GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2201A
Problem Set No. 4

Date Assigned: April 26, 1999
Date Due: April 30, 1999

Reading Assignment: In Oppenheim and Willsky, read pp. 583-610 and 514-534.

Homework Assignment: Hand in for grading only Problems 4.1*, 4.2* and 4.3*.

Practice Problems:
Look at Problem Set 4 of EE3230, Winter, Spring, Fall, 1998 and EE2823A of Winter 1999.

Problem 4.1*:
A signal $x(t) = 10 + 20\cos(200\pi t)$ is multiplied by a "window function" $w(t)$ to obtain:

$$v(t) = w(t)x(t) = w(t)[10 + 20\cos(200\pi t)]$$

(a) Give an equation for $V(j\omega)$ in terms of $W(j\omega)$.

(b) Now suppose that $w(t)$ is the "rectangular window" defined by

$$w(t) = \begin{cases} 1 & |t| \leq T_1 \\ 0 & |t| > T_1 \end{cases}$$

Determine the Fourier transform $W(j\omega)$ and make a careful sketch of it. The region $-\pi/T_1 \leq \omega \leq \pi/T_1$ is called the "main lobe" of $W(j\omega)$. Why?

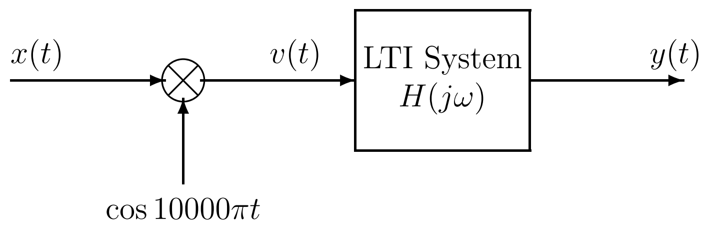
(c) Using the results of parts (a) and (b), plot $V(j\omega)$ for the case $T_1 = 0.1$ sec.

(d) What is the *minimum* value for T_1 such that the "main lobes" of the copies of $W(j\omega)$ in $V(j\omega)$ do not overlap?

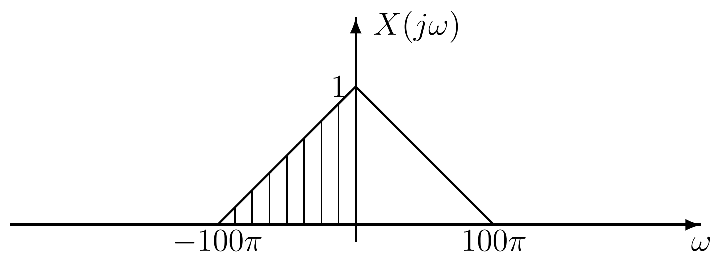
Problem 4.2*
Work Problem 8.23 in Oppenheim and Willsky.

Problem 4.3*

Consider the following modulation system:



Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below



and the linear system has frequency response

$$H(j\omega) = \begin{cases} 2 & 9900\pi < |\omega| < 10000\pi \\ 0 & \text{otherwise.} \end{cases}$$

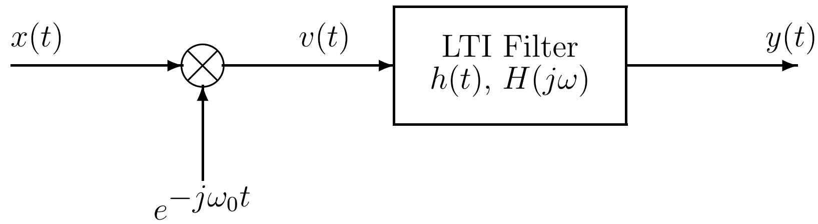
(a) Plot the Fourier transform $Y(j\omega)$ of the corresponding output signal $y(t)$. *Note that the negative frequency portion of the Fourier transform is shaded. Mark the corresponding region or regions in your plot of $Y(j\omega)$.*

(b) The resulting signal $y(t)$ is called an AM lower sideband single sideband (SSB) signal. Determine a system that would give an AM *upper sideband* single sideband signal as its output.

(c) Can $x(t)$ be recovered from the AM lower sideband SSB signal $y(t)$. If so, how?

Problem 4.4

Consider the following modulation/filtering system:



The impulse response of the LTI system is: $h(t) = \begin{cases} 1/T & |t| < T/2 \\ 0 & |t| > T/2 \end{cases}$

(a) Determine the frequency response of the LTI system and plot it.

(b) Suppose that $\omega_0 = 2\pi/T$ and the input signal is the periodic function

$$x(t) = A_0 + A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(2\omega_0 t + \phi_2)$$

Determine expressions for the Fourier transforms of $x(t)$ and $v(t)$. Plot the Fourier transform $V(j\omega)$ on the same axes as your plot of $H(j\omega)$.

(c) Determine the output $y(t)$ for the input $x(t)$ in part (b).

(d) Describe how you could use a system of this type to determine A_0, A_1, A_2, ϕ_1, and ϕ_2 for the given input signal.