GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

EE2823A
Quiz No. 1
February 3, 1999

Name: ______________________________

1. The exam is closed book. You may use one 8.5” by 11” sheet of notes and a calculator if you need it. A sheet of formulas is provide as the last page. Tear it off for ease of use.
2. Do all work in the space provided. If you need more room, use the *back* of the *previous* page. **Indicate your answer clearly by circling it or drawing a box around it.**
3. Most of the time the parts of the problems are independent, and individual parts generally count less than 10%. Therefore, your best strategy is not to spend too much time on any one part.
4. If you want to receive partial credit, you should clearly indicate your reasoning and method of attack on the problem.

Problem	**Points**	**Score**
1	10	
2	20	
3	10	
4	20	
5	15	
6	25	
TOTAL	100	

Problem Q1:W99-1: (10 %)

An LTI system has impulse response

$$h(t) = e^{a(t+t_0)}u(t+t_0)$$

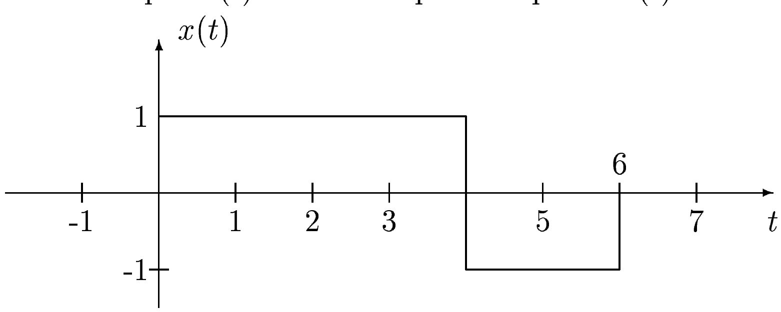
where a is a real number.

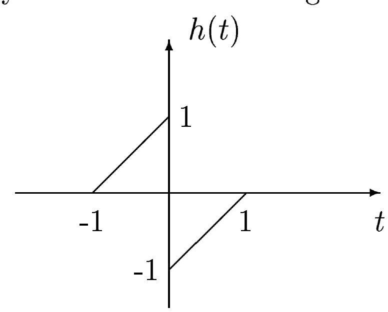
(a) Under what conditions on a and t_0 will the system be *causal*? Justify your answer to receive full credit.

(b) Under what conditions on a and t_0 will the system be *stable*? Justify your answer to receive full credit.

Problem Q1:W99-2: (15 %)

If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:





(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Determine the complete set of values of t such that the output $y(t) = 0$. *You do not need to find $y(t)$ at any other values of t.*

Problem Q1.W99-3 (10 %)

The frequency response of an LTI system is

$$H(j\omega) = \frac{1}{1+j\omega} + \frac{2}{2+j\omega} = \frac{4+3j\omega}{2+3j\omega+(j\omega)^2}$$

(a) What is the impulse response of the system?

(b) The input and output of this system satisfy a differential equation. What is that differential equation?

Problem Q1:W99-4: (20 %)

The input to an LTI system is $x(t) = 10 + 2\delta(t-2) + \dfrac{\sin(2000\pi t)}{\pi t}$

and the frequency response of the system is $H(j\omega) = \begin{cases} 10 & |\omega| < 1000\pi \\ 0 & |\omega| > 1000\pi. \end{cases}$

(a) Determine $X(j\omega)$, the Fourier transform of $x(t)$. (You may give part of your answer as a sketch if that is most convenient.)

(b) Using the specific $X(j\omega)$ determined in (a), obtain a simple expression for $Y(j\omega)$, the Fourier transform of the output of the LTI system, in terms of $H(j\omega)$.

(c) Determine $y(t)$, the output of the system for input $x(t)$. *Do not leave your answer in terms of* $h(t)$.

Problem Q1:W99-5: (15 %)

(a) If $x(t) = u(t)$ and $h(t) = \delta(t-2) + \delta^{(1)}(t)$, find $y(t) = x(t) * h(t)$.

(b) A linear time invariant system has frequency response

$$H(j\omega) = (j\omega)e^{-j\omega 3}$$

Describe in words what the system does to an input signal.

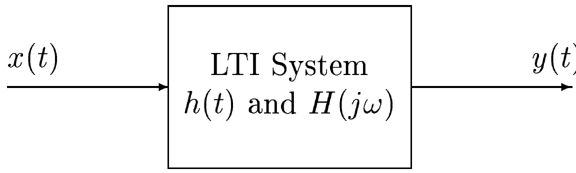
(c) A different system has frequency response

$$H(j\omega) = \frac{2\sin(10\omega)}{\omega(4 + j\omega)}$$

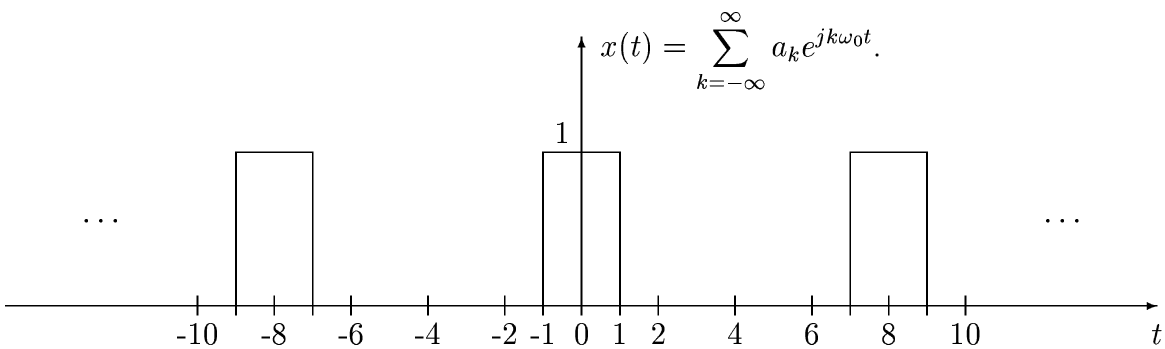
By separating $H(j\omega)$ into two recognizable parts, express the corresponding impulse response as the convolution of two impulse responses $h_1(t)$ and $h_2(t)$. Find $h_1(t)$ and $h_2(t)$, but do not evaluate their convolution.

Problem Q1.W99-6: (25 %)

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



(a) Determine ω_0 and the coefficients a_k in the Fourier series representation of $x(t)$.

(b) Determine an equation for the Fourier transform $X(j\omega)$ of the signal $x(t)$. Plot $X(j\omega)$ below for $-4\omega_0 \leq \omega \leq 4\omega_0$.

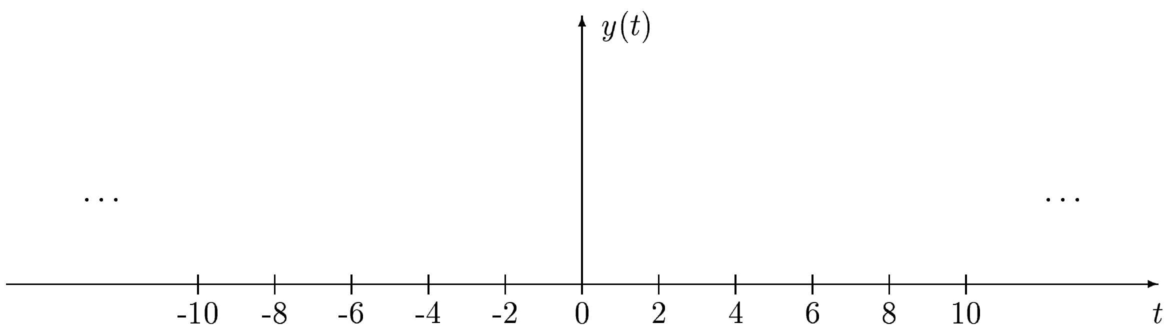
Problem Q1.W99-6 (continued): (25 %)

(c) If the frequency response of the system is the ideal lowpass filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < 0.5\pi \\ 0 & |\omega| > 0.5\pi \end{cases}$$

what is the output of the system when the input is $x(t)$ on the previous page? Give an equation for $y(t)$. *Use the plot of part (b) and a plot of the frequency response to help you solve this problem.*

(d) If the frequency response is $H(j\omega) = (j\omega)e^{-j\omega 3}$, plot the output of the system $y(t)$ when the input is $x(t)$ on the previous page.



FORMULAS FOR FIRST EXAM

Fourier Transform Pairs

$$x(t) = e^{-at}u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{1}{a + j\omega}$$

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

$$x(t) = \frac{\sin \omega_b t}{\pi t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \begin{cases} 1 & |\omega| < \omega_b \\ 0 & |\omega| > \omega_b \end{cases}$$

$$x(t) = \delta(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 1$$

$$x(t) = u(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$x(t) = \delta^{(1)}(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = j\omega$$

$$x(t) = 1 \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega)$$

$$x(t) = e^{j\omega_0 t} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = 2\pi\delta(\omega - \omega_0)$$

$$x(t) = \cos \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$

$$x(t) = \sin \omega_0 t \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = -j\pi\delta(\omega - \omega_0) + j\pi\delta(\omega + \omega_0)$$

$$\left.\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t} \\ a_k &= \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt \end{aligned}\right\} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Fourier Transform Theorems

$$a x_1(t) + b x_2(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad a X_1(j\omega) + b X_2(j\omega)$$

$$x(t - t_0) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad e^{-j\omega t_0} X(j\omega)$$

$$e^{j\omega_0 t} x(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad X(j(\omega - \omega_0))$$

$$\frac{dx(t)}{dt} \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad j\omega X(j\omega)$$

$$-jtx(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{dX(j\omega)}{d\omega}$$

$$y(t) = x(t) * h(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega) X(j\omega)$$

$$x(t)p(t) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(at) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$X(jt) \quad \stackrel{\mathcal{F}}{\longleftrightarrow} \quad 2\pi x(-\omega)$$