GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Problem Set #2

Assigned: 6 Oct 1998
Due Date: 12 Oct 1998 (MONDAY)

Quiz #1 will be held in lecture on Friday 16-Oct-98. It will cover material from Chapters 2 and 3, as represented in Prob. Sets #1 and #2. Closed book, calculators permitted.

Reading: In *DSP First*, Chapter 3 on *Spectrum Representation*, pages 48–68.

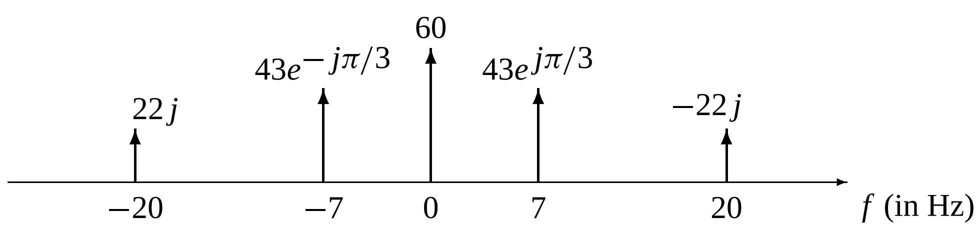
The web site: `http://webct.ece.gatech.edu/SCRIPT/AUT98EE2200/scripts/serve_home`
Your initial password = SSN(4:8). Please check the "Bulletin Board" often.

ALL of the **STARRED** problems will have to be turned in for grading.

Some of the problems have solutions that can be found on the CD-ROM. Next week a solution will be posted to the web on Tuesday, 13-Oct. After the HW posting late homework will be given a zero.

PROBLEM 2.1*:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$.

(b) Is $x(t)$ a periodic signal? If so, what is its period?

PROBLEM 2.2*:

The following MATLAB program makes a plot of the *amplitude modulated* signal that is "cosine-times-sine." (Actually the plot is of a finite time segment of the signal.)

```
tt = -1:0.01:1;
xx = cos(33*pi*tt) .* sin(pi*tt);
plot(tt,xx)
```

(a) Make a *sketch* of the plot that will be done by MATLAB. Label the time axis carefully.
Note: this can be done *without* running the MATLAB commands.

(b) The "spectrum" diagram gives the frequency content of a signal. Draw a sketch of the spectrum of the signal represented by `xx`. Label the frequencies and the complex amplitudes of each component.

PROBLEM 2.3*:

DSP First, Chapter 3, Problem 8, page 80.

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^{\#}$	D	$E^{\flat}$	E	F	$F^{\#}$	G	$G^{\#}$	A	$B^{\flat}$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(d) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The D Major chord is composed of the tones of $D\;\; F^{\#}\;\; A$ sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the D Major chord assuming that each note is realized by a pure sinusoidal tone. (You do not have to specify the complex amplitudes precisely.)

PROBLEM 2.4*:

A signal composed of sinusoids is given by the equation

$$x(t) = 44\cos(3\pi t + \pi/6) + 55\cos(6\pi t) - 33\sin(12\pi t)$$

(a) Sketch the spectrum of this signal indicating the complex size of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the smallest period?

(c) Now consider a new signal $y(t) = x(t) + 11\cos(5\pi t - \pi/6)$. Draw a carefully labelled sketch of the spectrum for $y(t)$. Is $y(t)$ still periodic? If so, what is the period?

(d) Finally, consider another new signal $w(t) = x(t) + 22\cos(18t + \pi/6)$. Draw a carefully labelled sketch of the spectrum for $w(t)$. Is $w(t)$ still periodic? If so, what is the period?

PROBLEM 2.5*:

DSP First, Chapter 3, Problem 7, page 79.
Consider a signal $x(t)$ such that

$$x(t) = 2\cos(\omega_1 t)\cos(\omega_2 t) = \cos[(\omega_2 + \omega_1)t] + \cos[(\omega_2 - \omega_1)t]$$

where $0 < \omega_1 < \omega_2$.

(a) What is the general condition that must be satisfied by $\omega_2 - \omega_1$ and $\omega_2 + \omega_1$ so that $x(t) = x(t + T_0)$; i.e., so that $x(t)$ is *periodic* with period T_0? In addition, determine T_0 in terms of ω_1 and ω_2.

(b) What does the result of part (a) imply about ω_1 and ω_2?

PROBLEM 2.6:

A discrete-time signal $x[n]$ is known to be a sinusoid:

$$x[n] = A\cos(\hat{\omega}_0 n + \phi)$$

The values of $x[n]$ are tabulated for $n = 0$, 1, 2, 3, 4, 5 and 6.

n	0	1	2	3	4	5	6
$x[n]$	-2.5000	-0.5226	1.5451	3.3457	4.5677	5.0000	4.5677

(a) Plot $x[n]$ vs. n in the format of a MATLAB "stem" plot.

(b) Prove (via phasors, not trig) the following identity for the cosine signal:

$$\beta = \frac{\cos(n+1)\hat{\omega}_0 + \cos(n-1)\hat{\omega}_0}{\cos n\hat{\omega}_0} \qquad \text{for all } n$$

Determine the value of the constant β. Note: β does not depend on n, but it might be a function of $\hat{\omega}_0$.

(c) Now determine the numerical values of A, ϕ and $\hat{\omega}_0$. (Hint: find $\hat{\omega}_0$ first.)

PROBLEM 2.7: (Q-2.1, Spring-96)

Suppose that MATLAB is used to plot a sinusoidal signal. The following MATLAB code generates the signal and makes the plot. Draw a sketch of the plot that will be done by MATLAB.

```
J = sqrt(-1);
dt = 1;
tt = -10 : dt : 20;
Fo = 0.05;
xx = 90*real( exp( J*(2*pi*Fo*tt  - pi/2) ) );
%
plot( tt, xx ),   grid
title( 'SECTION of a SINUSOID' ),    xlabel('TIME   (sec)')
```