partial-response ST-CPM. Section V presents simulation results verifying the developed ST-CPM rank and coding gain design optimization criteria. Section VI concludes the paper.

II. ST-CPM System Model

This paper considers a ST-CPM system with L_t transmit antennas and L_r receive antennas. As shown in Fig. 1, the K_b

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information symbols are input to a space-time (ST) encoder. The ST encoder uses the error control code $\mathcal{C}$ to encode information symbols into codeword vectors $\hat{\mathbf{u}} \in \mathcal{C}$ of length $N = N_c L_t$, and then maps these vectors onto an $L_t \times N_c$ matrix $\mathbf{U}$ in the following manner: codeword

$$\mathbf{u} \quad = \quad \left(u_1^{(1)}, \ldots, u_{N_c}^{(1)}, \ldots u_1^{(L_t)}, \ldots, u_{N_c}^{(L_t)}\right) \qquad (1)$$

is mapped onto the $L_t \times N_c$ matrix

$$\mathbf{U} = \begin{bmatrix} u_1^{(1)} & \cdots & u_{N_c}^{(1)} \\ \vdots & \ddots & \vdots \\ u_1^{(L_t)} & \cdots & u_{N_c}^{(L_t)} \end{bmatrix}, \qquad (2)$$

where $u_k^{(i)}$ is the code symbol assigned to i-th transmit antenna at time epoch k.

The outputs of the space-time encoder are L_t streams of symbols. Each stream of symbols is input to a tilted-phase CPM modulator. The CPM modulated signals are simultaneously transmitted from L_t transmit antennas.

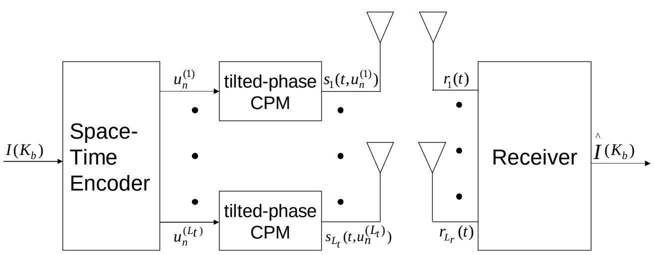

Fig. 1. Space-time coded CPM system.

The received signal at each receive antenna is a noisy superposition of the L_t transmitted signals, each affected by quasi-static flat Rayleigh fading, and independent zero-mean complex additive white Gaussian noise (AWGN). With these assumptions, the received signal can be represented as

$$\mathbf{r}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t), \qquad (3)$$

where $\mathbf{s}(t) = [s_1(t), \ldots, s_{L_t}(t)]^\top$ is the vector of transmitted signals, $\mathbf{r}(t) = [r_1(t), \ldots, r_{L_r}(t)]^\top$ is the vector of received signals, $\mathbf{n}(t) = [n_1(t), \ldots, n_{L_r}(t)]^\top$ is the noise vector that contains independent zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension, and $\mathbf{H}(t) = [h_{ij}(t)]_{L_r \times L_t}$ is the matrix of complex channel fading gains.

The performance of a ST-CPM system has a direct analogy to the performance of ST coded linear modulation [4]. Consequently, the rank and the determinant criterion for space-time linear modulation are directly applicable to ST-CPM, the only difference being the "signal" matrix $\mathbf{U}_s$, i.e.,

$$\begin{bmatrix} \int_0^{N_c T_c} |\Delta s_1(t)|^2 \mathrm{d}t & \cdots & \int_0^{N_c T_c} \Delta s_1(t) \Delta s_{L_t}^*(t) \mathrm{d}t \\ \vdots & \vdots & \vdots \\ \int_0^{N_c T_c} \Delta s_{L_t}(t) \Delta s_1^*(t) \mathrm{d}t & \cdots & \int_0^{N_c T_c} |\Delta s_{L_t}(t)|^2 \mathrm{d}t \end{bmatrix}, \qquad (4)$$

where $\Delta \mathbf{s}(t) = \mathbf{s}(t) - \hat{\mathbf{s}}(t)$ is the difference between CPM signals $\mathbf{s}(t)$ and $\hat{\mathbf{s}}(t)$, each corresponding to codewords $\mathbf{U}$ and $\hat{\mathbf{U}}$, respectively.

III. Linear Decomposition of CPM with Tilted Phase

Laurent [12] showed that a binary CPM signal can be decomposed into a linear combination of pulse amplitude modulated (PAM) waveforms. Mengali and Morelli [10] extended Laurent's CPM signal decomposition to M-ary CPM signals. These linear decomposition approaches [12], [10] use the CPM excess phase, but a decomposition based on the time-invariant CPM tilted-phase would be desirable.

The CPM tilted-phase complex envelope is [11] $s(t, \mathbf{u}) = \sqrt{2E_c/T_c} e^{j\psi(t,\mathbf{u})}$, $nT_c \le t \le (n+1)T_c$, where

$$\begin{aligned} \psi(t; \mathbf{u}) \quad &= \quad 2\pi h \sum_{k=0}^{n-L} u_k + 4\pi h \sum_{i=0}^{L-1} u_{n-i} \beta(t - (n-i)T_c) \\ &+ \quad \pi h W(t - nT_c), \end{aligned} \qquad (5)$$

is the tilted-phase, $\mathbf{u} = (u_0, \ldots, u_{N_c-1})$ is the information sequence with elements chosen from the M-ary alphabet $\{0, 1, \cdots, M-1\}$, h is the modulation index, T_c is the symbol duration, and E_c is the symbol energy. The term $W(t)$ in (5) is $W(t) = (M-1)[t/T_c + (L-1) - 2\sum_{i=0}^{L-1} \beta(t + iT_c)]$. The phase shaping pulse $\beta(t)$ is defined by $\beta(t) = \int_0^t q(\tau) d\tau$, where $q(t)$ is the frequency shaping pulse of length LT_c such that $q(LT_c) = 1/2$ for $t \ge LT_c$. The CPM waveform has full-response when $L = 1$ and partial-response when $L > 1$.

To derive the CPM tilted-phase decomposition, first note that some integer F exists such that $2^{F-1} < M \le 2^F$. Since u_k varies in the range $0 \le u_k \le (M-1) \le 2^F - 1$, u_k has the radix-2 representation $u_k = \sum_{l=0}^{F-1} \gamma_{k,l} 2^l$, where $\gamma_{k,l} \in \{0, 1\}$. Hence, the tilted phase in (5) can be rewritten as

$$\begin{aligned} &\psi(t, \mathbf{u}) = \pi h W_1(t - nT_c) + \qquad (6) \\ &\sum_{l=0}^{F-1} 2\pi h 2^l \left(\sum_{k=0}^{n-L} \gamma_{k,l} + 2 \sum_{i=0}^{L-1} (\gamma_{n-i,l} - 0.5) \beta(t - (n-i)T_c) \right), \end{aligned}$$

where $W_1(t) = (M-1)t/T_c + (M-1)(L-1)$. Then, the CPM tilted phase complex envelope has the form

$$\begin{aligned} &s(t, \mathbf{u}) = \sqrt{\frac{2E_c}{T_c}} e^{j\pi h W_1(t - nT_c)} \qquad (7) \\ &\prod_{l=0}^{F-1} e^{j\left(2\pi h 2^l \sum_{k=0}^{n-L} \gamma_{k,l} + 4\pi h 2^l \sum_{i=0}^{L-1} (\gamma_{n-i,l} - 0.5) \beta(t - (n-i)T_c)\right)}. \end{aligned}$$

The next step replaces the partial-response term associated with the $(n-i)$-th bit in (7) by an equivalent sum of two terms, such that only the second term depends upon the $(n-i)$-th bit. The partial-response term in (7) associated with the l-th bit of symbol u_k satisfies

$$\begin{aligned} &e^{j4\pi h 2^l (\gamma_l - 0.5)\beta(t)} = \qquad (8) \\ &\frac{\sin\left(\pi h 2^l - 2\pi h 2^l \beta(t)\right)}{\sin\left(\pi h 2^l\right)} + e^{j2\pi h 2^l (\gamma_l - 0.5)} \frac{\sin\left(2\pi h 2^l \beta(t)\right)}{\sin\left(\pi h 2^l\right)}. \end{aligned}$$

Define signals $s^{(l)}(t)$ as

$$\begin{cases} \exp\left(-j\pi h 2^l\right)\dfrac{\sin\left(2\pi h 2^l \beta(t)\right)}{\sin(\pi h 2^l)} & , \quad 0 \le t < LT_c \\ \dfrac{\sin\left(\pi h 2^l - 2\pi h 2^l \beta(t-LT_c)\right)}{\sin(\pi h 2^l)} & , \quad LT_c \le t \le 2LT_c \\ 0 & , \quad \text{otherwise} \end{cases} . \tag{9}$$

Therefore, when $0 \le t \le LT_c$,

$$e^{j4\pi h 2^l(\gamma_l - 0.5)\beta(t)} = s^{(l)}(t+LT_c) + e^{j2\pi h 2^l \gamma_l} s^{(l)}(t). \tag{10}$$

Similar to Mengali and Morelli's decomposition [10], M-ary partial-response CPM signals with tilted-phase have complex envelope

$$s(t,\mathbf{u}) = \sum_{k=0}^{R-1}\sum_{n=0}^{N_c-1} B_{k,n} g_k(t-nT_c), \tag{11}$$

where $R = (2^F-1)2^{(L-1)^F}$ and the functions $g_k(t)$ and symbols $B_{k,n}$ are defined below. Functions $g_k(t)$ are

$$g_k(t) \triangleq e^{j\pi h W_1(t)} \prod_{l=0}^{F-1} s^{(l)}(t + e_{j,l}^{(k-w_j)} T_c)$$
$$\prod_{i=1}^{L-1} s^{(l)}\left[t + \left(i + La_{d_{j,l},i} + e_{j,l}^{(k-w_j)}\right)T_c\right], \tag{12}$$

where $a_{d_{j,l},i} \in \{0,1\}$ are coefficients in the binary representation of the index $d_{j,l} = \sum_{q=1}^{L-1} a_{d_{j,l},q} 2^{q-1}$, $0 \le d_{j,l} \le 2^{L-1}-1$, and functions $s^{(l)}(t)$ are defined as in (9). Symbols $B_{k,n}$ are

$$\prod_{l=0}^{F-1}\exp\left[j2\pi h 2^l\left(\sum_{r=0}^{n-e_{j,l}^{(k-w_j)}}\gamma_{r,l} - \sum_{r=1}^{L-1}\gamma_{n-r-e_{j,l}^{(k-w_j)},l}\, a_{d_{j,l},r}\right)\right] \tag{13}$$

where $\gamma_{r,l} \in \{0,1\}$ are coefficients in the binary representation of the information symbol u_r. The integer j, used in (11) - (13), is chosen from the set $j \in \{0,\ldots,2^{F(L-1)}-1\}$ and satisfies $j = \sum_{l=0}^{F-1}(2^{l(L-1)})d_{j,l}$, where $0 \le d_{j,l} \le 2^{L-1}-1$. Finally, integers $e_{j,l}^{(k-w_j)}$, used in (11) - (13), are chosen to satisfy $0 \le e_{j,l}^{(k-w_j)} \le T_{j,l}-1$ and $\prod_{l=0}^{F-1} e_{j,l}^{(k-w_j)} = 0$, where $T_{j,l} = \min_i[L(2-a_{d_{j,l},i})-i]$ for $0 \le i \le L-1$, $0 \le l \le F-1$, and $w_j = \sum_{n=0}^{j-1}\left(\prod_{l=0}^{F-1} T_{n,l} - \prod_{l=0}^{F-1}(T_{n,l}-1)\right)$.

IV. Design Criteria for ST-CPM

The CPM modulator inputs in Fig. 1 are elements from the $L_t \times N_c$ matrix defined in (2), while the outputs are the signals $s(t,\mathbf{u}^{(i)})$ defined in (11). Assume that $h = K/P$, where K and P are relatively prime integers. For M-ary partial-response ST-CPM codes, we define

$$v_{n,k}^{(i)} \triangleq \left[\sum_{r=0}^{n-e_{j,l}^{(k-w_j)}} u_r^{(i)} - X^{(i)}\right]_{\text{mod P}}, \tag{14}$$

where $X^{(i)} = \sum_{l=0}^{F-1} 2^l \sum_{r=1}^{L-1} \gamma^{(i)}_{n-r-e_{j,l}^{(k-w_j)},l}\, a_{d_{j,l},r}$, $\gamma_{n,l}^{(i)} \in \{0,1\}$ are coefficients in the binary representation of the symbols $u_n^{(i)}$, and $a_{d_{j,l},i} \in \{0,1\}$ are coefficients in the binary representation of the index $0 \le d_{j,l} \le 2^{L-1}-1$. Note that $v_{n,k}^{(i)}$ can only assume values from the set $\{0,1,\cdots,P-1\}$. Define the matrix of accumulative values as $\mathbf{V}_k \triangleq [v_{n,k}^{(i)}]$, denote $e^{\mathbf{V}_k} \triangleq [e^{j2\pi h v_{n,k}^{(i)}}]$, and denote $\Delta\mathbf{V}_k \triangleq e^{\mathbf{V}_k} - e^{\hat{\mathbf{V}}_k}$. Finally, define the vector $\mathbf{g}_k(t) \triangleq [g_{0,k}(t), g_{1,k}(t),\ldots,g_{N_c-1,k}(t)]^T$, where $g_{i,k}(t) \triangleq g_k(t-iT_c)$. From (11) - (14), the ST-CPM signal $\mathbf{s}(t,\mathbf{V})$ can be represented as

$$s(t,\mathbf{u}) = \mathbf{s}(t,\mathbf{V}) = \sum_{k=0}^{R-1} e^{\mathbf{V}_k}\ \mathbf{g}_k(t). \tag{15}$$

The CPM difference signal matrix for two space-time codewords, $\mathbf{U}$ and $\hat{\mathbf{U}}$, is

$$\Delta\mathbf{s}(t) = \sum_{k=0}^{R-1} \Delta\mathbf{V}_k\ \mathbf{g}_k(t). \tag{16}$$

Using (4) and (16), and following the derivations in [9], the matrix $\mathbf{U}_s$ can be written as

$$\mathbf{U}_s = \sum_{k=0}^{R-1} \Delta\mathbf{V}_k \mathbf{G}_k \Delta\mathbf{V}_k{}^H, \tag{17}$$

where $\mathbf{G}_k \triangleq \int_0^{N_cT_c} \mathbf{g}_k(t)\mathbf{g}_k^H(t)\ \mathrm{d}t$.

A. Full-Diversity Design Criterion

The ST-CPM signals achieve full diversity if the matrix $\mathbf{U}_s$ defined in (17) has full rank for any two space-time codewords. The matrix $\mathbf{U}_s$ has full rank iff all functions in the differential matrix $\Delta\mathbf{s}(t)$ in (16) are linearly independent [4]. A sufficient condition for achieving full diversity is that the matrices $\Delta\mathbf{V}_k$ have full rank for any two space-time codewords $\mathbf{U}$, and $\hat{\mathbf{U}}$.

Lemma 1: If the complex matrices $\Delta\mathbf{V}_k = (e^{\mathbf{V}_k} - e^{\hat{\mathbf{V}}_k})$ have full rank, then the differential matrix $\Delta\mathbf{s}(t)$ has full rank, and the ST-CPM system achieves full spatial diversity L_t.
The proof is omitted for brevity.

Next we determine conditions under which matrices $\Delta\mathbf{V}_k$ have full rank. Observe that each 2^F-ary codeword $\mathbf{U}$, defined in (2), has the form $\mathbf{U} = \sum_{l=0}^{F-1} 2^l \Phi_l(\mathbf{U})$, where $\Phi_l(\cdot)$ denotes operation $(\mathbf{U}/2^l)_{\bmod 2}$.

Lemma 2: If the modulo-2 projection of the codeword $\mathbf{U}$, i.e., $\Phi_0(\mathbf{U})$, has full rank, then codeword $\mathbf{U}$ also has full rank.
The proof is omitted for brevity.

*Theorem 3 (**Rank Design Criterion**):* Denote $\mathcal{C}$ as a linear $L_t \times N_c$ space-time code over $\mathbb{Z}^*_{2^F}$ with $L_t \le N_c$, where $\mathbb{Z}^*_{2^F}$ denotes the commutative ring of nonnegative integers modulo-2^F. Suppose that all nonzero codewords $\mathbf{U} \in \mathcal{C}$ have different modulo-2 projections $\Phi_0(\mathbf{U})$ and that matrices $\Phi_0(\mathbf{U})$ have full rank over the binary field $\mathbb{F}$. Then for any partial-response 2^F-ary CPM scheme with $h = K/2^n$ (where $2^n \le 2^F$, and 2^n and K are relatively prime integers), the space-time code $\mathcal{C}$ achieves full spatial diversity L_t.
The proof is omitted for brevity.

The previous theorem showed that if the space-time code $\mathcal{C}$ satisfies the CPM rank criterion, the matrices $\Delta\mathbf{V}_k = (e^{\mathbf{V}_k} - e^{\hat{\mathbf{V}}_k})$ have full rank for any two different accumulative matrices $\mathbf{V}_k$ and $\hat{\mathbf{V}}_k$. Lemma 1 showed that $\Delta\mathbf{s}(t)$ has full rank if matrices $\Delta\mathbf{V}_k$ have full rank. Consequently, the overall ST-CPM system achieves full diversity.

B. Coding Gain Design Criterion

Once full diversity is guaranteed, the next objective is to maximize the coding gain, $\xi_{PEP}(\Delta\mathbf{s}(t))$, over all pairs of distinct codewords $\mathbf{U}$ and $\hat{\mathbf{U}}$. The coding gain is defined by geometric-mean of the nonzero eigenvalues of the matrix $\mathbf{U}_s$, i.e., $\xi_{PEP}(\Delta\mathbf{s}(t)) = (\prod_{i=1}^{L_t}\lambda_i)^{1/L_t} = |(\mathbf{U}_s)|^{1/L_t}$, where $|\cdot|$ denotes the determinant operation. From (17), the determinant $|(\mathbf{U}_s)|$ can be written as

$$|\mathbf{U}_s| = \left|\sum_{k=0}^{R-1}\Delta\mathbf{V}_k\mathbf{G}_k\Delta{\mathbf{V}_k}^H\right|. \tag{18}$$

Equation (18) shows that optimization of the coding gain for ST-CPM depends not only on the codewords $\mathbf{U}$ and $\hat{\mathbf{U}}$, as in linear modulation, but also on the frequency shaping function used in the vectors $\mathbf{g}_k(t)$. In [9] is shown that if matrices $\mathbf{G}_k$ are designed to be semi-identity matrices, i.e., $\mathbf{G}_k = Q_k\mathbf{I}_{N_c}$, where $\mathbf{I}_{N_c}$ is the $N_c\times N_c$ identity matrix, N_c is the rank of the matrices $\mathbf{G}_k$, and Q_k is some constant, then the coding gain is optimized if the matrices $\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H$ are designed to be semi-identity matrices with maximized traces $\mathrm{tr}(\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H)$.

Here, we first investigate conditions under which the partial-response CPM matrices $\mathbf{G}_k$ can be constructed as semi-identity matrices.

From (12), observe that functions $g_{i,k}(t)$ are non-zero only in the time interval $iT_c \le t \le (i+L+1)T_c$. Then, the matrix $\mathbf{G}_k$ defined in (17) has form

$$\mathbf{G}_k = \tag{19}$$
$$\begin{bmatrix} \int_0^{(L+1)T_c}|g_{k,0}(t)|^2\mathrm{d}t & \cdots & 0 \\ \int_{T_c}^{(L+1)T_c} g_{k,1}(t)g_{k,0}^*(t)\mathrm{d}t & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \int_{(N_c-1)T_c}^{(N_c+L)T_c}|g_{k,N_c-1}(t)|^2\mathrm{d}t \end{bmatrix},$$

where $(\cdot)^*$ denotes complex conjugate operation and functions $g_{k,i}(t)$ are defined as in (12). In general, the matrix $\mathbf{G}_k$ has equal diagonal elements, i.e., $\int_0^{(L+1)T_c}|g_k(t)|^2\mathrm{d}t = \int_{T_c}^{(L+2)T_c}|g_k(t-T_c)|^2\mathrm{d}t = \cdots = \int_{(N_c-1)T_c}^{(N_c+L)T_c}|g_k(t-(N_c-1)T_c)|^2\mathrm{d}t$. Since all diagonal elements of the matrix $\mathbf{G}_k$ are equal, we just need to define conditions when $\mathbf{G}_k$ is a diagonal matrix. For the indexes k for which at least one coefficient $a_{d_{j,l},L-1} = 1$, where $a_{d_{j,l},L-1}$ is defined as in (12), functions $g_{i,k}(t)$ are different from zero only in the time interval $iT_c \le t \le (i+1)T_c$. Because functions $g_{i,k}(t)$ and $g_{i+y,k}(t)$ do not overlap for $y > 0$, matrices $\mathbf{G}_k$ are diagonal for arbitrary phase shaping functions. However, for indexes k for which the coefficients $a_{d_{j,l},L-1} = 0$, a general analysis of the matrices $\mathbf{G}_k$ is difficult because functions $g_{i,k}(t)$ depend on the memory length L. Hence, we will analyze matrices $\mathbf{G}_k$ for cases of practical importance ($L = 2, 3$).

For $L = 2$ partial-response CPM,

$$g_{i,k}(t) = \exp\left(j\pi hW_1(t-iT_c)\right)\prod_{l=0}^{F-1}s^{(l)}\Big(t-\Big(i-e_{j,l}^{(k-w_j)}\Big)T_c\Big)$$
$$s^{(l)}\left(t-\left(i-1-2a_{d_{j,l},1}-e_{j,l}^{(k-w_j)}\right)T_c\right), \tag{20}$$

where $W_1(t) = (M-1)(t/T_c+1)$ and functions $s^{(l)}(t)$ are defined as in (9). For $L = 2$, $a_{d_{j,l},1}\in\{0,1\}$.

For indexes k having at least one coefficient $a_{d_{j,l},1} = 1$, $g_{i,k}(t) \ne 0$ only on the time interval $iT_c \le t \le (i+1)T_c$. Since there is no overlap among functions $g_{i,k}(t)$ and $g_{i+y,k}(t)$ for $y > 0$, matrices $\mathbf{G}_k$ are semi-identity for arbitrary phase shaping functions. For indexes k for which $\mathbf{G}_k$ are not semi-identity for an arbitrary phase shaping function, the integrals $I_{i,i+1} = \int_{(i+1)T_c}^{(i+3)T_c} g_{i,k}(t)g_{i+1,k}^*(t)\mathrm{d}t$ and $I_{i+1,i} = \int_{(i+1)T_c}^{(i+3)T_c} g_{i+1,k}(t)g_{i,k}^*(t)\mathrm{d}t$ for integers $e_{j,l}^{(k-w_j)}\in\{0,1\}$, and $I_{i,i+2} = \int_{(i+2)T_c}^{(i+3)T_c} g_{i,k}(t)g_{i+2,k}^*(t)\mathrm{d}t$ and $I_{i+2,i} = \int_{(i+2)T_c}^{(i+3)T_c} g_{i+2,k}(t)g_{i,k}^*(t)\mathrm{d}t$ for $e_{j,l}^{(k-w_j)} = 0$ should be minimized. The following lemma shows when the integrals $I_{i,i+1}$, $I_{i+1,i}$, $I_{i,i+2}$, and $I_{i,i+2}$ are approximately zero.

Lemma 4: For the raised cosine (2RC) frequency shaping function, $q(t)$, with the modulation indices $h = 1/2^n$, where $1 \le n \le F-1$, the integrals $I_{i,i+1}$, $I_{i+1,i}$, $I_{i,i+2}$, and $I_{i,i+2}$ are approximately zero.
The proof is omitted for brevity.

Using the similar reasoning as above, it can be shown that for $L = 3$ partial-response CPM signals with the raised cosine (3RC) frequency shaping function and the modulation indices $h = 1/2^n$, where $1 \le n \le F-1$, the matrices $\mathbf{G}_k$ are semi-identity matrices.

When the $\mathbf{G}_k$ have semi-identity form, the ST-CPM coding gain simplifies to

$$\xi_{PEP}(\Delta\mathbf{s}(t)) = |\mathbf{U}_s|^{1/L_t} = \left|\sum_{k=0}^{R-1}Q_k\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H\right|^{1/L_t}, \tag{21}$$

where $Q_k = \int_0^{2T_c}|g_k(t)|^2dt$. While it is difficult in general to maximize the exact coding gain $\xi_{PEP}(\Delta\mathbf{s}(t))$, it is possible to maximize the trace upper bound $|\mathbf{U}_s|^{1/L_t} \le \mathrm{tr}(\mathbf{U}_s)/L_t$ on the coding gain. Furthermore, in [3] is shown that for systems with $L_tL_r \ge 4$, maximization of trace $\mathrm{tr}(\mathbf{U}_s)$ is a sufficient condition for coding gain optimization. The trace $\mathrm{tr}(\mathbf{U}_s)$ is equal to

$$\mathrm{tr}(\mathbf{U}_s) = \sum_{k=0}^{R-1}Q_k\mathrm{tr}\Big(\Delta\mathbf{V}_k\Delta{\mathbf{V}_k}^H\Big) = \sum_{k=0}^{R-1}Q_kd_{E_k}^2\Big(e^{\mathbf{V}_k}, e^{\hat{\mathbf{V}}_k}\Big)$$
$$= \sum_{k=0}^{R-1}Q_k\sum_{i=1}^{L_t}\sum_{n=0}^{N_c-1}4\sin^2\left(\pi h\left(v_{n,k}^{(i)}-\hat{v}_{n,k}^{(i)}\right)\right), \tag{22}$$

where elements $v_{n,k}^{(i)}$ are defined in (14) and $d_{E_k}^2(e^{\mathbf{V}_k}, e^{\hat{\mathbf{V}}_k})$ is the squared Euclidean distance between the code matrices $e^{\mathbf{V}_k} = [e^{j2\pi hv_{n,k}^{(i)}}]$ and $e^{\hat{\mathbf{V}}_\mathbf{k}} = [e^{j2\pi h\hat{v}_{n,k}^{(i)}}]$. The maximization of $\mathrm{tr}(\mathbf{U}_s)$ is not straight forward because elements in matrices $\mathbf{V}_k$ and $\hat{\mathbf{V}}_k$ are obtained as linear combinations of elements in the codewords $\mathbf{U}$ and $\hat{\mathbf{U}}$. Using (14), the trace $\mathrm{tr}(\mathbf{U}_s)$ becomes

$$\mathrm{tr}(\mathbf{U}_s) = \sum_{k=0}^{R-1} Q_k d^2_{E_k}\left(e^{f_{k,P}(\mathbf{U})}, e^{f_{k,P}(\hat{\mathbf{U}})}\right) = \quad (23)$$
$$\sum_{k=0}^{R-1} Q_k \sum_{i=1}^{L_t}\sum_{n=0}^{N_c-1} 4\sin^2\left(\pi h\left(f_{k,P}\left(u_n^{(i)}\right) - f_{k,P}\left(\hat{u}_{n,}^{(i)}\right)\right)\right),$$

where functions $f_{k,P}(\ \cdot\ )$ are

$$f_{k,P}(\mathbf{U}) = \mathbf{V}_k = \quad (24)$$
$$\left[f_{k,P}\left(u_n^{(i)}\right)\right] = \left[\left(\sum_{r=0}^{n-e_{j,l}^{(k-w_j)}} u_r^{(i)} - X^{(i)}\right)_{\mathrm{mod\,P}}\right],$$

$X^{(i)} = \sum_{l=0}^{F-1} 2^l \sum_{r=1}^{L-1} \gamma^{(i)}_{n-r-e_{j,l}^{(k-w_j)},l} a_{d_{j,l},r}$, $\gamma^{(i)}_{n,l} \in \{0,1\}$ are coefficients in the binary representation of the symbols $u_n^{(i)}$, $a_{d_{j,l},i} \in \{0,1\}$ are coefficients in the binary representation of the index $0 \le d_{j,l} \le 2^{L-1}-1$, $h = K/P$, and $P = 2^H$ for $1 \le H \le F-1$. From (23) follows that the trace of the matrix $\mathbf{U}_s$ over all pairs of codewords $\mathbf{U} \ne \hat{\mathbf{U}} \in \mathcal{C}$ will be maximized if the squared minimum Euclidean distances $d^2_{E_k\min} = \min\{d^2_{E_k}(e^{f_{k,P}(\mathbf{U})}, e^{f_{k,P}(\hat{\mathbf{U}})}) : \mathbf{U} \ne \hat{\mathbf{U}} \in \mathcal{C}\}$ for $k \in \{0,\ldots,R-1\}$ are maximized.

In [13], is shown that the squared minimum Euclidean distance $d^2_{E\min}(\exp(j2\pi h\mathcal{C}))$ of a linear 2^H-ary block code $\mathcal{C}$ with codewords $c = \sum_{i=0}^{H-1} 2^i b_i$ is

$$d^2_{E\min}(e^{j2\pi h\mathcal{C}}) = \min_{i=0,\ldots,H-1}\left\{4\sin^2\left(\pi h 2^i\right) d_{H_i\min}\right\}, \quad (25)$$

where $\mathcal{B}_0, \mathcal{B}_1, \ldots, \mathcal{B}_{H-1}$ are binary block codes with the same block length, $b_i \in \mathcal{B}_i$, and $d_{H_i\min} = \min\{d_H(x_i,y_i) : x_i, y_i \in \mathcal{B}_i\}$ is the minimum Hamming distance for the code $\mathcal{B}_i$.

By observing that each code matrix $\mathbf{V}_k$ has the form

$$\mathbf{V}_k = f_{k,P}(\mathbf{U}) = \sum_{l=0}^{H-1} 2^l \Phi_l(f_{k,P}(\mathbf{U})), \quad (26)$$

where $\Phi_l(\cdot)$ denotes operation $(f_{k,P}(\mathbf{U})/2^l)_{\mathrm{mod}\,2}$ and using (25), the minimum trace of the matrix $\mathbf{U}_s$ over all pairs of codewords $\mathbf{U} \ne \hat{\mathbf{U}} \in \mathcal{C}$ can be written as

$$\min_{\mathbf{U}\ne\hat{\mathbf{U}}\in\mathcal{C}} \mathrm{tr}(\mathbf{U}_s) = \sum_{k=0}^{R-1} Q_k \min_{l=0,\ldots,H-1}\left\{4\sin^2\left(\pi h 2^l\right) d_{H_{l,k}\min}\right\}, \quad (27)$$

where $d_{H_{l,k}\min} = \min\left\{d_H\left(\Phi_l\left(f_{k,P}(\mathbf{U})\right), \Phi_l(f_{k,P}(\hat{\mathbf{U}}))\right)\right\}$ is the minimum Hamming distance over all code matrices $\Phi_l(f_{k,P}(\mathbf{U}))$ and $\Phi_l(f_{k,P}(\hat{\mathbf{U}}))$. From (27) follows that the trace of the matrix $\mathbf{U}_s$ over all pairs of codewords $\mathbf{U} \ne \hat{\mathbf{U}} \in \mathcal{C}$ will be maximized if all minimum Hamming distances $d_{H_{l,k}\min}$ are maximized. However, it is difficult to design codewords $\mathbf{U}$ that maximize all minimum Hamming distances $d_{H_{l,k}\min}$. Since most of the signal energy is concentrated in the matrix $\mathbf{G}_0$, the coefficient Q_0 is much larger then coefficients Q_i for $i = 1,\ldots,R-1$. Hence, the codewords $\mathbf{U}$ should be designed to maximize the minimum Hamming distances $d_{H_{l,0}\min} = \min\left\{d_H\left(\Phi_l\left(f_{0,P}(\mathbf{U})\right), \Phi_l(f_{0,P}(\hat{\mathbf{U}}))\right)\right\}$ for $0 \le l \le H-1$.

V. Simulation Results

Simulation results are presented to verify proposed design criteria. All simulations use a frame length of 130 with $L_r = 1$. Each spatial channel is modelled as being independently Rayleigh faded.

Fig. 2 compares the performance curves obtained for partial-response 16-ary CPM signals with 2RC frequency shaping function, $h = 1/4$, and $L_t = \{1,2,3,4\}$ transmit antennas. The space-time codewords are chosen from a (4×4) 16-ary CPM space-time code $\mathcal{C}$. Following the rank design criterion introduced in Theorem 3, the codewords $\mathbf{U} \in \mathcal{C}$ are constructed as $\mathbf{U} = f^{-1}_{0,16}(\sum_{l=0}^{3} 2^l \mathbf{U}_l)$, where $f^{-1}_{0,16}(\ \cdot\ )$ denotes the inverse of the function $f_{0,16}(\ \cdot\ )$ defined in (24) and $\mathbf{U}_l$ are binary codewords from linear (4×4) space-time codes $\mathcal{C}_l$. All codewords $\mathbf{U}_0 \in \mathcal{C}_0$ are designed to have full rank over $\mathbb{F}$ and all codewords $\mathbf{U}_0$ satisfy $\mathbf{U}_0 \ne \hat{\mathbf{U}}_0$. The codewords used to obtain the second, third, and fourth curve are obtained using the binary space-time code in [14] with the generator matrix $G = [1 \quad \alpha \quad \alpha^2 \quad \alpha^3]$, where α is a zero of the primitive polynomial $f(x) = x^4 + x + 1$ over $\mathbb{F}$. This binary code has the minimum Hamming distance $d_{Hmin} = 4$. The codewords used to obtain the fifth curve are obtained using the binary space-time code in [15] with the minimum Hamming distance $d_{Hmin} = 8$. Fig. 2 shows that full diversity and coding gain improvement are achieved when the space-time codes are constructed to satisfy the rank and coding gain design criteria.

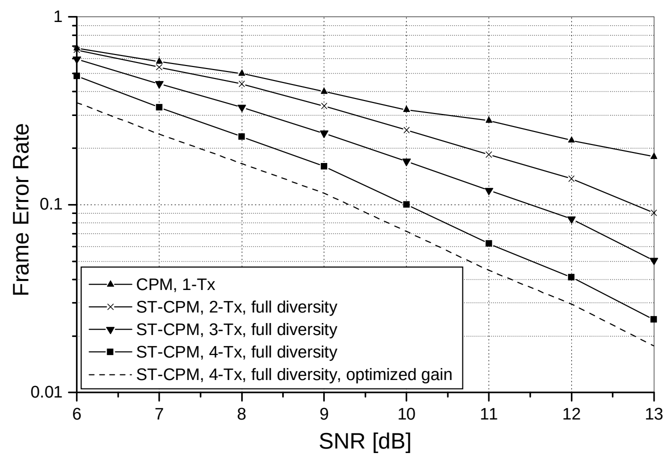

Fig. 2. Frame-error rate of 16-ary ST-CPM with 2RC and $h = 1/4$ in quasi-static fading.

We have shown that the raised cosine frequency shaping function satisfies the coding gain design criterion. Fig. 3 illustrates how much the coding gain is sacrificed if other frequency shaping functions are used instead of the raised cosine frequency shaping function. The results are obtained using the (2×2) 8-ary orthogonal space-time code [9] and modulation

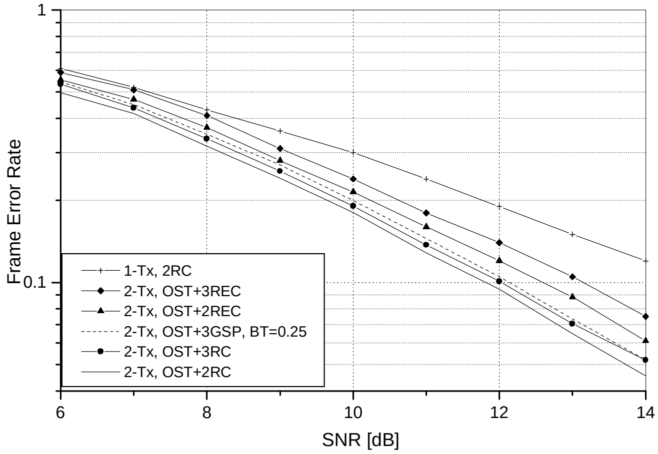

Fig. 3. Frame-error rate of 8-ary OST-CPM with Gaussian frequency shaping function, partial-response rectangular, and partial-response raised cosine frequency shaping function and $h = 1/4$ in quasi-static fading.

index $h = 1/4$. In Fig. 3, the second and the third curve are obtained using the rectangular frequency shaping function with $L = 3$ (3REC) and $L = 2$ (2REC), respectively. The fourth curve is obtained using the Gaussian frequency shaping function (GSP) with normalized filter bandwidth $BT = 0.25$. Finally, the fifth and the sixth curve are obtained using the raised cosine shaping function with $L = 3$ (3RC) and $L = 2$ (2RC), respectively. Results show that the Gaussian frequency shaping function with $BT = 0.25$ performs similar to the RC shaping function with $L = 3$. The RC shaping function with $L = 2$ improves the coding gain for approximately 0.47 dB compared to the Gaussian frequency shaping function and approximately 0.86 dB compared to the REC shaping function with $L = 2$.

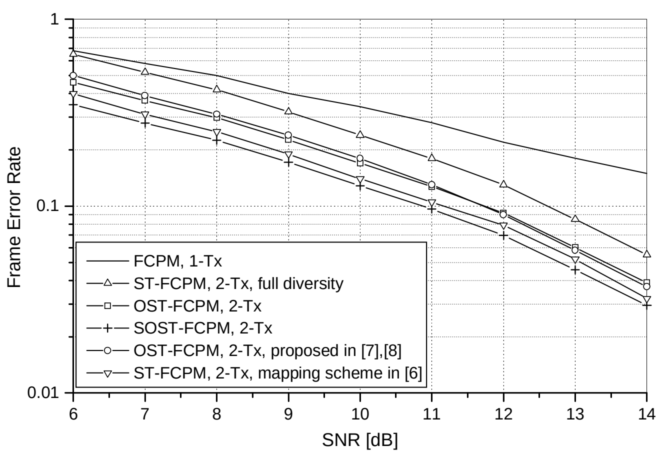

Fig. 4. Frame-error rate of (2×2) 8-ary ST-CPM with 1RC and $h = 1/4$ in quasi-static fading.

Fig. 4 compares our orthogonal space-time full-response CPM (OST-FCPM) and super-orthogonal space-time full-response CPM (SOST-FCPM) designs with OST-FCPM designs [7], [8] and ST-FCPM design with mapping scheme [6], respectively. The SOST-FCPM design uses the space-time code in [16]. Results show that our SOST-FCPM design has similar performance as ST-FCPM with mapping scheme [6] and performs better than the OST-FCPM designs [7],[8] and our OST-FCPM design.

VI. Conclusion

This paper derived sufficient conditions under which M-ary partial-response CPM space-time codes will attain both full spatial diversity and optimized coding gain. Using a linear decomposition of CPM signals in a tilted-phase representation, we have identified the rank criterion for the M-ary partial-response CPM that specifies the set of allowable modulation indices. The modulation indices and phase shaping functions that optimize the coding gain are specified and the optimization of ST-CPM codewords is discussed.

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