

GEORGIA INSTITUTE OF TECHNOLOGY  
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

**EE 2200 Fall 1998**  
**Problem Set #5**

Assigned: 2 Nov 1998

Due Date: 9 Nov 1998 (MONDAY)

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**Quiz #2 will be held in lecture on Friday 13-Nov-98.** It will cover material from Chapters 3, 4, 5, and 6, as represented in Problem Sets #3, #4 and #5.

**Closed book, calculators permitted, and one hand-written formula sheet** ( $8\frac{1}{2}'' \times 11''$ )

A quiz review session is planned for Wed, 11-Nov, at 11 AM in the ECE auditorium.

Reading: In *DSP First*, Chapter 6 on *Frequency Response of FIR Filters*.

⇒ The six(6) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted. Some similar problems solutions can be found on the CD-ROM, especially the “unstarred” problems.

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**PROBLEM 5.1\*:**

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] - x[n - 1] + x[n - 2] - x[n - 3]$$

- Find the frequency response  $\mathcal{H}(\hat{\omega})$ , and then express it as a mathematical formula, in polar form (magnitude and phase).
- Plot the magnitude and phase of  $\mathcal{H}(\hat{\omega})$  as a function of  $\hat{\omega}$  for  $-\pi < \hat{\omega} < \pi$ . Do this by hand, but you could check your answer by using the MATLAB function `freqz`.
- Find all frequencies,  $\hat{\omega}$ , for which the response to the input  $e^{j\hat{\omega}n}$  is zero.

**PROBLEM 5.2\*:**

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] - x[n - 1] + x[n - 2] - x[n - 3]$$

- Impulse Response:* Determine the impulse response of this system. Plot  $h[n]$  as a function of  $n$ .
- When the input to the system is  $x[n] = \exp(j\pi n/4)$  determine the functional form for the output signal  $y[n]$ . Find numerical values for the magnitude and phase.
- What is the output if the input is

$$x[n] = 4 + \cos[0.5\pi(n - 1)] - 3 \cos[0.25\pi n]$$

**PROBLEM 5.3\*:**

For the *Dirichlet* function:

$$\mathcal{D}(\hat{\omega}, 10) = \frac{\sin(5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

- Make a plot of  $\mathcal{D}(\hat{\omega}, 10)$  over the range  $-4\pi \leq \hat{\omega} \leq +4\pi$ . Label all the zero crossings.
- Determine the period of  $\mathcal{D}(\hat{\omega}, 10)$ . Is it equal to  $2\pi$ ; why, or why not?
- Find the maximum value of the function.

NOTE: the *Dirichlet* function is defined via:  $\mathcal{D}(\hat{\omega}, L) = \frac{\sin(L\hat{\omega}/2)}{\sin(\frac{1}{2}\hat{\omega})}$

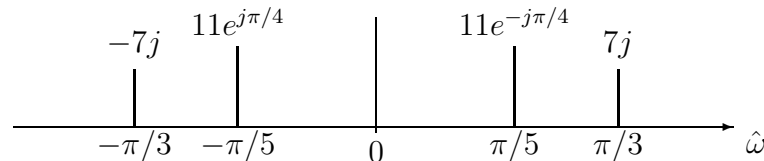
In MATLAB consult help on `diric` for more information.

**PROBLEM 5.4\*:**

An FIR filter is characterized by the following frequency response:

$$\mathcal{H}(\hat{\omega}) = \frac{\sin(5\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

- If the input to the filter is a signal with the following spectrum, determine a formula for the input signal,  $x[n]$  for  $-\infty < n < \infty$ .



- Using the input signal from part (a), determine the output,  $y[n]$  for  $-\infty < n < \infty$ .

**PROBLEM 5.5\*:**

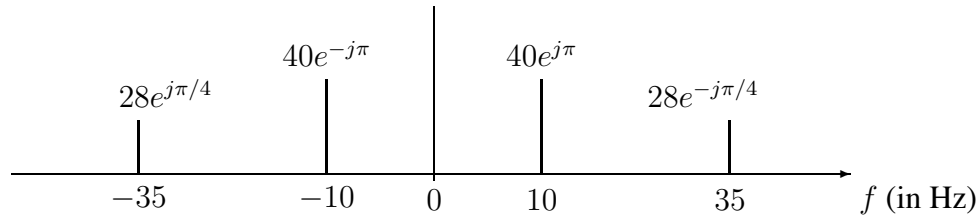
The frequency response of a linear time-invariant filter is given by the formula

$$\mathcal{H}(\hat{\omega}) = (1 + e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}})$$

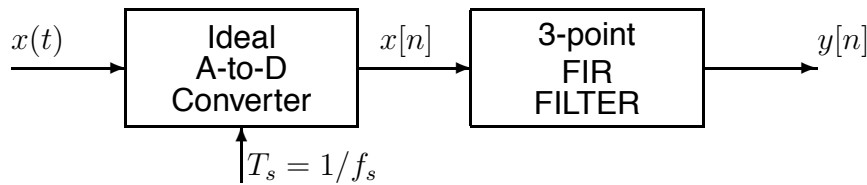
- Write the difference equation for the FIR filter that gives the relation between the input  $x[n]$  and the output  $y[n]$ . Give numerical values for the filter coefficients.
- What is the output of this FIR filter if the input is  $x[n] = \delta[n]$ ?
- Evaluate the frequency response  $\mathcal{H}(\hat{\omega})$  at the frequencies  $\hat{\omega} = \pi$  and  $\hat{\omega} = \pi/3$ .
- If the input is of the form  $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$ , for what values of  $-\pi \leq \hat{\omega} \leq \pi$  will  $y[n] = 0$  for all  $n$ ?

**PROBLEM 5.6\*:**

A signal  $x(t)$  has the two-sided spectrum representation shown below.



- Write an equation for  $x(t)$ . Make sure to express  $x(t)$  as a real-valued signal.
- If the signal is sampled at a rate of  $f_s = 25$  Hz, sketch the “digital” spectrum of this signal. Indicate the complex phasor value at each frequency. Only the range  $-\pi < \hat{\omega} \leq \pi$  needs to be shown.
- If the length-3 FIR filter (below) has filter coefficients  $\{b_k\} = \{1, b_1, 1\}$ , show that  $b_1 = -2 \cos(0.8\pi) = 1.618$  will make the output signal  $y[n]$  equal to zero.



**PROBLEM 5.7:**

Suppose that  $\mathcal{S}$  is a linear, time-invariant system whose exact form is unknown. It needs to be tested by running some inputs into the system, and then observing the output signals. Suppose that the following input/output pairs are the result of the tests:

$$\begin{aligned} x[n] = \delta[n] - \delta[n - 1] &\longrightarrow y[n] = 4\delta[n] - 4\delta[n - 4] \\ x[n] = \cos(\pi n/2) &\longrightarrow y[n] = 0 \\ x[n] = \cos(\pi n/3) &\longrightarrow y[n] = 6.93 \cos(\pi n/3 - \pi/2) \end{aligned}$$

- Make a plot of the signal:  $x[n] = 4\delta[n] - 4\delta[n - 4]$ .
- Use linearity and time-invariance to find the output of the system when the input is

$$x[n] = 3\delta[n] - 3\delta[n - 3]$$

- Determine the output when the input is  $x[n] = 7 \cos(\pi(n - 2)/3)$ .
- Determine the output when the input is  $x[n] = 9 \sin(\pi n/2)$

**PROBLEM 5.8:**

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

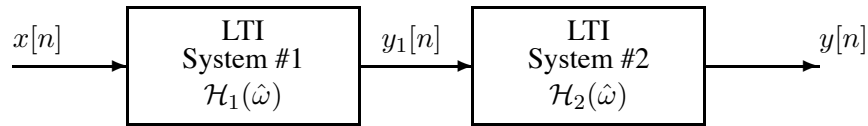


Figure 1: Cascade connection of two LTI systems.

- (a) Show that if the input is  $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$ , then the corresponding output of the overall system is

$$y[n] = \mathcal{H}_2(\hat{\omega})\mathcal{H}_1(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n} = \mathcal{H}(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n}$$

where  $\mathcal{H}_1(\hat{\omega})$  is the frequency response of the first system and  $\mathcal{H}_2(\hat{\omega})$  is the frequency response of the second system. That is, show that the overall frequency response of a cascade of two LTI system is the product of the individual frequency responses, and therefore the cascade system is equivalent to a single system with frequency response  $\mathcal{H}(\hat{\omega}) = \mathcal{H}_2(\hat{\omega})\mathcal{H}_1(\hat{\omega})$ .

- (b) Use the result of part (a) to show that the order of the systems is not important; i.e., show that for the same input  $x[n]$  into the systems of Figs. 1 and 2, the overall outputs are the same ( $w[n] = y[n]$ ).

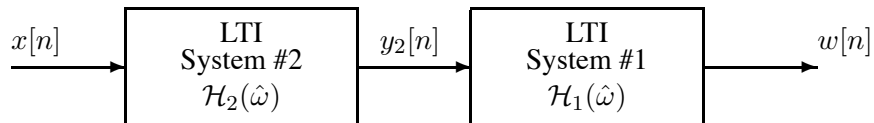


Figure 2: Equivalent system to system of Figure 1.

- (c) Suppose that System #1 is described by the difference equation  $y_1[n] = x[n] + x[n - 2]$ , and System #2 is described by the frequency response function  $\mathcal{H}_2(\hat{\omega}) = (1 - e^{-j\hat{\omega}2})$ . Determine the frequency response function of the overall cascade system.
- (d) Sketch the frequency response (magnitude and phase) of the overall cascade system for  $-\pi \leq \hat{\omega} \leq \pi$ .
- (e) Obtain a single difference equation that relates  $y[n]$  to  $x[n]$  in Fig. 1 and  $w[n]$  to  $x[n]$  in Fig. 2.