Performance Analysis of a System using Coordinate Interleaving and Constellation Rotation

Nauman F. Kiyani †, Jos H. Weber †, Alenka G. Zajić ‡, and Gordon L. Stüber ‡
†Faculty of Electrical Engineering, Mathematics and Computer Science
Delft University of Technology, 2600 GA Delft, The Netherlands
‡ School of Electrical and Computer Engineering
Georgia Institute of Technology, Atlanta, GA, 30332 USA

***Abstract*— Diversity can play an important role in the performance improvement of a communication system in fading channels. The achievable performance with signal space diversity (SSD) is analyzed and a closed form expression for the upper bound of average probability of bit error rate (P_b) for M-ary phase shift keying (MPSK) in Rayleigh fading channel is presented. The problem of calculating P_b of coherent MPSK over a Rayleigh fading channel has been studied previously in the literature. A solution based on the nearest neighbors was given. In this paper we show that the results with the nearest neighbor approximation represent an expurgated bound and are only valid for a small range of rotational angles. Exact pair-wise error probability (PEP) is calculated for Rayleigh fading channels. It is shown that Gray signal constellation mapping is not necessarily the best option for a system employing coordinate interleaving and constellation rotation. Optimum rotation angles are found by finding the minimum of the upper bound of P_b. It is shown that the calculated bound is tight for the entire range of rotational angles at high signal-to-noise ratio. Furthermore, the performance of the system in case of phase estimation error is also investigated by simulations.**

I. Introduction

One of the most powerful techniques to mitigate the performance degradation on fading channels is by the use of diversity. Any diversity technique (e.g., space, time and frequency) tries to provide statistically independent copies of the transmitted sequence at the receiver for reliable detection. Signal space diversity (SSD) can provide performance improvement over fading channels without using extra bandwidth and power expansion [1]–[3]. The basic premise of SSD is that multidimensional signal constellations are used and the components of the each signal constellation point are transmitted over independent fading channels. The independence of the fading channels can easily be accomplished by interleaving.

In [2], the union bound of average probability of symbol error (P_s) for a system employing SSD is calculated for uncorrelated Rayleigh fading channels. The rotation angles are calculated at high signal-to-noise ratio (SNR) to maximize the minimum product distance of the rotated constellations. In [3], the average probability of bit error (P_b) is approximated by considering only the nearest neighbors. The rotation angles are also chosen based on this approximation.

In this paper, we present the closed form analytical expressions of the union bound of P_b for M-ary phase shift keying (MPSK) signal constellations. The entire analysis can be extended to any multi-dimensional signal constellation. We calculate the exact pair-wise error probability (PEP) for Rayleigh fading channels. We show that the P_b calculated in [3] is an expurgated bound which is not valid for the entire range of signal constellation rotation. Optimum rotation angles are calculated by minimizing the upper bound on P_b and it is shown that optimum angles are different from [2], [3]. An interesting aspect of signal constellation mapping is highlighted and it is shown that the Gray signal constellation mapping is not necessarily the best option for the entire range of rotational angle. Simulation results obtained are in close agreement with the calculated analytical expressions. Furthermore, the effect of the phase estimation error on the system performance is also presented.

The paper is organized as follows. Section II briefly outlines the main blocks of the system model. Section III presents the calculations of the upper bound of P_b for a system employing SSD in a Rayleigh fading channel. Simulation and analytical results are presented in Section IV, followed by the conclusions in Section V.

II. System Model

In MPSK/MQAM constellations the inphase (I) and the quadrature (Q)-channels are orthogonal and can be separated at the receiver. It is assumed that the channel state information (CSI) is available at the receiver. It is shown in the subsequent analysis that a diversity gain is achieved if I and Q channels experience independent fades. The diversity gain is achieved by rotating the signal constellation and separately interleaving the I and the Q component, without effecting its bandwidth efficiency [2].

Figure 1 shows a block diagram of a system employing coordinate interleaving and constellation rotation (SSD), hereafter referred as an "SSD system". The concept of coordinate interleaving and constellation rotation is generic to all MPSK/MQAM constellations. We confine ourselves in this report to MPSK signal constellations. A conventional MPSK signal constellation is denoted by $\mathcal{S}_M = \{s_l = e^{j2\pi(l/M)} : l = 0, 1, \ldots, M-1\}$, where the energy has been constrained to unity and each symbol corresponds to $m = \log_2 M$ bits. Anti-clockwise rotation over an angle θ leads to the constellation

$$\mathcal{S}_M^{\theta} = \{s_l = e^{j(2\pi(l/M)+\theta)} : l = 0, 1, \ldots, M-1\}. \qquad (1)$$

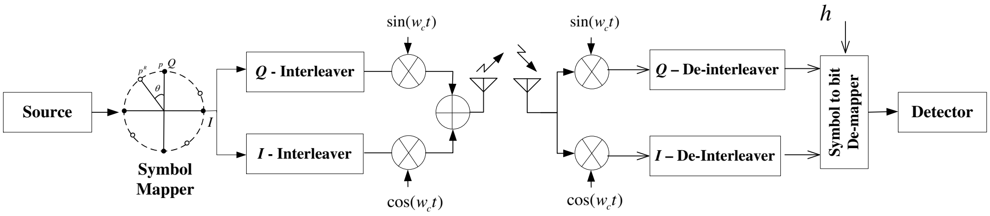

Fig. 1. System model (referred to as SSD system).

The symbol mapper can be represented by a one-to-one mapping function $\wp : \{0,1\}^m \rightarrow \mathcal{S}_M^\theta, s = \wp(\mathbf{b})$, where, $\mathbf{b} = (b_1, \cdots, b_m), b_j \in \{0,1\}$ represents the binary sequence and s is chosen from the set $\mathcal{S}_M^\theta$ consisting of M complex signal points. In case of N symbol transmission, let the sequence of rotated I and Q - components be denoted as $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1})$ and $\mathbf{y} = (y_0, y_1, \ldots, y_{N-1})$, respectively. Let η and μ represent the I and Q interleavers, resulting in sequences $\tilde{\mathbf{x}} = \eta(\mathbf{x}) = (\tilde{x}_0, \tilde{x}_1, \ldots, \tilde{x}_{N-1})$ and $\tilde{\mathbf{y}} = \mu(\mathbf{y}) = (\tilde{y}_0, \tilde{y}_1, \ldots, \tilde{y}_{N-1})$, respectively. The transmitted waveform for the rotated and interleaved system is given by

$$\begin{aligned} \tilde{s}(t) &= \sum_{i=0}^{N-1} \tilde{x}_i g(t - iT_s)\cos(2\pi f_c t) \\ &+ \sum_{i=0}^{N-1} \tilde{y}_i g(t - iT_s)\sin(2\pi f_c t). \end{aligned} \quad (2)$$

where

$$g(t) = \begin{cases} 1, & 0 \leq t \leq T_s, \\ 0, & \text{otherwise}, \end{cases}$$

T_s is the symbol period and f_c is the carrier frequency.

As we have two orthogonal channels, let the squared Euclidean distances between two different signal constellation points in the I and Q-directions be represented by d_I^2 and d_Q^2, respectively. The distances are given as

$$\begin{aligned} d_I^2 &= \left(\cos(\phi_1 + \theta) - \cos(\phi_2 + \theta)\right)^2, \\ d_Q^2 &= \left(\sin(\phi_1 + \theta) - \sin(\phi_2 + \theta)\right)^2, \end{aligned} \quad (3)$$

where ϕ_1, ϕ_2 represent the phase of the two signal constellation points under consideration, respectively.

The communication channel is assumed to be frequency non-selective slowly fading with a multiplicative factor representing the fading effect and an additive term representing the additive white Gaussian noise. The received signal samples in baseband can be given as $\tilde{r_i} = |\tilde{\alpha_i}| e^{j\tilde{\phi_i}} \tilde{s_i} + \tilde{n_i}$, for $i = 1, \cdots, N$ where $\tilde{\alpha_n} = |\tilde{\alpha_n}| e^{j\tilde{\phi_n}}$ represents a zero-mean complex Gaussian process, $\tilde{\phi_n}$ represents the phase shift that is introduced by the fading channel, $\tilde{s_i}$ is the transmitted symbol and $\tilde{n_i}$ are complex Gaussian random variables with zero mean and a variance of $N_0/2$ in each dimension. As CSI is available at the receiver, the phase shift, therefore, can be removed without any error. Thus after phase removal the received sample takes the form $\tilde{r_i} = |\tilde{\alpha_i}| \tilde{s_i} + \tilde{z_i}$, where $\tilde{z_i} = \tilde{z}_i^I + j\tilde{z}_i^Q$ represents the complex white Gaussian noise. The received sequences $\tilde{\mathbf{r}}^{\mathbf{I}}$ and $\tilde{\mathbf{r}}^{\mathbf{Q}}$ are de-interleaved resulting in $\mathbf{r}^{\mathbf{I}} = \eta^{-1}(\tilde{\mathbf{r}}^{\mathbf{I}})$ and $\mathbf{r}^{\mathbf{Q}} = \rho^{-1}(\tilde{\mathbf{r}}^{\mathbf{Q}})$. The fading sequence $\tilde{\alpha} = (\tilde{\alpha}_0, \tilde{\alpha}_1, \ldots, \tilde{\alpha}_{N-1})$ are also de-interleaved resulting in $\alpha^{\mathbf{I}} = \eta^{-1}(\tilde{\alpha})$ and $\alpha^{\mathbf{Q}} = \rho^{-1}(\tilde{\alpha})$. The receiver then performs a maximum likelihood (ML) detection.

III. Performance Analysis of Uncoded System over an Uncorrelated Rayleigh Fading Channel

The probability density function (pdf) of Rayleigh distribution is given as $p(\alpha) = \frac{\alpha}{b_0} e^{\left\{\frac{-\alpha^2}{2b_0}\right\}}$. The average envelope power is assumed to be $\mathcal{E}[\alpha^2] = 2b_0 = 1$. Hence, the pdf can be rewritten as

$$p(\alpha) = 2\alpha e^{-\alpha^2} \quad \alpha \geq 0. \quad (4)$$

Furthermore, the channel coefficients are assumed to independent and identically distributed (i.i.d). In order to highlight the diversity gain that is achieved by coordinate interleaving coupled with constellation rotation we firstly consider the conventional QPSK system (i.e., a system without coordinate interleaving and signal constellation rotation) and analyze its performance.

A. Probability of Bit Error Rate for Conventional QPSK System

Assuming perfect CSI, the average probability of error for a conventional QPSK system is calculated by averaging the conditional probability of error on the fading statistic as given in [4]

$$P_b = \int_0^\infty Q\left(\sqrt{\bar{\gamma}\alpha^2(d_{\min}^2)}\right) p(\alpha) d\alpha, \quad (5)$$

where $d_{\min}^2$ represents the minimum squared Euclidean distance between any two different signal constellations points. The $d_{\min}^2$ is a constant having a value of 2. $d_{\min}^2$ can also be represented as a sum of $d_{\min}^2 = d_{I_{\min}}^2 + d_{Q_{\min}}^2$. This implies that for $d_{\min}^2$ only the nearest neighbors (NN) are to be considered. Therefore, (3) can be simplified as,

$$\begin{aligned} d_{I_{\min}}^2 &= 1 \pm \sin(2\theta), \\ d_{Q\min}^2 &= 1 \mp \sin(2\theta). \end{aligned} \quad (6)$$

In (5), $\bar{\gamma}$ is the average SNR per bit and $Q(x)$ is the Gaussian probability function defined as [5]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{\frac{-x^2}{2\sin^2(\psi)}} d\psi. \quad (7)$$

Using (4) and (7) in (5) we have

$$
\begin{aligned}
P_b &= \frac{1}{\pi}\int_0^{\pi/2}\int_0^{\infty} e^{\left(\frac{-\alpha^2\bar{\gamma}(d^2_{I_{\min}}+d^2_{Q_{\min}})}{2\sin^2(\psi)}\right)} 2\alpha e^{-\alpha^2} d\alpha d\psi \\
&= \frac{1}{2}\left(1-\sqrt{\frac{\frac{\bar{\gamma}}{2}(d^2_{I_{\min}}+d^2_{Q_{\min}})}{1+\frac{\bar{\gamma}}{2}(d^2_{I_{\min}}+d^2_{Q_{\min}})}}\right) \\
&= \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right). \qquad (8)
\end{aligned}
$$

Equation (8) is widely reported in the literature, e.g., in [4], [6] and [7]. As it is evident from (8) that the system in the absence of coordinate interleaving is invariant to constellation rotation. This invariance to constellation rotation is due to the fact that the I and the Q-channels experience the same fade. In the subsequent analysis we will consider a scenario where the I and the Q-channels experience different fades.

B. Probability of Bit Error Rate for MPSK using Coordinate Interleaving and Constellation Rotation

For an arbitrary two-dimensional (2-D) signal constellation, a standard approach of evaluating the error probability of a signal set $\mathcal{S}_M^{\theta}$ is based on the union bound [6] and the average probability of symbol error P_s is thus upper bounded as

$$P_s \le P_s^{UB} = \frac{1}{M}\sum_{s\in\mathcal{S}_M^{\theta}}\sum_{\substack{\hat{s}\in\mathcal{S}_M^{\theta}\\ s\neq\hat{s}}} P(s\to\hat{s}), \qquad (9)$$

where $\mathcal{S}_M^{\theta}$ is the signal constellation of size $|\mathcal{S}_M^{\theta}| = M = 2^m$ and $P(s\to\hat{s})$ is the unconditional pairwise error probability (PEP) that the receiver estimated $\hat{s}$ when s was transmitted; given that s and $\hat{s}$ are the only two signal constellation points under-consideration. The bound can be modified to evaluate the average bit error probability by considering the number of bits per symbol (m) and the mapping rule specifying the Hamming distance associated with each PEP calculation [8]. Let $a(s,\hat{s})$ represent the Hamming distance between the sequences of bits of s and $\hat{s}$ under consideration. Then, P_b can be upper bounded as

$$P_b \le P_b^{UB} = \frac{1}{m2^m}\sum_{s\in\mathcal{S}_M^{\theta}}\sum_{\substack{\hat{s}\in\mathcal{S}_M^{\theta}\\ s\neq\hat{s}}} a(s,\hat{s})P(s\to\hat{s}). \qquad (10)$$

If we consider only the nearest neighbors, the P_b can be approximated as

$$P_b \approx P_b^{NN} = \frac{1}{m2^m}\sum_{s\in\mathcal{S}_M^{\theta}}\sum_{\hat{s}\in\mathcal{N}(s)} a(s,\hat{s})P(s\to\hat{s}), \qquad (11)$$

where $\mathcal{N}(s)$ is the set of the nearest neighbors of s in $\mathcal{S}_M^{\theta}$. In the literature, for convenience the calculation of PEP is also approximated by using bounds (e.g., Chernoff bound). We, on the other hand in the subsequent analysis, evaluate the exact PEP for MPSK constellations.

Coordinate interleaving is employed so that the I and the Q- channels experience independent fades. Let α_1 and α_2 be the Rayleigh distributed random variables with pdf given in equation (4). In order to calculate the average probability of error for a system employing coordinate interleaving, the conditional PEP needs to be averaged over α_1 and α_2 given as

$$
\begin{aligned}
P(s\to\hat{s}) &= \int_0^{\infty}\int_0^{\infty} Q(\sqrt{\bar{\gamma}(\alpha_1^2 d_I^2+\alpha_2^2 d_Q^2})\\
&\quad p(\alpha_1)p(\alpha_2)d\alpha_1 d\alpha_2 \qquad (12)
\end{aligned}
$$

using (7) in (12) we have

$$
\begin{aligned}
P(s\to\hat{s}) &= \frac{1}{\pi}\int_0^{\pi/2}\int_0^{\infty}\int_0^{\infty} 2\alpha_1 e^{-\alpha_1^2\frac{2\sin^2(\psi)+\bar{\gamma}d_I^2}{2\sin^2(\psi)}} \\
&\quad 2\alpha_2 e^{-\alpha_2^2\frac{2\sin^2(\psi)+\bar{\gamma}d_Q^2}{2\sin^2(\psi)}} d\alpha_1 d\alpha_2 d\psi \\
&= \frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^4(\psi)}{(\sin^2(\psi)+\frac{\bar{\gamma}d_I^2}{2})(\sin^2(\psi)+\frac{\bar{\gamma}d_Q^2}{2})}d\psi \\
&= \frac{1}{2}-\frac{d_I^2}{2(2d_I^2-C)}\left(\sqrt{\frac{\bar{\gamma}d_I^2}{2+\bar{\gamma}d_I^2}}\right)+ \\
&\quad \frac{C-d_I^2}{2(2d_I^2-C)}\left(\sqrt{\frac{\bar{\gamma}(C-d_I^2)}{2+\bar{\gamma}(C-d_I^2)}}\right), \qquad (13)
\end{aligned}
$$

where $C = d_I^2 + d_Q^2$.

C. Nearest Neighbor Approach

The nearest neighbor (NN) approach is exact for a conventional QPSK system (8). Using (13) in (11) and with $C = 2$, P_b^{NN} for Gray mapped signal constellation can be given as

$$
\begin{aligned}
P_b^{NN} &= \frac{1}{4(d_I^2-1)}\left\{d_I^2\left(2-\sqrt{\frac{\bar{\gamma}d_I^2}{(2+\bar{\gamma}d_I^2)}}-\right.\right.\\
&\quad\left.\sqrt{\frac{\bar{\gamma}(2-d_I^2)}{(2+\bar{\gamma}(2-d_I^2))}}\right)\\
&\quad\left.-2\left(1-\sqrt{\frac{\bar{\gamma}(2-d_I^2)}{(2+\bar{\gamma}(2-d_I^2))}}\right)\right\}. \qquad (14)
\end{aligned}
$$

Equation (14) is also reported in [3]. Using (14) P_b^{NN} is calculated and compared with the simulation results. Figure 2 shows the P_b^{NN} versus the rotation angle θ at $E_b/N_0 = 15$ dB. For θ in the range $22.5° \le \theta \le 67.5°$, P_b^{NN} matches the simulation results. However, outside this range there is a divergence from the simulation results. This divergence is due to the NN approximation as not all the error events are considered. This also shows that for a system employing coordinate interleaving, the NN approximation is not as tight as for a conventional QPSK system.

It is evident from (13) & (14) that the P_b of a system employing coordinate interleaving is dependent upon the constellation rotation (θ). It shows that if I and Q channels are exposed to independent fades, the system performance varies with constellation rotation. Furthermore, the worst case scenario for a system employing constellation rotation and coordinate interleaving is when one branch has been completely

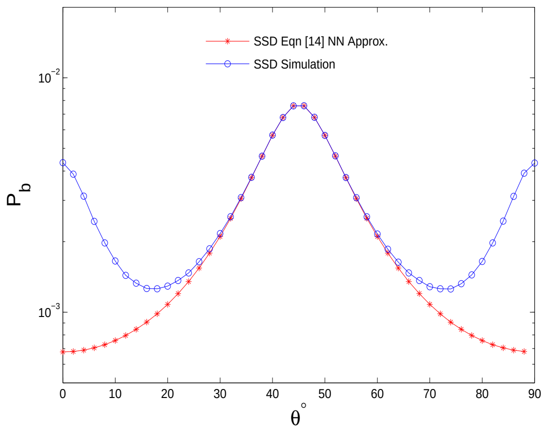

Fig. 2. Average probability of error P_b and P_b^{NN} versus rotational angle θ of an SSD system using QPSK signal constellation over Rayleigh fading channel with perfect CSI at $E_b/N_0 = 15$ dB. Gray signal constellation mapping is used.

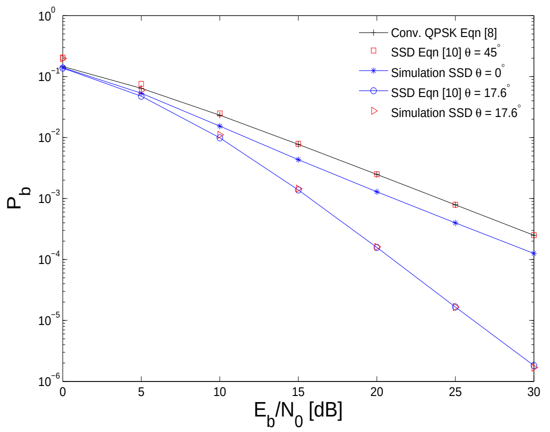

Fig. 3. Average probability of error P_b of conventional and SSD systems using QPSK signal constellation over Rayleigh fading channels with perfect CSI. Gray signal constellation mapping is used.

removed, i.e., d_I^2 or $d_Q^2 = 0$. In such a case, (14) reduces to (8), showing that the choice of the rotation angle plays a very vital role.

D. Finding the Optimal θ

In order to find the optimum rotation angle θ for P_b, we need to find the minimum of (10). This can be accomplished by taking the derivative of the objective function (10) with respect to θ. In (10), we need to take the derivative of (13). The derivative of (13) is given by (15). In (15), $d_I' = -\sin(\phi_1 + \theta) + \sin(\phi_2 + \theta)$.

IV. Results

We take QPSK as the candidate constellation in MPSK signal constellations and present the analytical and simulation results but we also emphasize the fact that the analysis presented in Section III is applicable for all MPSK signal constellations.

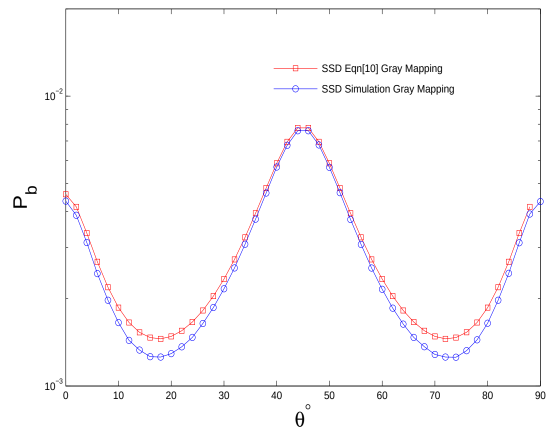

Fig. 4. Average probability of error P_b and P_b^{UB} versus rotational angle θ of an SSD system using QPSK signal constellation over Rayleigh fading channel with perfect CSI at $E_b/N_0 = 15$ dB. Gray signal constellation mapping is used.

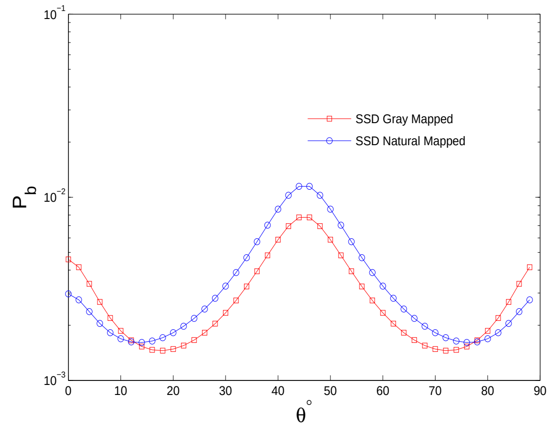

Fig. 5. Average probability of error P_b for Gray and Natural mapped QPSK signal constellation over Rayleigh fading channel with perfect CSI at $E_b/N_0 = 15$ dB.

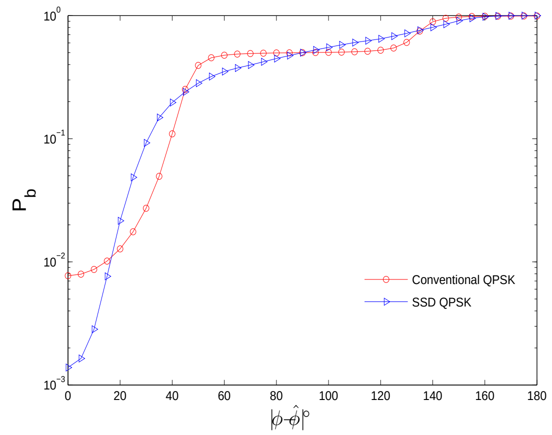

Fig. 6. The effect of phase estimation error on the performance of an SSD system ($\theta = 22.5°$) with respect to a conventional QPSK system at $E_b/N_0 = 15$ dB. At the receiver coherent detection is carried out. Gray signal constellation mapping is employed.

$$\frac{\partial P(s \rightarrow \hat{s})}{\partial \theta} = \frac{-\left(d_I^2\left(\frac{-2\bar{\gamma}^2 d_I^3 d_I'}{(2+\bar{\gamma}d_I^2)^2} + \frac{2\bar{\gamma}d_I d_I'}{2+\bar{\gamma}d_I^2}\right)\right)}{4(-C+2d_I^2)\sqrt{\frac{\bar{\gamma}d_I^2}{2+\bar{\gamma}d_I^2}}} + \frac{\left((C-d_I^2)\left(\frac{2\bar{\gamma}^2 d_I(C-d_I^2)d_I'}{(2+\bar{\gamma}(C-d_I^2))^2} - \frac{2\bar{\gamma}d_I d_I'}{2+\bar{\gamma}(C-d_I^2)}\right)\right)}{4(-C+2d_I^2)\sqrt{\frac{\bar{\gamma}(C-d_I^2)}{2+\bar{\gamma}(C-d_I^2)}}} + \frac{2d_I^3\sqrt{\frac{\bar{\gamma}d_I^2}{2+\bar{\gamma}d_I^2}}d_I'}{(-C+2d_I^2)^2} - \frac{d_I\sqrt{\frac{\bar{\gamma}d_I^2}{2+\bar{\gamma}d_I^2}}d_I'}{-C+2d_I^2} - \frac{2d_I(C-d_I^2)\sqrt{\frac{\bar{\gamma}(C-d_I^2)}{2+\bar{\gamma}(C-d_I^2)}}d_I'}{(-C+2d_I^2)^2} - \frac{d_I\sqrt{\frac{\bar{\gamma}(C-d_I^2)}{2+\bar{\gamma}(C-d_I^2)}}d_I'}{(-C+2d_I^2)} \quad (15)$$

a) Bit error rate Performance: Figure 3 shows the comparison of a conventional system (refer to (8)) with a system employing SSD at various rotation angles. The simulation results are in good agreement with the closed form analytical expression (10). The figure also shows a gain of 10.7 dB for SSD over the conventional QPSK system at a bit error rate of 3×10^{-4}.

b) Union Bound: In order to calculate (10) the squared Euclidean distances (SED) along I and Q- channels, for all possible symbol combinations, need to be calculated. Figure 4 shows the upper bound on the average probability of bit error performance calculated by using (10) and by simulation. The figure shows that the average probability of bit is upper bounded by the union bound. Gray labeled QPSK signal constellation employing coordinate interleaving and constellation rotation is used. The figure shows that the bound is tight.

c) Mapping Effect: Figure 5 shows the comparison of QPSK with Gray and Natural signal constellation mapping. The figure shows that Gray mapped signal constellation is not necessarily the best choice for the entire θ range. At $\theta = 0°$, for instance, Natural mapped signal constellation outperforms Gray mapped signal constellation. This provides a possibility to design signal constellation mapping for higher MPSK signal constellations which are capable of outperforming Gray signal constellation mapping at a wider range of θ.

d) Optimal Theta: The steepest descent algorithm is used to find the optimum angles for QPSK signal constellations using (10) and (15). The optimum angle of $\theta = 17.6°$ is calculated for QPSK which leads to a gain of 10.7 dB for SSD over the conventional QPSK system at a bit error rate of 3×10^{-4} as shown in Figure 3. In [3], the reported angle is $22.5°$, and in [2] for P_s the reported angle is $\theta = 31.7°$.

e) Phase estimation effect: A system employing constellation rotation is very sensitive to phase estimation error as it is evident from the derivation of P_b. Let $\hat{\phi}$ be the estimated channel phase and $|\phi - \hat{\phi}|$ be the difference between the estimated and the actual channel phase. Figure 6 shows the simulation results for the sensitivity of the system employing coordinate interleaving and constellation rotation to the phase estimation error when coherent detection is carried out at the receiver. The figure shows that as the channel phase estimation error becomes larger ($> 20°$) the system starts to perform worse than a conventional QPSK system.

V. CONCLUSIONS

In this paper, we have investigated the performance of a system employing signal space diversity. An expression for the upper bound of P_b is calculated for MPSK signal constellation. It is shown that the P_b calculated in [3] is an expurgated bound. A new bound is calculated and it is shown that the new calculated bound is tight for the entire range of the constellation rotation angle at high SNR. Furthermore, we have shown that the Gray mapping in a system employing constellation rotation and coordinate interleaving is not necessarily the best option over the entire range of θ. Finally, we have shown that the system under-consideration is more robust against small phase estimation error than the conventional system.

ACKNOWLEDGEMENT

This work was supported by STW under McAT project DTC.6438.

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