GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Problem Set #1

Assigned: 29 Sept 1998
Due Date: 5 Oct 1998 (MONDAY)

Reading: In *DSP First*, Chapter 2 on *Sinusoids*, pages 9–43.

The web site for the course uses Web-CT: `http://webct.ece.gatech.edu`
Your initial password is digits 4 through 8 of your SSN.

ALL of the **STARRED** problems will have to be turned in for grading.

Some of the problems have solutions that can be found on the CD-ROM. Next week a solution will be posted to the web on Tuesday, 6-Oct. After the HW posting late homework will be given a zero.

PROBLEM 1.1*:

Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -3 - j4$ and $z_2 = 2e^{j\pi/4}$.

(a) Conjugate: z_1^*
(b) jz_2
(c) z_2/z_1
(d) z_2^2
(e) $z_1^{-1} = 1/z_1$
(f) $z_1 z_1^*$
(g) $z_1 + z_2^*$
(h) z_1/z_2
(i) $z_1 z_2$

Note: z^* means the "conjugate" of z.

PROBLEM 1.2: (P-2.5, F-94)

Simplify the following complex-valued expressions:

(a) $3e^{j\pi/3} + 4e^{-j\pi/6}$

(b) $(\sqrt{3} - j3)^{10}$

(c) $(\sqrt{3} - j3)^{-1}$

(d) $(\sqrt{3} - j3)^{1/3}$ (How many different answers can be found?)

(e) $\Re e\{je^{-j\pi/3}\}$

Give the answers in *both* Cartesian $(x + jy)$ and polar form $(re^{j\theta})$.

PROBLEM 1.3: (P-2.1, F-94)

Evaluate the following and give the answer in rectangular form.

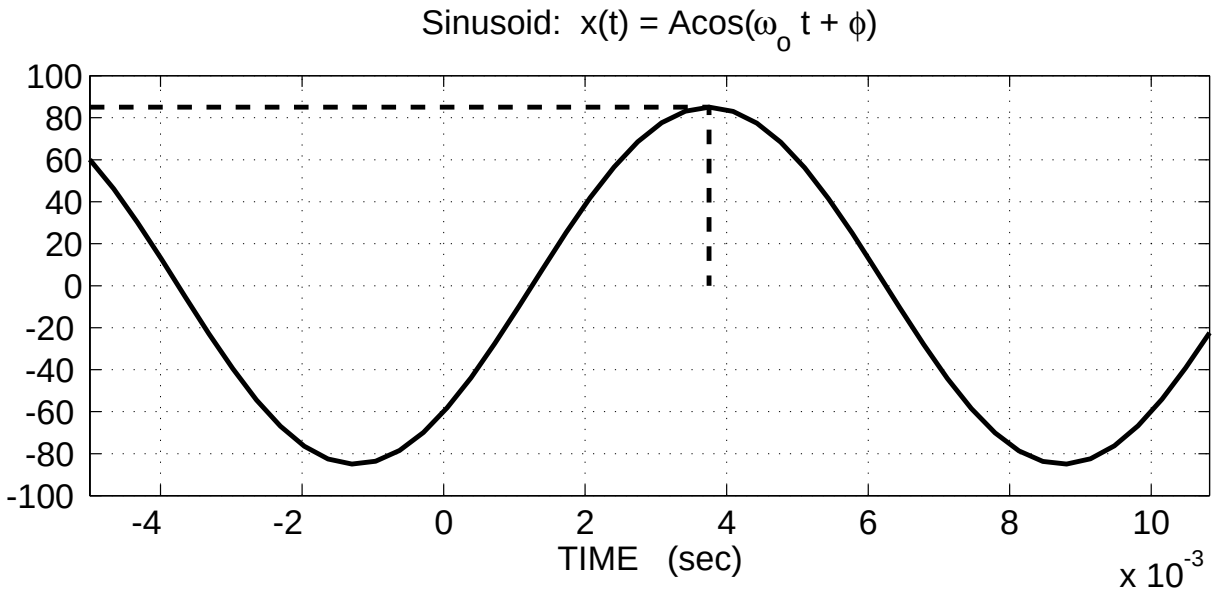
(a) j^{37}
(b) $e^{j(\pi/2+m\pi)}$ (m an integer)
(c) $j^{1/5}$ (find 5 answers)
(d) $(1-j)^{1+j}$ (is the answer unique?)

PROBLEM 1.4*:

The figure below is a plot of a sinusoidal signal. From the plot, determine values for the amplitude (A), phase (ϕ), and frequency ($\omega_\circ$) needed in the formula:

$$x(t) = A\cos(\omega_\circ t + \phi)$$

Give the answer as numerical values *including the units* where applicable. Since you must make approximate measurements on the figure, your final answers will be estimates.



PROBLEM 1.5*:

The phase of a sinusoid can be related to time shift:

$$x(t) = A\cos(2\pi f_\circ t + \phi) = A\cos(2\pi f_\circ (t - t_1)) \tag{1}$$

In the following parts, assume that the frequency of the sinusoidal wave is $f = 60$ Hz.

(a) "When $t_1 = -1/300$ sec, the value of the phase is $\phi = \pi/5$."
Explain whether this is TRUE or FALSE.

(b) "When $t_1 = 1/300$ sec, the value of the phase is $\phi = -2\pi/5$."
Explain whether this is TRUE or FALSE.

(c) "When $t_1 = 1/50$ sec, the value of the phase is $\phi = -2\pi/5$."
Explain whether this is TRUE or FALSE.

PROBLEM 1.6*:

Define $x(t)$ as

$$x(t) = 4\cos(\omega_0 t + 3\pi/4) + 2\sin(\omega_0 t)$$

(a) Find a complex-valued signal $z_1(t)$ such that $\Re e\{z_1(t)\} = 4\cos(\omega_0 t - \pi/4)$.

(b) Find a complex-valued signal $z_2(t)$ such that $\Re e\{z_2(t)\} = 2\sin(\omega_0 t)$.

(c) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$

(d) Assume that $\omega_0 = 0.4\pi$. Make a plot of $x(t)$ over the range $-5 \leq t \leq 10$. How many periods are included in the plot?

(e) Find a complex-valued signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$.

PROBLEM 1.7*:

Consider the complex signal $z(t) = Ze^{j10\pi t}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as $\dot{z}(t) = Qe^{j10\pi t}$ and determine an expression for the phasor Q in terms of Z.

(b) Prove that the angle of Q will always be equal to the angle of Z plus a constant, and determine the constant.

(c) If $Z = -3 - j4$, plot the phasors Z and Q in order to verify the angle relationship between Z and Q.

PROBLEM 1.8*:

Solve the following simultaneous equations via the phasor method. Is the answer for A_1, A_2, ϕ_1, ϕ_2 unique? Provide a geometric (phasor) diagram to explain the answer.

$$\begin{aligned} \cos(\omega_0 t - 2\pi/3) &= 5A_1\cos(\omega_0 t + \phi_1) - 6A_2\cos(\omega_0 t + \phi_2) \\ 2\cos(\omega_0 t) &= -4A_1\cos(\omega_0 t + \phi_1) + 5A_2\cos(\omega_0 t + \phi_2) \end{aligned}$$

PROBLEM 1.9: (P-2.3, F-94)

Solve the following equation for M and ψ. Obtain *all* possible answers. Use the phasor method, and provide a geometric diagram to explain the answer.

$$5\cos(\omega_0 t) - 4\cos(\omega_0 t + \psi) = M\cos(\omega_0 t + \pi/4)$$

HINT: describe the figure in the z-plane given by the set $\{z : z = 5 - 4e^{j\psi}\}$ where $0 \leq \psi \leq 2\pi$.