GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2823A
Problem Set No. 6

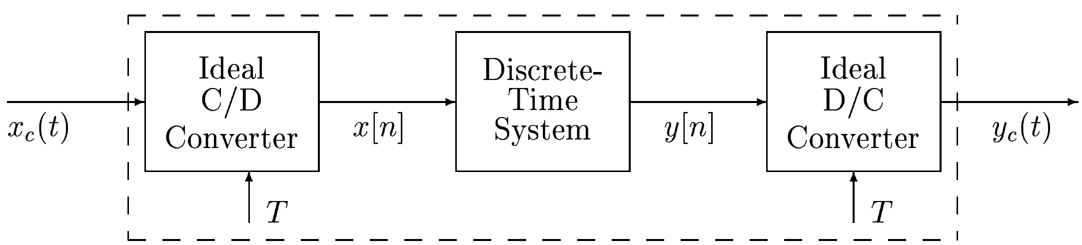
Date Assigned: February 12, 1999
Date Due: February 19, 1999

Reading Assignment: In Kamen and Heck, read pp. 582-610 and 514-545.

Homework Assignment: Turn in for grading only the starred problems: 6.1* and 6.3*.

Practice Problems:
Look at Problems on Problem Sets 5 and 6 of EE3230, Winter, Spring, Fall of 1998.

Problem 6.1*
All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

Assume that $X_c(j\omega) = 0$ for $|\omega| \geq 400\pi$.

(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?

(b) If the input is $x_c(t) = \cos(400\pi t - \pi/4)$, the sampling frequency is $2\pi/T = 600\pi$, and $y[n] = x[n]$, plot $Y_c(j\omega)$ and give an equation for $y_c(t)$.

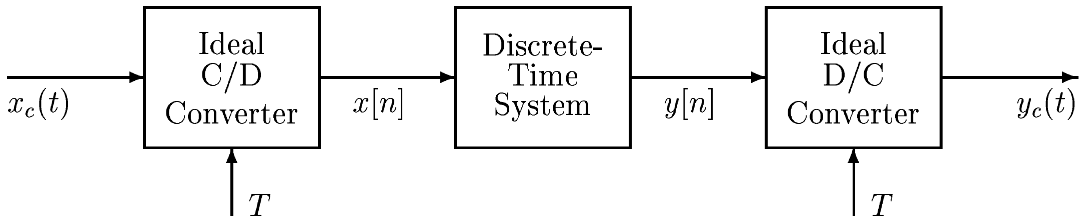
(c) The input/output relation for the discrete-time system is

$$y[n] = 0.25x[n] + 0.5x[n-2] + 0.25x[n-4]$$

For the value of T chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{\text{eff}}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{\text{eff}}(\omega)$, and plot $|H_{\text{eff}}(j\omega)|$ and $\arg[H_{\text{eff}}(j\omega)]$.

Problem 6.2

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t - nT)}{\frac{\pi}{T}(t - nT)}$$

Assume that $X_c(\omega) = 0$ for $|\omega| \geq 1000\pi$.

(a) Suppose that the discrete-time system is defined by $y[n] = x[n]$. What is the *minimum* value of $2\pi/T$ such that $y_c(t) = x_c(t)$?

(b) Determine the relationship between $y[n]$ and $x[n]$ so that if the sampling rate satisfies the condition of (a), then $y_c(t) = x_c(t - 10T)$.

(c) If the input is $x_c(t) = \cos(1200\pi t + pi/3)$, the sampling frequency is $2\pi/T = 2000\pi$, and $y[n] = x[n]$, what is $y_c(t)$?

(d) The input/output relation for the discrete-time system is

$$y[n] = \frac{1}{3}\left(x[n] - x[n-1] + x[n-2]\right)$$

For the value of T chosen in part (a), the input and output Fourier transforms are related by an equation of the form $Y_c(j\omega) = H_{eff}(j\omega)X_c(j\omega)$. Find an equation for the overall effective frequency response $H_{eff}(j\omega)$. Plot the magnitude and phase of $H_{eff}(j\omega)$. Use MATLAB to do this or sketch it by hand.

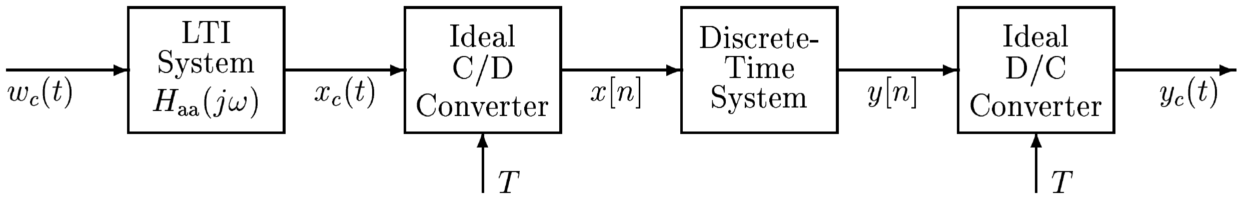
(e) Suppose that the frequency response of the discrete-time system is defined over one period ($-\pi \leq \hat{\omega} \leq \pi$) by

$$H(e^{j\hat{\omega}}) = \begin{cases} 0 & |\hat{\omega}| < \pi/2 \\ e^{-j\hat{\omega}10} & \pi/2 < |\hat{\omega}| \leq 3\pi/4 \\ 0 & 3\pi/4 < |\hat{\omega}| \leq \pi \end{cases}$$

where $\hat{\omega} = \omega T$. Plot the magnitude and phase of $H_{eff}(j\omega)$.

Problem 6.3*

All parts of this problem are concerned with the following system.



$$x[n] = x_c(nT) \qquad\qquad y_c(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin \frac{\pi}{T}(t-nT)}{\frac{\pi}{T}(t-nT)}$$

(a) The first LTI system has frequency response such that $H_{aa}(j\omega) = 0$ for $|\omega| \geq 1000\pi$. This system is a lowpass filter, but not necessarily an *ideal* lowpass filter; i.e., its passband may not be perfectly flat. A filter used like this in a sampling system is often called an *anti-aliasing filter*. Why?

(b) How should T be chosen if we want $y_c(t) = x_c(t)$ when the discrete-time system is defined by the difference equation $y[n] = x[n]$?

(c) If the filter $H_{aa}(j\omega)$ satisfies the condition given in part (a) and the sampling rate is chosen to avoid aliasing as in part (b), and the discrete-time system is defined by $y[n] = x[n]$, give an expression for $y_c(t)$ in terms of $w_c(t)$ and the impulse response $h_{aa}(t)$ of the anti-aliasing filter.

(d) If the filter $H_{aa}(j\omega)$ satisfies the condition given in part (a) and the sampling rate is chosen to avoid aliasing as in part (b), and the discrete-time system now is defined by $y[n] = x[n] + x[n-1]$, what is the effective frequency response $H_{\text{eff}}(j\omega)$ of the overall system from $w_c(t)$ to $y_c(t)$; i.e., what is $H_{\text{eff}}(j\omega)$ in the equation

$$Y_c(j\omega) = H_{\text{eff}}(j\omega) W_c(j\omega)$$

Be as specific as you can.