GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 3

Date Assigned: October 9, 1998
Date Due: October 16, 1998

Reading Assignment: In Oppenheim and Willsky, read pp. 1 231-244 and read all of Chapter 4.

Homework Assignment: Turn in for grading only the starred problems: 3.1*, 3.2*, and 3.4*.

Practice Problems:

For practice try Problems 4.1, 4.2, 4.3, 4.4 and 4.6 in Oppenheim and Willsky. These problems have answers in the back of the book.

Practice Problems without Answers:

Work Problem 4.22.(a), (b) and (c) in O & W.

Work Problem 3.35 in Oppenheim and Willsky.

Problem 3.1*

Consider the signal $x(t)$, whose Fourier transform is

$$X(j\omega) = \begin{cases} 10 & -2\pi < \omega < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

(a) $x(t)$ is the input to a linear time-invariant system whose impulse response is

$$h(t) = \frac{\sin[\pi(t-2)]}{\pi(t-2)}$$

Use Fourier transforms to determine an equation for the output $y(t) = x(t) * h(t)$ of the LTI system.

(b) Another signal is formed by modulation: $y(t) = x(t)\cos(10\pi t)$. Plot the Fourier transform $Y(j\omega)$ of this signal.

(c) Still another signal is $v(t) = (x(t))^2$. Plot the Fourier transform $V(j\omega)$ of this signal.

Problem 3.2*:

Consider the periodic signal $x(t)$, which is defined over one period by

$$x(t) = \begin{cases} 1 & -2 < t < 0 \\ 0 & 0 < t < 2 \end{cases}$$

The period of the signal is $T = 4$.

(a) The signal $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k. Sketch the spectrum of the input signal as a function of ω.

(b) The frequency response of a LTI highpass filter is

$$H(j\omega) = \begin{cases} 0 & |\omega| < 5\pi/4 \\ e^{-j\omega} & 5\pi/4 < |\omega| \end{cases}$$

Plot the magnitude of the frequency response, $|H(j\omega)|$ on the same graph as your spectrum plot. What is the effect of the factor $e^{-j\omega}$ on the output waveform?

(c) Determine the output of the system for the given input $x(t)$. Give the simplest possible equation for your answer.

Problem 3.3:

Consider the following periodic signal, which is the input to a LTI system:

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n4)$$

(a) The input $x(t)$ can be expressed in the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

Determine the the fundamental frequency ω_0 and the Fourier coefficients a_k for all k.

(b) The impulse response of the LTI system is

$$h(t) = \alpha e^{-\alpha(t-1)} u(t-1)$$

Use convolution to obtain an equation for the output $y(t)$ when the input is the signal in part (a). *Hint: Use superposition and time invariance to find the output due each impulse.* Make a sketch of the output signal as a function of time for the case $\alpha = 2$.

(c) Determine the frequency response of the LTI system. Sketch $|H(j\omega)|$ as a function of ω. How does the shape of the frequency response depend on α?

(d) Use the frequency response and the Fourier series result of part (a) to determine a Fourier series expression for the output of the system for the given input $x(t)$. How would you choose α if you wanted the output to be essentially equal to a constant?

Problem 3.4*:

Consider an LTI system with

$$h(t) = \delta(t-1) - e^{-(t-1)}u(t-1)$$

(a) Use the tables on pp. 328-329 to find the frequency response $H(j\omega)$ of this system.

(b) For a particular input $x(t)$, the Fourier transform of the output of the system is

$$Y(j\omega) = \frac{3}{2}\frac{e^{-j\omega 2}}{j\omega + 3} - \frac{1}{2}\frac{e^{-j\omega 2}}{j\omega + 1}.$$

Use the tables on pp. 328-329 to determine the input $x(t)$.