GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE3230
Homework Assignment No. 6

Date Assigned: May 18, 1998
Date Due: May 27, 1998

Reading Assignment: Read Chapter 9 of Oppenheim and Willsky.

Homework Assignment: Hand in Problems 6.1*, 6.3*, 6.5*, 6.6* and 6.7*.

REMEMBER: The third quiz will be given on May 28. It will cover through this problem set.

Practice Problems:

Work Problems 9.3, 9.6, and 9.9 in O and W. These have answers at the back of the book.

Problem 6.1*:

Determine the Laplace transform of each of the following signals. Be sure to state the region of convergence.

(a) $x_a(t) = 2e^{-2t}u(t) - u(t)$.

(b) $x_b(t) = e^{-2|t|} = e^{-2t}u(t) + e^{2t}u(-t)$.

(c) $x_c(t) = e^{-t}\cos(t)u(t)$.

(d) $x_d(t) = u_1(t-1) + \delta(t-1)$.

(e) $x_d(t) = 2e^{-2t}u(t) - e^{2t}u(t)$

Problem 6.2:

Determine the inverse Laplace transform of each of the following. Do parts (a) and (b) by the partial fraction expansion method. Check your answer using the table of transforms on p. 692 of O and W. Then you should be able to do most of the rest of the work by applying the properties of the Laplace transform in the table on p. 691.

(a) $X_a(s) = \dfrac{1}{s^2+9} \qquad \mathcal{R}e\{s\} > 0$

(b) $X_b(s) = \dfrac{1}{s^2+9} \qquad \mathcal{R}e\{s\} < 0$

(c) $X_c(s) = \dfrac{s}{s^2+9} \qquad \mathcal{R}e\{s\} > 0$

(d) $X_d(s) = \dfrac{se^{-s2}}{s^2+9} \qquad \mathcal{R}e\{s\} > 0$

(e) $X_e(s) = \dfrac{s^2}{s+1} + s + 1 \qquad \mathcal{R}e\{s\} > -1$

Problem 6.3*:

Determine the inverse Laplace transform of each of the following. You should be able to do most of the rest of the work by applying the properties of the Laplace transform in the table on p. 691.

(a) $X_a(s) = \dfrac{1}{s(s+4)} \qquad \mathcal{R}e\{s\} > 0$

(b) $X_b(s) = \dfrac{1}{s(s+4)} \qquad \mathcal{R}e\{s\} < -4$

(c) $X_b(s) = \dfrac{1}{s(s+4)} \qquad -4 < \mathcal{R}e\{s\} < 0$

(d) $X_d(s) = \dfrac{e^{-s2}}{s(s+4)} \qquad \mathcal{R}e\{s\} > 0$

Problem 6.4:

The system function of a *causal* LTI system is

$$H(s) = \frac{s^2}{s^2+4} = \frac{s^2}{(s+j2)(s-j2)}$$

(a) What is the region of convergence of $H(s)$?

(b) Is the system stable?

(c) Determine the impulse response, $h(t)$, of the system.

(d) Determine the input $x(t)$ such that the corresponding output of the system is

$$y(t) = u_1(t-1) = \frac{d\delta(t-1)}{dt}$$

Problem 6.5*

A linear time-invariant system with input $x(t)$ and output $y(t)$ has system function

$$H(s) = \frac{s-2}{s+2}$$

(a) If it is known that the system is causal, determine the impulse response $h(t)$.

(b) Is the system stable? How do you know?

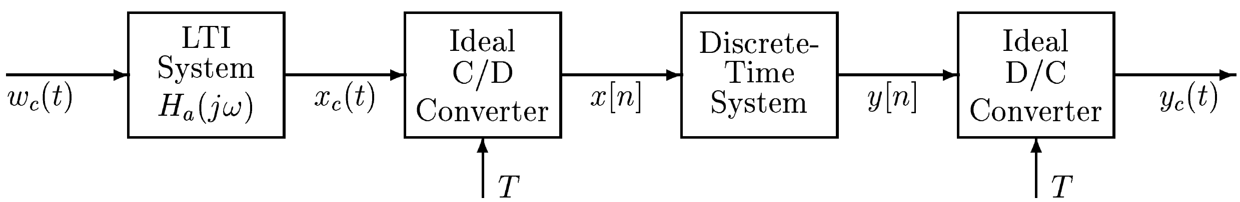
(c) Suppose that the input to the system is $x(t) = u(-t)$. Determine the corresponding output $y(t)$.

(d) Determine the Laplace transform $X(s)$ of the input $x(t)$ so that $y(t) = \delta(t-5)$.

If it is known that $\int_{-\infty}^{\infty} |x(t)| dt < \infty$, what is the region of convergence of $X(s)$?

Problem 6.6*

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)}$$

(a) If $T = 1/1000$, what condition on $H_a(j\omega)$ will guarantee that $y_c(t) = x_c(t)$ if the discrete-time system is defined by the difference equation $y[n] = x[n]$?

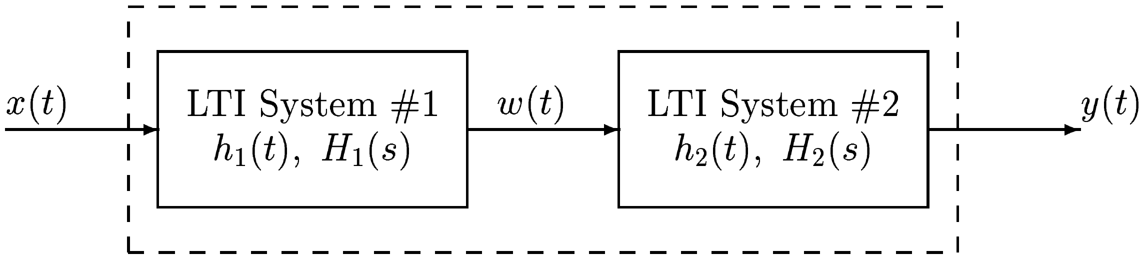
(b) If $T = 1/1000$ and the system $H_a(j\omega)$ is as chosen in part (a), determine $y_c(t)$ in terms of $x_c(t)$ if the discrete-time system is defined by the difference equation $y[n] = x[n-100]$.

(c) If the filter $H_a(j\omega)$ satisfies the condition determined in part (a) and the discrete-time system now is defined by $y[n] = x[n] + x[n-1]$, what is the frequency response $H_c(j\omega)$ of the overall system from $w_c(t)$ to $y_c(t)$; i.e., what is $H_c(j\omega)$ in the equation

$$Y_c(j\omega) = H_c(j\omega)W_c(j\omega)$$

Be as specific as you can.

Problem 6.7*



The first system has impulse response $h_1(t) = \delta(t) - 2e^{-2t}u(t)$, and the overall cascade system is a causal system with system function

$$H(s) = \frac{s(s+1)}{(s+2)(s+5)}$$

(a) Determine the system function, $H_2(s)$ [including the region of convergence], of the second system.

(b) Determine the differential equation that is satisfied by the input $x(t)$ and the overall output $y(t)$.

(c) Determine the step response (i.e., $y(t)$ when $x(t) = u(t)$) for the overall cascade system.