Statistical Modeling of a Shielded Wireless Charging Device

Alenka Zajić * and Zoya Popović #
* Georgia Institute of Technology, Atlanta, GA 30332 USA
# University of Colorado-Boulder, Boulder CO 80309 USA

***Abstract*— In this paper, a geometrical propagation model for wireless powering channel in an over-moded cavity loaded with conductive objects is presented. The cavity is designed for wireless charging of multiple objects simultaneously in a shielded environment. A conductive resonator approximately 50 cm $\times$ 28 cm $\times$ 18 cm in size is excited by a 10-GHz probe and the power distribution inside the charging box is measured using a rectenna and power management circuit for various positions of other conductive objects within the resonator. Based on the geometrical model, a simulation model for multipath fading in this cavity is developed and compared with measured statistical data with 2000 samples. The results show good agreement between measured and simulated received power levels and distributions.**

I. Introduction

Research in wireless powering devices has grown rapidly in popularity over the past decade [1]. Methods of wireless power transfer include near-field coupling for high-power vehicle applications [2], lower power inductive charging [3], [4], far-field power beaming for powering aircraft or beaming power though the atmosphere [5], [6], and far-field scavenging/harvesting to power embedded or hard-to-reach sensors where a battery is not easily changed [7].

It is well documented that near-field wireless power transfer has efficiency and sensitivity limitations due to coil orientation and de-tuning from undesired coupling, e.g., [8]. In addition, FCC/IEEE electromagnetic emission guidelines constrain the power level that can be used even in relatively short-range inductive powering [9]. In contrast, far-field powering implies plane-wave propagation between antennas at longer range. It can be done without line of sight, and is less sensitive to the orientation and position relative to the transmitting antenna. This approach differs from RF identification devices (RFIDs) in that powering is independent of signal transmission and is done at different time scales, power levels, and frequencies.

In addition, far-field powering can be used for watt-level shielded (e.g. in a metal enclosure) wireless powering of multiple electronic devices. A photograph of such a cavity is shown in Figure 1, where a transmit probe is shown along with a receive probe which is connected to a rectifer. In [10], a metal enclosure at 2.2 GHz was demonstrated with the goal of watt-level shielded wireless powering of multiple electronic devices is investigated. A similar approach was used in [11], where frequency-dependent meta-material cavity walls were implemented in order to control magnetic field leakage. Recently, experimental characterization of an over-moded metal waveguide cavity that operates at 10 GHz is presented in [12]. In this work, mechanical stirring, conductive and absorptive object loading, and frequency modulation are investigated in order to increase power density uniformity within the cavity. While the results show that all these methods increase the power density uniformity, predicting the optimal position of mechanical stirrers or conductive and absorptive loading was not performed. Full-wave simulations for a statistical analysis would be very time consuming, so an analytical model is desired.

As a first step in this direction, this paper proposes a geometrical propagation model for wireless powering channel in an over-moded cavity loaded with conductive objects. Based on the geometrical model, a simulation model for multipath fading in this cavity is developed and compared with measured data. The results show good agreement between measured and simulated received power levels and distributions.

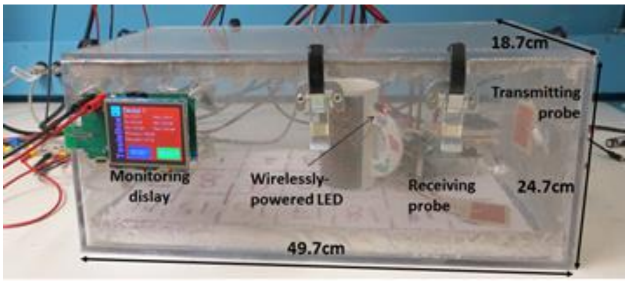


Fig. 1. Photograph of an over-moded cavity for wireless powering of multiple electronic devices. The cavity walls are made of a metal mesh that is reflective at the excitation frequency and semi-transparent optically. The transmitting probe is fed by a 0.25 W amplifier at 10 GHz and two devices are powered in the photograph: a receive probe connected to a rectifier and a rectenna-fed LED mounted on a coffee-cup.

The remainder of the paper is organized as follows. Section II introduces the geometrical model and presents a parametric model for the over-moded cavity loaded with conductive objects. Section III describes over-moded cavity characterization and derives space-time correlation function. Section IV compares measured and simulated received power levels and distributions.

II. Statistical Model for Over-moded Cavity Loaded with Conductive Objects

This paper considers propagation between a stationary transmitter (T_x) and a single stationary receiver (R_x) placed in an over-moded cavity loaded with conductive objects. For this analysis, both the $\mathrm{T_x}$ and $\mathrm{R_x}$ probes are assumed to be omni-directional.

An over-moded cavity is an electrically large shielded metal box that, when operated at high enough frequency, permits the excitation of a large number of modes with closely proximate resonant frequencies. A diagram of an over-moded cavity is shown in Fig. 2. It is desirable to operate at frequencies that have very high mode density (i.e. the number of modes that can be excited per given bandwidth of frequencies should be large). The field distribution of the excited modes creates locations of high and low field magnitudes called hot and cold spots, respectively. In order to increase power density uniformity within the cavity, some method of relocating the hot and cold spots between measurement samples is needed. In this paper, we choose to perturb the field within the over-moded cavity using randomly placed conductive objects.

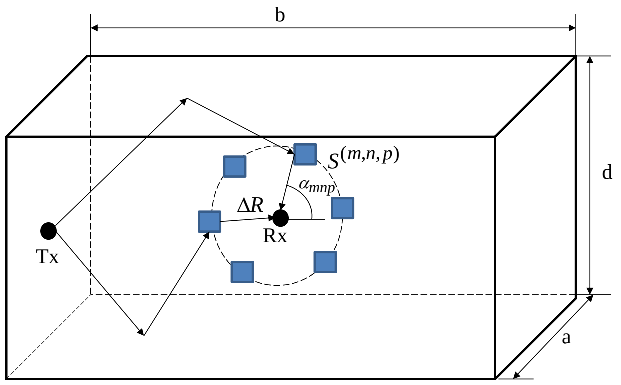


Fig. 2. The geometrical model of over-moded cavity excited with one transmit probe Tx and loaded with conductive scatterers (blue squares). All walls of the parallelepiped are assumed to be PEC, while the Tx and Rx probes are assumed to be isotropic.

To model the impact of conductive objects in the cavity, it is assumed that every mode generated in the cavity, before it arrives at the $\mathrm{R_x}$, impinges on one of the conductive objects, i.e. scatterers $S^{(m,n,p)}$, uniformly distributed on a circle around the $\mathrm{R_x}$. For the model presented here, theoretically the scatterers should be positioned on a sphere around the $\mathrm{R_x}$, but due to practical constraints regarding positions of the $\mathrm{T_x}$ and $\mathrm{R_x}$ and loading objects (it is unlikely that they will be hanging in the air), we assume only a two-dimensional distribution of scatterers.

In the model, it is assumed that N_s fixed omnidirectional scatterers are uniformly distributed around the circle with radius ΔR. The number of scatterers N_s depends on the number of modes generated in the over-moded cavity, because each of the modes will be perturbed by the scatterer in the vicinity of the receiver. The number of modes in a rectangular cavity is a function of frequency and can be calculated as [13]

$$N_s(f) \approx \frac{8\pi f^3 V}{3c_0^3}, \tag{1}$$

where f is the frequency of excitation, $V = a \cdot b \cdot d$ is the volume of the rectangular cavity, and c_0 is the speed of light in a vacuum.

The radius of the circle ΔR represents the additional excess path traveled by each mode in the over-moded cavity. This excess path can be related to frequency deviation required to change the excited mode distribution as follows:

$$\Delta R = \frac{c_0}{\Delta f}, \tag{2}$$

where frequency deviation Δf can be calculated from [14]

$$\Delta f \approx \frac{c_0^3}{8\pi V f^2}. \tag{3}$$

Finally, the symbol $\alpha_{m,n,p}$ denotes the angles of arrivals of the waves that scatter from the scatterers $S^{(m,n,p)}$ before arriving at the receiver.

To describe the electromagnetic field at the receiver in the over-moded cavity, we start with the dyadic green's function for a rectangular cavity [15] given by

$$\vec{E}_c(\vec{r},t) = \frac{1}{N_s}\sum_m\sum_n\sum_p E_{m,n,p}(\vec{r}_{Rx})e^{j\omega t}, \tag{4}$$

where $E_{m,n,p}(\vec{r}_{Rx})$ is defined as

$$E_{m,n,p}(\vec{r}_{Rx}) = \frac{E_0}{8}\sum_{i=1}^{8}(-1)^{i+1}\frac{e^{-jkR_{i,m,n,p}}}{R_{i,m,n,p}}. \tag{5}$$

The amplitude of the electric field E_0 can be obtained in terms of cavity parameters, i.e.,

$$E_0 = \sqrt{\frac{QP_t}{\omega\epsilon_0 V}}, \tag{6}$$

where P_t is the power transmitted into the cavity, $Q = f/\Delta f$ is the cavity quality factor, V is the volume of the cavity, ω is the angular frequency of the transmitted signal, and ϵ_0 is the dielectric constant in vacuum. The distance $R_{i,m,n,p}$ can be calculated from cavity geometry and positions of the $\mathrm{T_x}$ and $\mathrm{R_x}$ as follows:

$$R_{i,m,n,p} = \sqrt{(x_i^2 + 2ma)^2 + (y_i + 2nb)^2 + (z_i + 2pd)^2}, \tag{7}$$

where spatial coordiantes x_i, y_i, and z_i are obtained as follows:

$$x_i = \begin{cases} x_T - x_R, & i = 1,2,3,8 \\ x_T + x_R, & i = 4,5,6,7 \end{cases}, \tag{8}$$

$$y_i = \begin{cases} y_T - y_R, & i = 1,2,5,6 \\ y_T + y_R, & i = 3,4,7,8 \end{cases}, \tag{9}$$

$$z_i = \begin{cases} z_T - z_R, & i = 1,3,5,7 \\ z_T + z_R, & i = 2,4,6,8 \end{cases}, \tag{10}$$

and vectors (x_T,y_T,z_T) and (x_R,y_R,z_R) denote the coordinates of the $\mathrm{T_x}$ and the $\mathrm{R_x}$, respectively.

To add effect of the conductive objects in the resonant cavity, we propose to modify the dyadic green's function for rectangular cavity by accounting for additional random excess path and phase change due to wave interaction with conductive objects. The electromagnetic field in the over-moded cavity with conductive objects can be described as follows:

$$\vec{E}(\vec{r},t) = \frac{1}{N_s}\sum_m\sum_n\sum_p E_{m,n,p}(\vec{r}_{Rx}) \times e^{-jk\cos\alpha_{m,n,p}\Delta R + j\phi_{m,n,p} + j\omega t}, \quad (11)$$

where k is the wave number, $\alpha_{m,n,p}$ is the random angle of arrival, $\phi_{m,n,p}$ is the random phase and ΔR is the excess path defined in (2). It is assumed that the phases $\phi_{m,n,p}$ and angles of arrival $\alpha_{m,n,p}$ are independent random variables uniformly distributed on the interval $[-\pi,\pi)$. Using the assumptions introduced above and the Central Limit Theorem, we can conclude that $\vec{E}(\vec{r},t)$ is independent zero-mean complex Gaussian random processes.

III. Characterization of Over-Moded Cavity with Conductive Loading

The averaged power received by the probe can be obtained as [13]

$$< P_r >= \frac{|\vec{E}(\vec{r},t)|^2}{Z_0} < A_e > \quad (12)$$

where Z_0 is the impedance of free space and $< A_e >$ is the antenna effective area averaged over incidence angles and polarization obtained as

$$< A_e >= \frac{\lambda^2}{8\pi}(1-|\Gamma|^2)\eta_A, \quad (13)$$

where $|\Gamma|$ is the probe impedance mismatch, and η_A is an antenna efficiency factor.

To this point, field properties at a point have been considered. Real antennas and test objects have significant spatial extent and the spatial correlation function of the fields is important in understanding responses of extended objects in the over-moded cavity. Here, we derive a spatial correlation function of the electric field in (11).

The normalized space-time correlation function between the electromagnetic fields $\vec{E}(\vec{r}_1,t)$ and $\vec{E}(\vec{r}_2,t+\Delta t)$ is defined as

$$\rho(\vec{r}_1,\vec{r}_2,\Delta t) = \frac{\mathrm{E}\left[\vec{E}(\vec{r}_1,t)\vec{E}(\vec{r}_2,t+\Delta t)^*\right]}{\sqrt{\mathrm{E}[|\vec{E}(\vec{r}_1,t)|^2]\mathrm{E}[|\vec{E}(\vec{r}_2,t)|^2]}}, \quad (14)$$

where $\vec{r}_1$ and $\vec{r}_2$ are two arbitrary locations, $(\cdot)^*$ denotes complex conjugate operation, and $\mathrm{E}[\cdot]$ is the statistical expectation operator.

Substituting (11) into (14), the space-time correlation function can be written as

$$\rho(\vec{r}_1,\vec{r}_2,\Delta t) \quad (15)$$
$$= \frac{1}{N_s}\sum_{m,n,p}\mathrm{E}\left[|E_{m,n,p}(\vec{r}_{Rx})|^2 e^{jk\cos\alpha_{m,n,p}(\vec{r}_2-\vec{r}_1)-j\omega\Delta t}\right].$$

The expectation in (15) can be evaluated by averaging over all spatial angles as follows [16]

$$\rho(\vec{r}_1,\vec{r}_2,\Delta t) \quad (16)$$
$$= \frac{1}{N_s}\sum_{m,n,p}|E_{m,n,p}(\vec{r}_{Rx})|^2 e^{-j\omega\Delta t}\int_{-\pi}^{\pi}\frac{1}{2\pi}e^{jk(\vec{r}_2-\vec{r}_1)\cos\alpha}d\alpha$$
$$= \frac{1}{N_s}\sum_{m,n,p}|E_{m,n,p}(\vec{r}_{Rx})|^2 e^{-j\omega\Delta t}J_0(k(\vec{r}_2-\vec{r}_1)),$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

Note that the correlation function derived here can be a useful tool to predict the optimal position of the conductive and/or dielectric scatterers in the over-moded cavity.

IV. Comparison with Measured Data

In this section, we compare the averaged receive power obtained using the model in Section II with the measured averaged received power inside a cavity with inside dimensions of $a = 24.7$ cm, $b = 49.7$ cm, and $d = 18.7$ cm, at 10 GHz [12]. The 0.25-W X-band transmitter probe is placed at $(x_T, y_T, z_T) = (a/2, 0, 0.05\,\mathrm{cm})$, while a single receiver probe is placed at $(x_R, y_R, z_R) = (0, b/2, d/2)$. Using Eq. 1, the number of modes is calculated to be $N_s = 7122$, and the frequency deviation is found to be $\Delta f \approx \pm 467$ kHz. Knowing the number of modes, Eq. (11) is used to calculate the electric field and subsequently the average received power from Eq. (12).

Since any change in the volume of the cavity will change the excited mode distribution, a set of experiments examining the effect of placing random scattering objects inside the cavity is performed. During the experiment, 11 metal scatterers of various shapes and sizes are placed at numerous positions and configurations within the cavity, and one of the measured scenarios is shown in Figure 3. The Matlab simulations and measurements are repeated for 2000 configurations, and the power is measured and simulated at the probe for each scenario. In simulations, the total number of modes N_s is equally distributed over all three coordinates, i.e., m = 20, n = 19, and p = 19. The measured results are shown in Fig. 4(a), while the simulated results are shown in Fig. 4(b). Figure 4 shows good agreement between simulated and measured power distributions in the over-moded cavity. We can also observe that the scatterers mixed the modes to a wide range of measured power densities incident on the static Rx probe. Additionally, it was observed that even a small change in location of a metal object, such as block A in Figure 3, had an effect on the power incident on the static receive probe, but that the metal scatterers did not affect the power level at the peak of the distribution from the 4 dBm center measured for an empty cavity with a single receive probe.

V. Conclusions

A geometrical propagation model for wireless powering channel in an over-moded cavity loaded with conductive

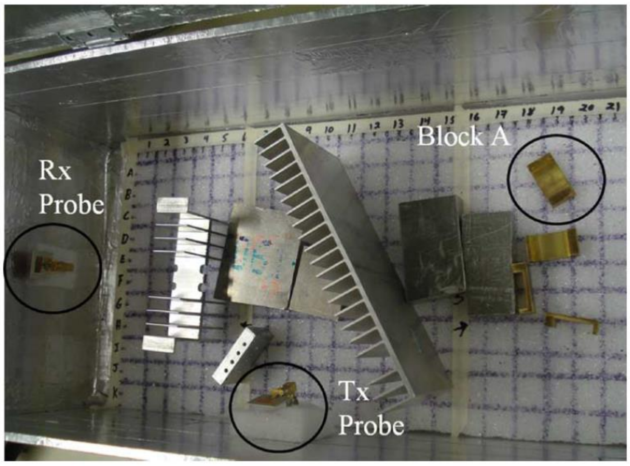
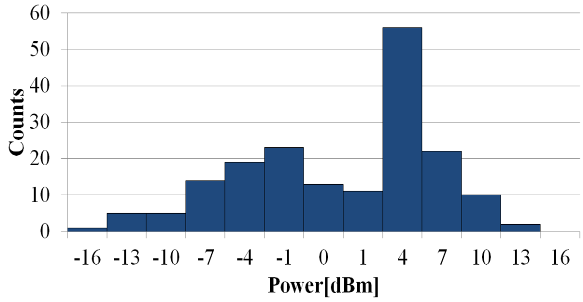


Fig. 3. Cavity with a multitude of metal scattering surfaces. Many shapes and sizes are placed in a mixed fashion throughout the box; shorting to the walls, blocking the transmit probe, blocking the receive probe, and stacking the metal.

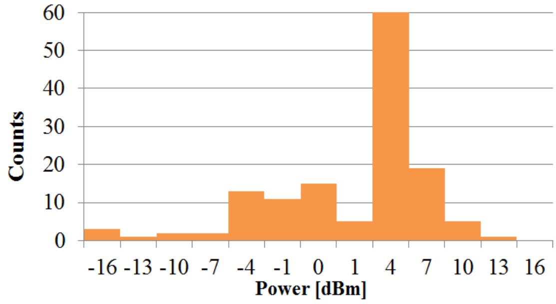
objects is shown. Based on the geometrical model, a simulation model for multipath fading in a rectangular cavity with dimensions of $\lambda_0 \times \lambda_0 \times \lambda_0$ is developed and compared with measured data for a 0.25-W transmitted power at 10GHz. The results show good agreement between measured and simulated received power levels and distributions. Specifically, the power level at the peak of the distribution of 4 dBm is well predicted, and the shape of the power distribution exhibits a secondary maximum in both measurement and simulations.

The higher power level distribution is better predicted than the lower power level shape, which could be attributed to the assumptions used in the simulations, e.g. the isotropic approximation for the probes. Note that presented analysis is done for conductive scatterers, but it can be extended for the cavity loaded with several devices being charged, i.e., when scatterers have more dielectric then conductive properties.

The wireless powering cavity discussed in this paper offers the possibility of efficient and shielded powering of multiple electronic devices. The maximum number of devices that allow efficient charging will be determined by the statistical power distribution throughout the cavity when multiple objects are randomly placed throughout the volume. The approach scales with frequency, size, power and number of powered devices, and the presented model allows a statistical prediction of the power levels and distribution. The model is validated by experimental data and can be used for future designs of scalable over-moded cavities.



(a) Measured received power in over-moded cavity with metal object steering.



(b) Simulated received power in over-moded cavity with metal object steering.

Fig. 4. Comparison of histograms of measured and simulated power from static probe with metal scatterers randomly placed inside the cavity.

References

[1] K. Wu, D. Choudhury, and H. Matsumoto, "Wireless power transmission, technology, and applications [scanning the issue]," *Proc. IEEE*, vol. 101, no. 6, pp. 1271-1275, June 2013.

[2] "Commercial near field wireless power transfer system for public bus systems," Wireless Advanced Vehicle Electrification, Oct. 1, 2013 [Online]. Available: http://www.waveipt.com/

[3] E. G. Kilinc, B. Canovas, F. Maloberti, and C. Dehollaini, "Intelligent cage for remotely powered freely moving animal telemetry systems," *Proc. IEEE Int. Symp. Circuits Syst.*, May 20-23, 2012, pp. 2207-2210.

[4] "Qi Standard for Interoperability for Wireless Power Transfer Systems", Wireless Power Consortium, Oct. 1, 2013 [Online]. Available: http://www.wirelesspowerconsortium.com/

[5] T. W. R. East, "A self-steering array for the SHARP microwave-powered aircraft," *IEEE Transactions on Antennas and Propagation*, vol. 40, no. 12, pp. 1565-1567, December 1992.

[6] Y. Pinhasi, I. M. Yakover, A. L. Eichenbaum, and A. Gover, "Efficient electrostatic-accelerator free-electron masers for atmospheric power beaming," *IEEE Transactions on Plasma Sci.*, vol. 24, no. 3, pp. 1050-1057, June 1996.

[7] Z. Popovic, E. Falkenstein, D. Constinett, R. Zane, "Low-power far-field wireless powering for wireless sensors," *Proceedings of the IEEE, Special Issue on Wireless Powering*, Vol. 101, No. 6, pp. 1397–1409, June 2013.

[8] M. Kiani and M. Ghovanloo, "Afigure-of-merit for design of high performance inductive power transmission links for implantable microelectronic devices," *Proc. Annu. Int. Conf. IEEE Eng. Med. Biol. Soc.*, Aug.Sep. 28–1, 2012, pp. 847–850.

[9] A. Christ, M. G. Douglas, J. M. Roman, E. B. Cooper, A. P. Sample, B. H. Waters, J. R. Smith, and N. Kuster, "Evaluation of wireless resonant power transfer systems with human electromagnetic exposure limits," *IEEE Trans. Electromagn. Compatibil.*, vol. 55, no. 2, pp. 265–274, Apr. 2013.

[10] S. Rahimizadeh, S. Korhummel, B. Kaslon, and Z. Popovic, "Scalable adaptive wireless powering of multiple electronic devices in an over-moded cavity," *Proc. IEEE Wireless Power Transfer Conf.*, May 15–16, 2013, pp. 84–87.

[11] C.-L. Liou, C.-G. Kuo, M.-L. Lee, and S.-G. Mao, "Wireless charging system of mobile handset using metamaterial-based cavity resonator,"*IEEE MTT-S Int. Microw. Symp. Dig.*, pp. 1–3, 17–22 June 2012.

[12] S. Korhummel, A. Rosen, and Z. Popovic, "Over-moded cavity for multiple-electronic-device wireless charging," *IEEE Transactions on Microwave Theory and Techniques*, vol. 62, no. 4, pp. 1074–1079, April 2014.

[13] D. A. Hill, Electromagnetic Fields in Cavities. Hoboken, NJ, USA:Wiley, 2009.

[14] D. A. Hill, *Motivation for statistical approaches, in Electromagnetic Fields in Cavities.* Hoboken, NJ, USA:Wiley, 2009.
[15] M. A. K. Hamed and W. A. Johnson, “Ray-optical solution for the dyadic green’s function in a rectangular cavity,” *IEEE Electronic Letters*, vol. 6, no. 10, pp. 317–319, May 1970.
[16] A. G. Zajić, *Mobile-to-Mobile Wireless Channels*. Artech House, Boston MA, January 2013.