GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2200 Fall 1998
Problem Set #7

Assigned: 23 Nov 1998
Due Date: 4 Dec 1998 (FRIDAY)

Final Exam is scheduled for Period #15: 11-Dec (Friday).

Review Sessions are planned for Tues, Wed, & Thurs evenings of Finals week. More details later.

$\Longrightarrow$ The five(5) **STARRED** problems will have to be turned in for grading.

A solution will be posted early in the week of 7-Dec. Some similar problems solutions can be found on the CD-ROM, especially the "unstarred" problems.

PROBLEM 7.1*:

Given a feedback filter defined via the recursion:

$$y[n] = -y[n-3] + x[n] \qquad \text{(DIFFERENCE EQUATION)}$$

(a) Determine the impulse response $h[n]$, assuming the "at rest" initial condition.

(b) Prove that the impulse response signal is periodic for $n > 0$, and determine the period.

(c) When the input to the system is the signal:

$$x[n] = \delta[n] + 2\delta[n-3] + \delta[n-6]$$

determine the output signal $y[n]$, assuming the "at rest" initial condition (i.e., the output signal is zero for $n < 0$). Present your final answer as a plot of all of $y[n]$.

PROBLEM 7.2*:

For the following system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

determine the time-domain (n) and frequency-domain $(\hat{\omega})$ behavior.

(a) The inverse z-transform of $H(z)$ is the impulse response $h[n]$. Determine the inverse z-transform for $H(z)$ as a mathematical formula, and sketch the first five values of the impulse response, $h[n]$.

(b) Make a sketch of the magnitude of the frequency response over the appropriate range for $\hat{\omega}$. Label the peak value and the locations of any zeros. Is the filter low-pass or high-pass?

(c) (Optional) Use `freqz` or the `pez` GUI from the lab to verify your answer.

PROBLEM 7.3*:

A linear time-invariant filter is described by the difference equation

$$y[n] = 0.8y[n-1] + 4x[n] - 5x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} (negative powers of z).

(b) Plot the poles and zeros of $H(z)$ in the z-plane.
Hint: express $H(z)$ as a ratio of polynomials in positive powers of z.

(c) Show that $|H(e^{j\hat{\omega}})|^2$ is a constant for all $\hat{\omega}$; and determine the value of the constant.
Hint: From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) (Optional) Use `freqz` or the `pez` GUI from the lab to verify your answer.

PROBLEM 7.4*:

Suppose that a system is defined by the following operator

$$H(z) = (1 + z^{-1})\frac{1 + z^{-2}}{1 + 0.5z^{-1}}$$

(a) Write the time-domain description of this system—in the form of a difference equation.

(b) This system can "null" certain input signals. Determine *all* input frequencies $\hat{\omega}_\circ$, for which the response to $x[n] = \cos(\hat{\omega}_\circ n)$ is equal to zero.

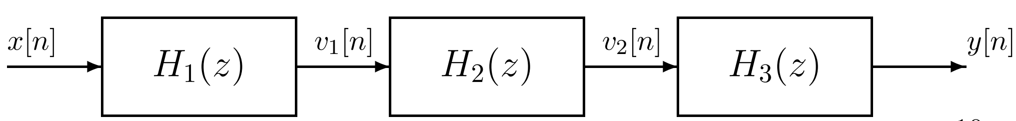
(c) When the input to the system is the unit-step signal $x[n] = u[n]$ determine the output signal $y[n]$ in the form:

$$y[n] = K_1 \alpha^n u[n] + K_2 u[n] + K_3 \delta[n-1]$$

Give numerical values for the constants K_1, K_2 and α. Verify that K_2 is equal to $H(z)$ at $z = 1$.
Hint: this system stable, so the value for $|\alpha|$ must be less than one. Thus, $y[n] \to K_2$, as $n \to \infty$

PROBLEM 7.5*:

In the following cascade of systems, all of the individual transfer functions are known.

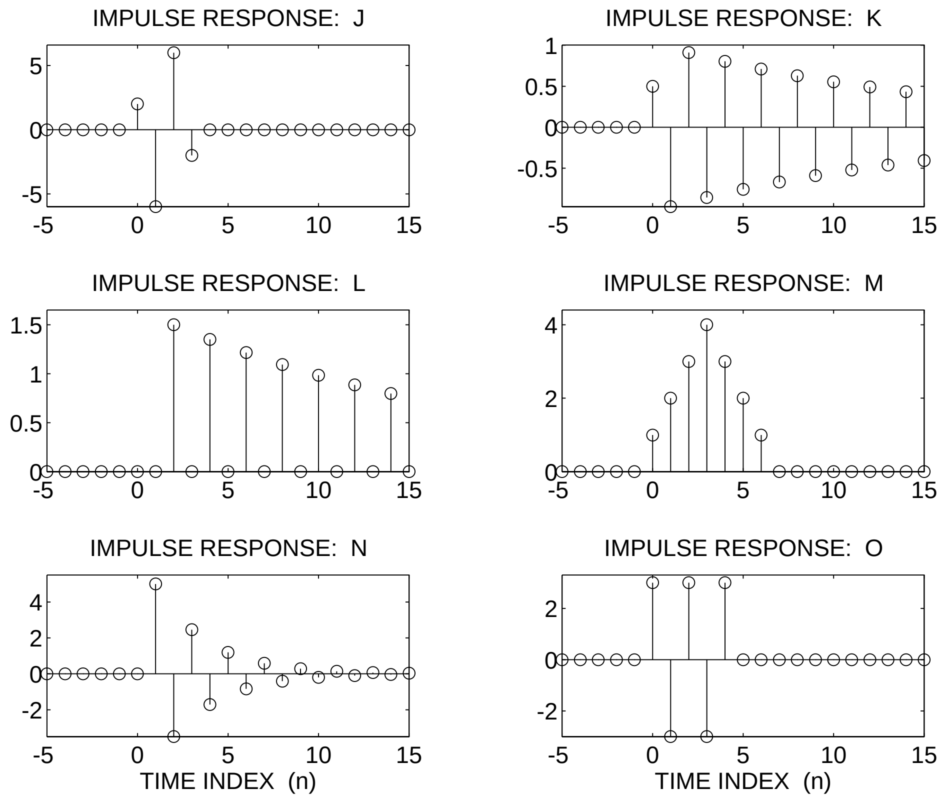


$H_1(z) = z^{-2} + z^{-3}$ $\quad$ $H_2(z) = 4 - 3z^{-1}$ $\quad$ $H_3(z) = \dfrac{10}{8 - 10z^{-1} + 3z^{-2}}$

(a) Determine $H(z)$ the z-transform of the cascaded system. Simplify $H(z)$ by cancelling common factors in the numerator and denominator.

(b) Consider the impulse response of the cascaded system, i.e., the response $y[n]$ when the input is $x[n] = \delta[n]$. Prove that the impulse response has the form $h[n] = G\,\alpha^n$ for $n \geq 3$. Find values for α and G.

PROBLEM 7.6:



For each of the impulse-response plots (J, K, L, M, N, O), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the impulse response.

$\mathcal{S}_0 :\quad y[n] = 0.90y[n-2] + 1.5x[n-2]$

$\mathcal{S}_1 :\quad y[n] = -0.7y[n-1] + 5x[n-1]$

$\mathcal{S}_2 :\quad y[n] = -0.7y[n-1] + 7x[n] + 10x[n-1]$

$$\mathcal{S}_3 :\quad H(z) = \frac{\frac{1}{2}(1-z^{-1})}{1+0.94z^{-1}}$$

$\mathcal{S}_4 :\quad H(z) = 2(1-z^{-1})^3$

$\mathcal{S}_5 :\quad H(z) = 3(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$

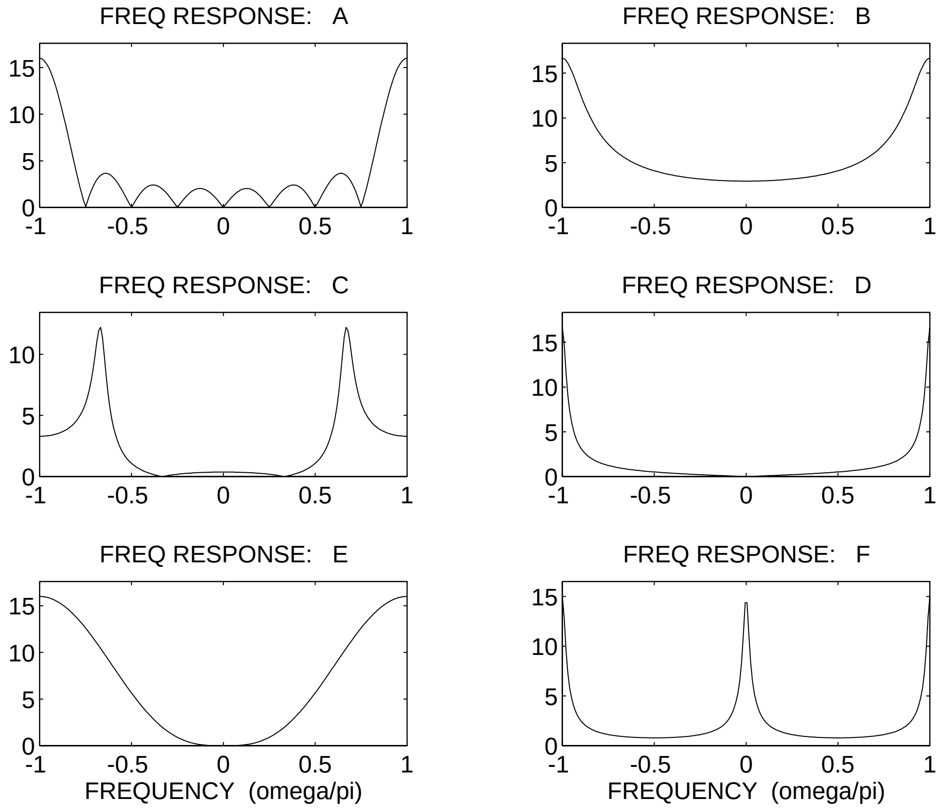
$\mathcal{S}_6 :\quad y[n] = 8x[n] - 8x[n-1]$

$$\mathcal{S}_7 :\quad y[n] = 2\sum_{k=0}^{7}(-1)^k x[n-k]$$

$\mathcal{S}_8 :\quad y[n] = x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 3x[n-4] + 2x[n-5] + x[n-6]$

$$\mathcal{S}_9 :\quad H(z) = \frac{1 - z^{-1} + z^{-2}}{1 + 0.9z^{-1} + 0.81z^{-2}}$$

PROBLEM 7.7:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$ or a difference equation) matches the frequency response. NOTE: frequency axis is normalized; it is $\hat{\omega}/\pi$.

$\mathcal{S}_0 :\quad y[n] = 0.90y[n-2] + 1.5x[n-2]$

$\mathcal{S}_1 :\quad y[n] = -0.7y[n-1] + 5x[n-1]$

$\mathcal{S}_2 :\quad y[n] = -0.7y[n-1] + 7x[n] + 10x[n-1]$

$$\mathcal{S}_3 :\quad H(z) = \frac{\frac{1}{2}(1 - z^{-1})}{1 + 0.94z^{-1}}$$

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$\mathcal{S}_6 :\quad y[n] = 8x[n] - 8x[n-1]$

$$\mathcal{S}_7 :\quad y[n] = 2\sum_{k=0}^{7}(-1)^k x[n-k]$$

$\mathcal{S}_8 :\quad y[n] = x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 3x[n-4] + 2x[n-5] + x[n-6]$

$$\mathcal{S}_9 :\quad H(z) = \frac{1 - z^{-1} + z^{-2}}{1 + 0.9z^{-1} + 0.81z^{-2}}$$