GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Problem Set No. 4

Date Assigned: April 24, 1998
Date Due: May 1, 1998

Reading Assignment: In Oppenheim and Willsky, read all of Chapter 4 and read pp. 516-534 and 583-597 in Chapter 8.

Homework Assignment: Hand in for grading only Problems 4.1*, 4.2* 4.3*, and 4.5*.

Practice Problems:

(a) Work Problem 4.23 in O & W. (See solution to Problem 4.3, winter 98.)

(b) Work Problem 4.26(b) in O & W. (See solution to Problem 4.4, winter 98.)

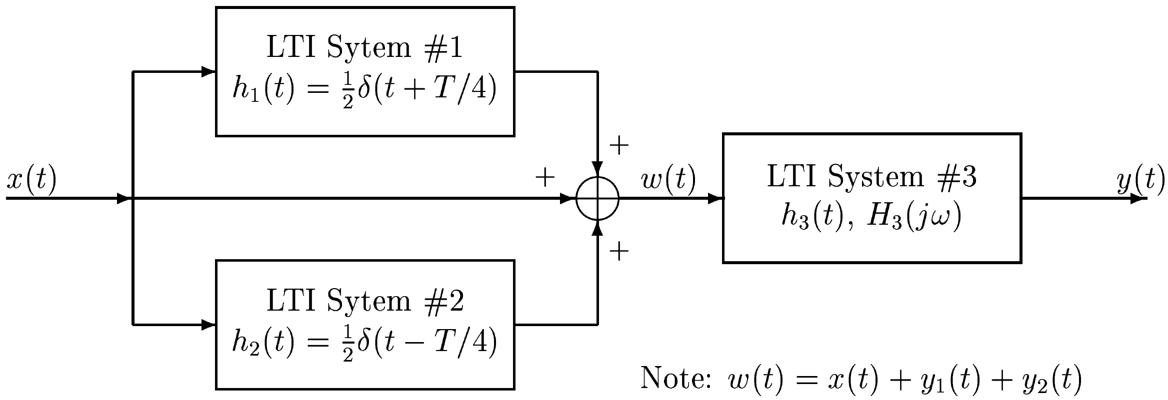
(c) Work Problem 4.28(a) and 4.28(b-ii) and (b-viii) in O & W. (See solution to Problem 4.5, winter 98.)

(d) Work Problem 8.22 in O & W. (See solution to Problem 5.4, winter 98.)

Problem 4.1*:
Work Problem 8.23 in O & W.

Problem 4.2*:

The following system is a LTI system.



Note: $w(t) = x(t) + y_1(t) + y_2(t)$

The frequency response of LTI System #3 is: $H_3(j\omega) = \begin{cases} T/4 & |\omega| < 4\pi/T \\ 0 & |\omega| > 4\pi/T \end{cases}$

(a) Determine the impulse response, $h_3(t)$, of LTI System #3.

(b) First, give an expression in terms of $h_3(t)$ for the impulse response $h(t)$ of the overall system. Then use your result from part (a) to find an equation for the overall impulse response $h(t)$. Sketch your answer showing the value of $h(0)$ and the times at which $h(t) = 0$.

(c) Determine the frequency response, $H(j\omega)$, for the overall system. Express your answer in terms of $H_3(j\omega)$ and manipulate it into a simple form so that you can easily plot it. **Plot it.**

Problem 4.3*

Consider the signal $x(t) = w(t)\cos(\omega_c t)$, where $\omega_c = 2\pi/T$ and

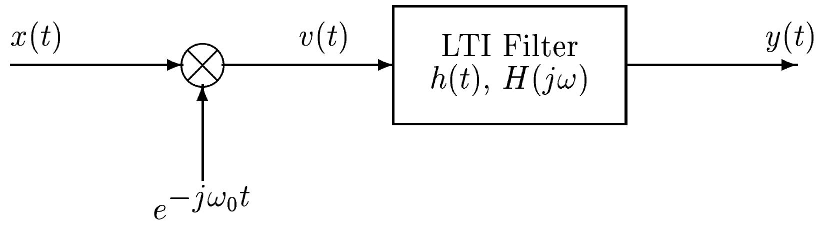
$$w(t) = \begin{cases} 1 & -3T < t < 3T \\ 0 & \text{otherwise} \end{cases}$$

(a) Carefully sketch $x(t)$.

(b) Determine an equation for $X(j\omega)$ in terms of $W(j\omega)$, and then determine $W(j\omega)$ and substitute it into your equation. Plot $X(j\omega)$ carefully marking all significant amplitudes and frequencies.

Problem 4.4

Consider the following modulation/filtering system:



The impulse response of the LTI system is: $h(t) = \begin{cases} 1/T & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}$

(a) Determine the frequency response of the LTI system and plot it.

(b) Suppose that $\omega_0 = 2\pi/T$ and the input signal is the periodic function

$$x(t) = A_0 + A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(2\omega_0 t + \phi_2)$$

Determine expressions for the Fourier transforms of $x(t)$ and $v(t)$. Plot the Fourier transform $V(j\omega)$ on the same axes as your plot of $H(j\omega)$.

(c) Determine the output $y(t)$ for the input $x(t)$ in part (b).

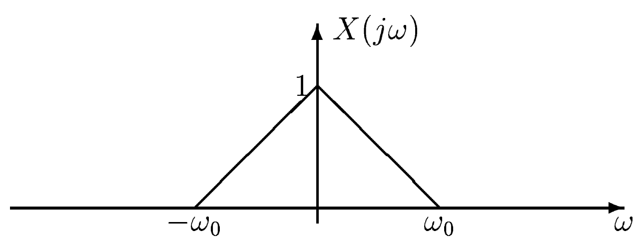
(d) Describe how you could use a system of this type to determine A_0, A_1, A_2, ϕ_1, and ϕ_2 for the given input signal.

Problem 4.5*

A signal $y(t) = x(t)p(t)$ is formed by modulating a periodic squarewave $p(t)$ given by

$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{with} \quad a_k = \begin{cases} \dfrac{\sin(\pi k/2)}{\pi k} & k \neq 0 \\ 0 & k = 0 \end{cases}$$

where $\omega_0 = 2\pi/T$. Assume that the input signal $x(t)$ has a Fourier transform as depicted below



(a) Give an equation for $Y(j\omega)$ in terms of $X(j\omega)$.

(b) For the Fourier transform $X(j\omega)$ given in the graph above, plot $Y(j\omega)$ for frequencies $-6\omega_0 < \omega < 6\omega_0$.

(c) From your plot in (b), you should be able to see how to recover the original signal $x(t)$ by using a combination of ideal filters and modulators. Draw a block diagram of the demodulator system showing all filter frequency responses (cutoff frequencies, gains), and modulator frequencies.