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# A New Simulation Model for Mobile-to-Mobile Rayleigh Fading Channels

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***Abstract*— A new statistical sum-of-sinusoids simulation model is proposed for mobile-to-mobile Rayleigh fading channels and compared with existing simulation models. The new proposed model has a lower variance of the auto-correlation functions, i.e., it converges faster and has a lower correlation between the in-phase and quadrature components of the complex faded envelope than existing simulation models. This model yields adequate statistics with only 30 simulation runs.**

## I. Introduction

Mobile-to-mobile (M-to-M) communication channels, where both the transmitter and the receiver are in motion and equipped with low elevation antennas, find application in mobile ad-hoc wireless networks, intelligent transportation systems, and relay-based cellular networks. M-to-M channels differ from conventional fixed-to-mobile (F-to-M) cellular radio channels, where the base-station is stationary, elevated, and relatively free of local scattering. Akki and Haber [1], [2] showed that the received signal envelope of M-to-M channels is Rayleigh faded under non line-of-sight conditions, but the statistical properties differ from F-to-M channels. They were the first to propose a mathematical reference model for M-to-M Rayleigh fading channels. Vatalaro and Forcella [3] extended their models to account for scattering in three dimensions (3-D). Channel measurements for outdoor-to-indoor narrow-band mobile-to-mobile communications have been presented in [4] while those for outdoor-to-outdoor communications are given in [5]. More recently, measurements for wide-band mobile-to-mobile communications have been reported in [6].

Several methods for simulating M-to-M channels have been proposed in the literature. Wang and Cox [7] described a model that approximates the continuous Doppler spectrum by a discrete line spectrum. However, the correlation functions are periodic functions of the time delay, and method requires numerical integration of the Doppler spectrum [8]. Patel *et al.* [8] proposed two sum-of-sinusoids (SoS) models for M-to-M channels. Generally, SoS models approximate the underlying random processes by the superposition of a finite number of properly selected sinusoids. They can be classified as either statistical or deterministic. Deterministic SoS models have sinusoids with fixed phases, amplitudes, and Doppler frequencies for all simulation trials. Statistical SoS models leave at least one of the parameter sets (amplitudes, phases, or Doppler frequencies) as random variables that vary with each simulation trial. The statistical properties of the statistical SoS models vary for each simulation trial, but converge to the desired properties when averaged over a large number of simulation trials. An ergodic statistical model is one that converges to the desired properties in a single simulation trial.

Patel *et al.* [8] used the "double ring" concept, proposed in [9], to derive their SoS models for mobile-to-mobile channels. They first modified the Method of Exact Doppler Spread (MEDS) proposed by Pätzold *et al.* for F-to-M channels [10]. The statistical correlation functions of the faded envelope match those of the reference model only for a small range of normalized time delays ($0 \leq f_1 T_s \leq 3$). To improve the properties of their ergodic statistical model, Patel *et al.* [8] also modified a statistical SoS model proposed by Zheng *et al.* for F-to-M channels [11]. However, the model requires a large number of simulation trials (at least 50) to obtain adequate ensemble averaged statistical properties. Moreover, existing models have a notable difficulties in producing time averaged auto- and cross-correlation functions that match those of the reference model.

This paper proposes a new statistical SoS model for M-to-M Rayleigh fading channels. We employ "double ring" model, where orthogonal functions are chosen as the in-phase (I) and quadrature (Q) components of the complex faded envelope. Moreover, our new model is designed to directly generate *multiple* uncorrelated complex faded envelopes, a lacking feature in the existing models reported in [7] and [8]. The statistical properties of our model are derived and verified by simulation. Compared to existing models, this paper shows our new model has more rapidly converging ensemble average statistics, has a lower variance of the auto-correlation functions, has less correlated I and Q components, and produces uncorrelated multiple faded envelopes.

The remainder of the paper is organized as follows. Section II presents the mathematical reference model. Section III reviews the existing SoS models for M-to-M channels. Section IV describes our new statistical SoS simulation model. Section V compares the new model to previously reported models. Finally, Section VI provides some concluding remarks.

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## II. The Mathematical Reference Model

Akki and Haber's M-to-M reference model [1] defines the complex faded envelope as

$$g(t) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} \exp\{j[\omega_1 t\cos(\alpha_n) + \omega_2 t\cos(\beta_n) + \phi_n]\}, \quad (1)$$

where $N$ is the number of propagation paths, $\omega_1$ and $\omega_2$ are the maximum angular Doppler frequencies, $\alpha_n$ and $\beta_n$ are the angle of departure and the angle of arrival of the $n^{th}$ propagation path measured with respect to the Tx and Rx velocity vectors, respectively, and $\phi_n$ is the phase associated with the $n^{th}$ propagation path. It is assumed that $\alpha_n$, $\beta_n$, and $\phi_n$ are mutually independent random variables and that $\phi_n$ is uniformly distributed on the interval $[-\pi, \pi)$. Invoking the Central Limit Theorem [12], the real part $g_i(t) = \Re\{g(t)\}$ and the imaginary part $g_q(t) = \Im\{g(t)\}$ of the complex faded envelope are Gaussian random processes as $N \to \infty$. Therefore, the envelope $|g(t)|$ is Rayleigh distributed and the phase $\Theta_g(t)$ is uniformly distributed. The auto- and cross-correlation functions of the reference model, assuming omni-directional antennas and a 2-D isotropic scattering environment, are in the limit $N \to \infty$ [1], [2]

$$R_{g_i g_i}(\tau) = E[g_i(t+\tau)g_i(t)] = J_0(\omega_1\tau)J_0(\omega_2\tau), \quad (2)$$

$$R_{g_q g_q}(\tau) = E[g_q(t+\tau)g_q(t)] = J_0(\omega_1\tau)J_0(\omega_2\tau), \quad (3)$$

$$R_{g_i g_q}(\tau) = R_{g_q g_i}(\tau) = E[g_i(t+\tau)g_q(t)] = 0, \quad (4)$$

$$R_{gg}(\tau) = \frac{1}{2}E[g(t+\tau)g(t)^*] = J_0(\omega_1\tau)J_0(\omega_2\tau), \quad (5)$$

where $E[\cdot]$ is the statistical expectation operator, $(\cdot)^*$ denotes the complex conjugate operator, and $J_0(\cdot)$ is the zero-th order Bessel function of the first kind. Note that auto-correlation functions of M-to-M channels are a product of two Bessel functions in contrast to F-to-M channel which the corresponding functions involve single Bessel functions [12].

The objective of the channel simulators discussed in this paper is to reproduce the above reference model properties as faithfully and efficiently as possible.

## III. Existing Mobile-to-Mobile SoS Simulation Models

Patel *et al.* [8] were the first to propose sum-of-sinusoids (SoS) models for M-to-M channels, and used a "double ring" concept to derive their models. The "double ring" model defines two rings of uniformly spaced fixed scatterers placed, one around the transmitter (Tx) and another around the receiver (Rx). Assuming omni-directional antennas at both ends, the waves from the Tx antenna first arrive at the scatterers located on the Tx ring. Considering these fixed scatterers as "virtual base-stations" (VBS), the communication link from each VBS to the Rx is modelled as conventional F-to-M link. The signals from each VBS arrive at the Rx antenna uniformly from all directions in the horizontal plane due to isotropic scatterers located on the Rx end ring. Using this model, the received complex faded envelope is

$$g(t) = \sqrt{\frac{2}{NM}} \sum_{n=1}^{N}\sum_{m=1}^{M} \exp\{j[\omega_1 t\cos(\alpha_n) + \omega_2 t\cos(\beta_m) + \phi_{nm}]\}, \quad (6)$$

Note that the single summation in (1) is replaced by a double summation, because each wave on its way from the Tx to the Rx is reflected twice. The channel characteristics remain the same as the reference model in (1), because each path will undergo a Doppler shift due to the motion of the Tx and the Rx.

Patel *et al.* [8] first proposed an ergodic statistical SoS model. By choosing only the phases to be random variables, the statistical properties of this model converge to the desired properties in a single simulation trial. We refer to this model as Model I.

*Model I*: The complex faded envelope is $g(t) = g_i(t) + jg_q(t)$, where

$$g_i(t) = \sqrt{\frac{2}{N_i M_i}} \sum_{m=1}^{M_i}\sum_{n=1}^{N_i} \cos\left(\omega_1 t\cos\left(\alpha_n^i\right) + \omega_2 t\cos(\beta_m^i) + \phi_{nm}^i\right), \quad (7)$$

$$g_q(t) = \sqrt{\frac{2}{N_q M_q}} \sum_{m=1}^{M_q}\sum_{n=1}^{N_q} \cos\left(\omega_1 t\cos\left(\alpha_n^q\right) + \omega_2 t\cos(\beta_m^q) + \phi_{nm}^q\right), \quad (8)$$

and where $\phi_{nm}^i$ and $\phi_{nm}^q$ are independent random phases uniformly distributed on the interval $[-\pi, \pi)$. The $n^{th}$ angle of departure is equal to $\alpha_n^{i/q} = \pi(n-0.5)/(2N_{i/q})$, for $n = 1, 2, \ldots, N_{i/q}$. The $m^{th}$ angle of arrival is equal to $\beta_m^{i/q} = \pi(m-0.5)/(M_{i/q})$, for $m = 1, 2, \ldots, M_{i/q}$.

This model has the disadvantage that the statistical properties match those of the reference model only for a small range of normalized time delays ($0 \le f_1 T_s \le 3$). To improve these statistical properties, Patel *et al.* [8] also proposed a statistical SoS model. By allowing all three parameter sets (amplitudes, phases, and Doppler frequencies) to be random variables, the statistical properties of this model vary for each simulation trial, but they converge to desired ensemble averaged properties when averaged over a large number of simulation trials. We call this Model II.

*Model II*: The complex faded envelope is $g(t) = g_i(t) + jg_q(t)$, where

$$g_i(t) = \sqrt{\frac{2}{N_0 M}} \sum_{n=1}^{N_0}\sum_{m=1}^{M} \cos\left(\omega_1 t\cos\left(\alpha_n\right) + \omega_2 t\cos(\beta_m) + \phi_{nm}^i\right), \quad (9)$$

$$g_q(t) = \sqrt{\frac{2}{N_0 M}} \sum_{n=1}^{N_0}\sum_{m=1}^{M} \cos\left(\omega_1 t\sin\left(\alpha_n\right) + \omega_2 t\cos(\beta_m) + \phi_{nm}^q\right), \quad (10)$$

and where $\phi_{nm}^i$ and $\phi_{nm}^q$ are independent random phases uniformly distributed on the interval $[-\pi, \pi)$. Model II assumes $N_0$ scatterers located on the Tx ring and $M$ scatterers located on the Rx. The $n^{th}$ angle of departure is $\alpha_n = (2\pi n - \pi + \theta)/(4N_0)$, where $\theta$ is an independent random variable uniformly distributed in the interval $[-\pi, \pi)$. The $m^{th}$ angle of arrival is equal to $\beta_m = (2\pi m - \pi + \psi)/(2M)$, where $\psi$ is an independent random variable uniformly distributed in the interval $[-\pi, \pi)$.

Model II obtains better statistical properties than Model I. However, this model requires a large number of simulation trials (at least 50) to obtain adequate ensemble averaged statistical properties.

## IV. A New Statistical Simulation Model

Existing models have a noted difficulty in producing time averaged auto- and cross-correlation functions that match those of the reference model. We solve this problem by using a "double ring" concept and by choosing orthogonal functions for the in-phase (I) and quadrature (Q) components of the complex faded envelope. Also, this model is designed to directly generate multiple uncorrelated complex envelopes.

The following function is considered as the $k^{th}$ complex faded envelope

$$g_k(t) = \sum_{n=1}^{N}\sum_{m=1}^{M} C \exp\{j[\omega_1 t\cos(\alpha_{nk}) + \omega_2 t \cos(\beta_{mk}) + \phi_{nmk}]\}, \quad (11)$$

where $C = 2/\sqrt{MN}$, $\omega_1$, $\omega_2$, $\alpha_{nk}$, $\beta_{mk}$, and $\phi_{nmk}$ are the maximum radian Doppler frequencies, the random angle of departure, the random angle of arrival, and the random phase, respectively. It is assumed that $P$ independent complex faded envelopes are required ($k = 0, \ldots, P-1$) each consisting of $NM$ sinusoidal components.

The number of sinusoidal components needed for simulation can be reduced by choosing $N_0 = N/4$ to be an integer, by taking into account shifts of the angles $\alpha_{nk}$ and $\phi_{nmk}$, and by splitting the sum in (11) into four terms, *viz.*,

$$\begin{aligned} g_k(t) &= C\sum_{m=1}^{M} e^{j\omega_2 t\cos(\beta_{mk})}\sum_{n=1}^{N_0} e^{j(\omega_1 t\cos(\alpha_{nk})+\phi_{nmk})} \quad (12)\\ &+C\sum_{m=1}^{M} e^{j\omega_2 t\cos(\beta_{mk})}\sum_{n=1}^{N_0} e^{j(\omega_1 t\cos(\alpha_{nk}+\pi/2)+\phi_{nmk}+\pi/2)}\\ &+C\sum_{m=1}^{M} e^{j\omega_2 t\cos(\beta_{mk})}\sum_{n=1}^{N_0} e^{j(\omega_1 t\cos(\alpha_{nk}+\pi)+\phi_{nmk}+\pi)}\\ &+C\sum_{m=1}^{M} e^{j\omega_2 t\cos(\beta_{mk})}\sum_{n=1}^{N_0} e^{j(\omega_1 t\cos(\alpha_{nk}+3\pi/2)+\phi_{nmk}+3\pi/2)}. \end{aligned}$$

Equation (12) simplifies as

$$\begin{aligned} g_k(t) &= \frac{2}{\sqrt{N_0 M}} \quad (13)\\ &\{\sum_{n=1}^{N_0}\sum_{m=1}^{M}\cos(\omega_2 t\cos(\beta_{mk}))\cos(\omega_1 t\cos(\alpha_{nk})+\phi_{nmk})\\ &+j\sum_{n=1}^{N_0}\sum_{m=1}^{M}\sin(\omega_2 t\cos(\beta_{mk}))\sin(\omega_1 t\sin(\alpha_{nk})+\phi_{nmk})\}. \end{aligned}$$

Based on $g_k(t)$, we define our new statistical simulation model.

*Definition*: The $k^{th}$ complex faded envelope is $g_k(t) = g_{ik}(t) + jg_{qk}(t)$, where

$$g_{ik}(t) = \frac{2}{\sqrt{N_0 M}}\sum_{n=1}^{N_0}\sum_{m=1}^{M}\cos(\omega_2 t\cos(\beta_{mk})) \cos(\omega_1 t\cos(\alpha_{nk})+\phi_{nmk}), \quad (14)$$

$$g_{qk}(t) = \frac{2}{\sqrt{N_0 M}}\sum_{n=1}^{N_0}\sum_{m=1}^{M}\sin(\omega_2 t\cos(\beta_{mk})) \sin(\omega_1 t\sin(\alpha_{nk})+\phi_{nmk}). \quad (15)$$

It is assumed that $P$ independent complex envelopes are desired ($k = 0, \ldots, P-1$), each having $MN_0$ sinusoidal terms in the I and Q components. The angles of departures and the angles of arrivals are chosen as follows:

$$\alpha_{nk} = \frac{2\pi n}{4N_0} + \frac{2\pi k}{4PN_0} + \frac{\theta - \pi}{4N_0}, \quad (16)$$

$$\beta_{mk} = 0.5\left(\frac{2\pi m}{M} + \frac{2\pi k}{PM} + \frac{\psi - \pi}{M}\right), \quad (17)$$

for $n = 1, \ldots, N_0$, $m = 1, \ldots, M$ $k = 0, \ldots, P-1$. The angles of departures and the angles of arrivals in the $k^{th}$ complex faded envelope are obtained by rotating the angles od departures and the angles of arrivals in the $(k-1)^{th}$ complex envelope by $(2\pi)/(4PN_0)$ and $(2\pi)/(2PM)$, respectively. The parameters $\phi_{nmk}$, $\theta$, and $\psi$ are independent random variables uniformly distributed on the interval $[-\pi, \pi)$.

Our statistical model can be shown to exhibit properties (2)-(5) of the reference model. For brevity, only the derivation of the auto-correlation function of the in-phase component is presented in Appendix A. Other properties can be derived in an analogous fashion. Figures 1 and 2 show that, for $N_0 = M = P = 8$ and $N_{stat} = 30$ trials, the auto- and cross-correlations of the I and Q components, and the auto- and cross-correlations of the complex faded envelopes approach those of the reference model.

## V. Performance Evaluation

This section compares the performance and complexity of our new model with Models I and II. In all simulations, we use a normalized sampling period $f_1 T_s = 0.01$ ($f_1 = f_2$ are the maximum Doppler frequencies and $T_s$ is the sampling period) and $M = N_0 = P = 8$. For Model II, we use $N_i = M_i = 8$

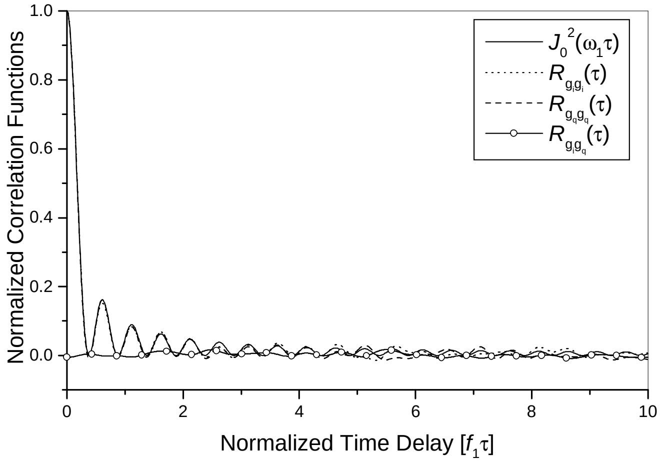


Fig. 1. Theoretical and simulated ($N_{stat} = 30$) auto-correlation functions and the cross-correlation function of the in-phase and the quadrature component of the new statistical M-to-M model.

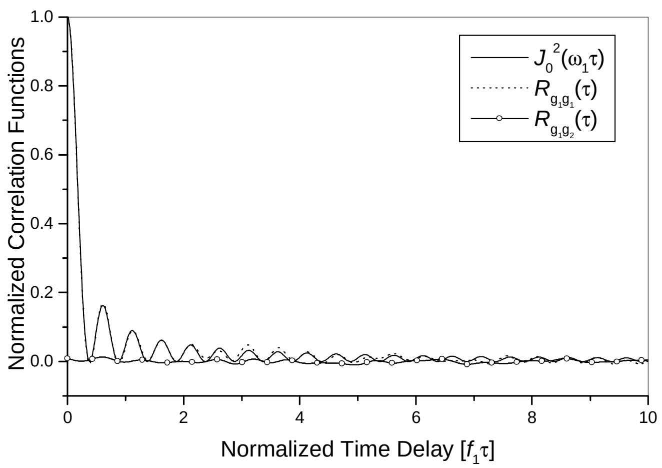


Fig. 2. Theoretical and simulated ($N_{stat} = 30$) auto-correlation functions and the cross-correlation function of the first and the second faded envelopes in the new statistical M-to-M model.

and $N_q = M_q = 9$ to obtain a complex envelope with uncorrelated quadrature components. Using these parameters, we have calculated the mean square errors (MSE) and maximum deviations (MAX) from the theoretical value (zero) for the normalized cross-correlations of the I and Q components, and for the normalized cross-correlations of the first and the second faded envelopes. The results are shown in Table I. Note that different simulation trials yield slightly different simulation results. To estimate these differences, the variances are computed by averaging over $10^3$ simulation trials. The variances of the normalized cross-correlations of the I and Q components, and the variances of the normalized cross-correlations of the first and the second faded envelopes are also shown in Table I. From Table I, we conclude that our new statistical model with $N_{stat} = 1$ has cross-correlations similar to Model I and Model II with $N_{stat} = 1$. The new statistical model with $N_{stat} = 30$ performs similar to Model II with $N_{stat} = 50$ and significantly better than Model I. Increasing the number of simulation trials to $N_{stat} = 50$ yields a lower cross-correlation between the I and Q components of the complex faded envelope.

TABLE I

MEAN SQUARE ERROR, MAXIMUM DEVIATION AND VARIATIONS.

| Simulators | Model I | Model II $N_{stat} = 1$ | Model II $N_{stat} = 50$ | New Stat. Model $N_{stat} = 1$ | New Stat. Model $N_{stat} = 30$ | New Stat. Model $N_{stat} = 50$ |
|---|---|---|---|---|---|---|
| MSE($R_{g_ig_q}$) | $1.8 \cdot 10^{-3}$ | $1.1 \cdot 10^{-3}$ | $3.16 \cdot 10^{-5}$ | $8.81 \cdot 10^{-4}$ | $4.39 \cdot 10^{-5}$ | $2.05 \cdot 10^{-5}$ |
| Max($R_{g_ig_q}$) | $9.61 \cdot 10^{-2}$ | $5.91 \cdot 10^{-2}$ | $1.27 \cdot 10^{-2}$ | $6.84 \cdot 10^{-2}$ | $1.28 \cdot 10^{-2}$ | $7.0 \cdot 10^{-3}$ |
| Var($R_{g_ig_q}$) | $2.74 \cdot 10^{-6}$ | $4.59 \cdot 10^{-6}$ | $6.67 \cdot 10^{-9}$ | $3.11 \cdot 10^{-6}$ | $5.69 \cdot 10^{-9}$ | $2.51 \cdot 10^{-9}$ |
| MSE($R_{g_1g_2}$) | $5.8 \cdot 10^{-3}$ | $1.1 \cdot 10^{-3}$ | $1.31 \cdot 10^{-5}$ | $7.02 \cdot 10^{-4}$ | $3.59 \cdot 10^{-5}$ | $1.26 \cdot 10^{-5}$ |
| Max($R_{g_1g_2}$) | $15.9 \cdot 10^{-2}$ | $6.34 \cdot 10^{-2}$ | $0.63 \cdot 10^{-2}$ | $6.57 \cdot 10^{-2}$ | $0.96 \cdot 10^{-2}$ | $5.6 \cdot 10^{-3}$ |
| Var($R_{g_1g_2}$) | $1.24 \cdot 10^{-4}$ | $1.77 \cdot 10^{-6}$ | $7.38 \cdot 10^{-10}$ | $1.22 \cdot 10^{-6}$ | $3.0 \cdot 10^{-9}$ | $4.39 \cdot 10^{-10}$ |

Figures 3 and 4 compare the variance of the auto-correlation functions of the quadrature components averaged over $10^3$ simulation trials, for the reference model, Models I and II, and the new statistical model. For the statistical models, the variance can be defined as [13] $\mathrm{Var}[R(\cdot)] = E[|\hat{R}(\cdot) - \lim_{N\to\infty} R(\cdot)|^2]$, where $\hat{R}(\cdot)$ denotes the time averaged correlation function and $R(\cdot)$ denotes the correlation function defined as in (2)-(5). For Model I, the equivalent quantity is the squared error $|\hat{R}(\cdot) - R(\cdot)|^2$. The variance for the reference model is obtained using $\mathrm{Var}[R_{g_ig_i}(\tau)] = \mathrm{Var}[R_{g_qg_q}(\tau)] = \left[1 + J_0(2\omega_1\tau)J_0(2\omega_2\tau) - 2J_0^2(\omega_1\tau)J_0^2(\omega_2\tau)\right]/(2N)$, as defined in [8]. The variance provides a measure of the usefulness of the model in simulating the desired channel with a finite $N$. A lower variance means that a smaller number of simulation trials are needed to achieve the desired statistical properties and, hence, the corresponding model is better. Since the reference model does not exploit the symmetry of the "double ring" model as the new statistical model and Models I and II, for fair comparison, we use $N = 4N_0 \times 2M = 512$ sinusoids for simulation of the reference model. From Figure 3, we conclude that our new statistical model with $N_{stat} = 1$ has a similar variance as Model I and a lower variance than Model II with $N_{stat} = 1$. However, Figure 3 shows that all three models do not perform as well as the reference model. An increase in the number of simulation trials to $N_{stat} = 30$ in the new statistical model yields a significantly lower variance of the auto-correlation function of the quadrature component. Figure 4 shows that the new statistical model with $N_{stat} = 30$ outperforms Models I and II and the reference model.

Table I and Figures 3 and 4 show that our new statistical model converges faster than other statistical models, has a lower variance of the auto-correlation functions, and has a lower correlation between the I and Q components of the complex faded envelope than Models I and II. Adequate statistics can be achieved with only 30 trials using our new statistical model.

## VI. CONCLUSIONS

This paper proposed a new statistical SoS model for M-to-M channels. The properties of the proposed model have been derived and verified using simulation. The statistics of

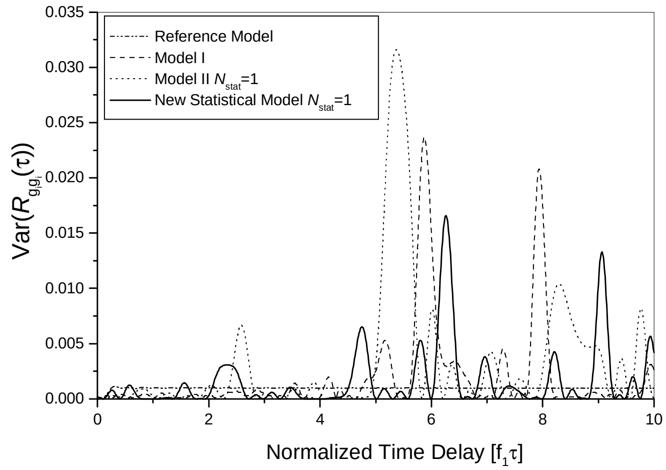


Fig. 3. Variance of the auto-correlation function of the quadrature component.

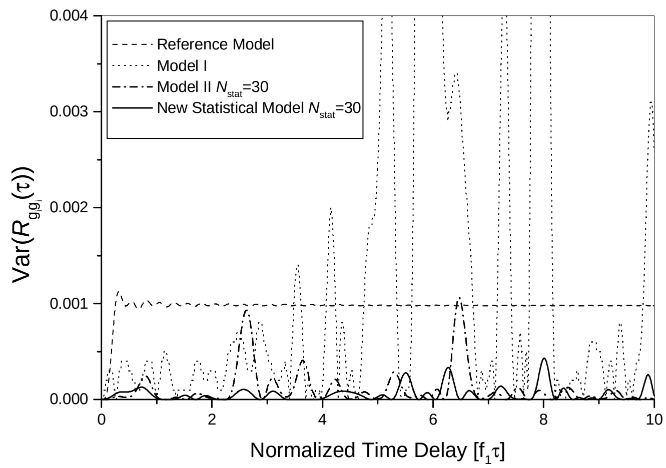


Fig. 4. Variance of the auto-correlation function of the quadrature component.

the new model match those of the reference model for a large range of normalized time delays ($0 \leq f_1 T_s \leq 8$). Our new statistical model converges faster, has a lower variance of the auto-correlation functions and has a lower correlation between the I and Q components of the complex faded envelope than other models discussed in this paper. Finally, unlike existing M-to-M models, our model generates multiple uncorrelated faded envelopes as well.

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## APPENDIX A

### DERIVATION OF AUTO-CORRELATION FUNCTION OF THE IN-PHASE COMPONENT

The auto-correlation function of the I component of the $k^{th}$ complex faded envelope is

$$R_{g_{ik}g_{ik}}(\tau) = E[g_{ik}(t+\tau)g_{ik}(t)] = \frac{4}{N_0 M} \tag{18}$$

$$\sum_{n,m=1}^{N_0,M} \sum_{p,r=1}^{N_0,M} E[\cos(\omega_2(t+\tau)\cos(\beta_{mk}))\cos(\omega_2 t\cos(\beta_{pk}))$$

$$\cos(\omega_1(t+\tau)\cos(\alpha_{nk}) + \phi_{nmk})\cos(\omega_1 t\cos(\alpha_{rk}) + \phi_{prk})]$$

$$= \frac{1}{N_0 M}\sum_{n=1}^{N_0}\sum_{m=1}^{M} E\left[\cos(\omega_2\tau\cos(\beta_{mk}))\cos(\omega_1\tau\cos(\alpha_{nk}))\right]$$

$$= \frac{1}{M}\sum_{m=1}^{M}\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(\omega_2\tau\cos(\frac{2\pi m}{2M} + \frac{2\pi k}{2PM} + \frac{\psi-\pi}{2M}))d\psi$$

$$\frac{1}{N_0}\sum_{n=1}^{N_0}\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(\omega_1\tau\cos(\frac{2\pi n}{4N_0} + \frac{2\pi k}{4PN_0} + \frac{\theta-\pi}{4N_0}))d\theta.$$

As in [14], the derivation can be completed by replacing the variables of integration, $\theta$ and $\psi$, with $\gamma_{nk} = (2\pi n)/(4N_0) + (2\pi k)/(4PN_0) + (\theta-\pi)/(4N_0)$ and $\delta_{mk} = (2\pi m)/(2M) + (2\pi k)/(2PM) + (\psi-\pi)/(2M)$, respectively. Finally,

$$\lim_{N_0,M\to\infty} R_{g_{ik}g_{ik}}(\tau) = J_0(\omega_1\tau)J_0(\omega_2\tau). \tag{19}$$

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