GEORGIA INSTITUTE OF TECHNOLOGY

School of Electrical Engineering

Quiz #1

Date: October 19, 1995

Course: EE 3230

Name: ______________________________
Last, First

- Closed book, closed notes, no calculators, one $8\frac{1}{2}'' \times 11''$ handwritten sheet is allowed. Eighty minute time limit.
- None of the problems require involved calculations. Reconsider your approach before doing something tedious.
- All work should be performed on the quiz itself. If more space is needed, use the backs of the pages.

Problem	*Score*
1	
2	
3	
4	
5	
Total	

Problem 1:

Let $h(t)$ denote the impulse response of a linear, time-invariant system. Suppose that the system is causal and $h_e(t)$, the even part of $h(t)$ for $t > 0$ is given by:

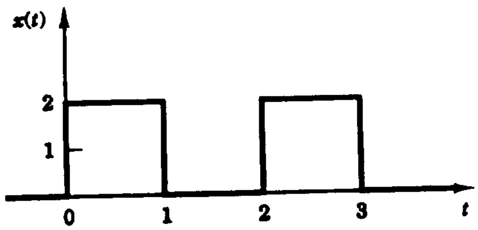
$$h_e(t) = t[u(t) - u(t-1)] + 2u(t-2), \quad t > 0$$

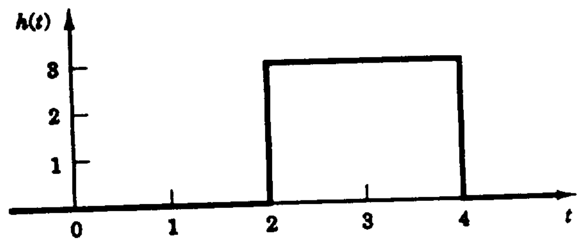
(a) Sketch $h_e(t)$ for all t.

(b) Find and sketch $h(t)$ for all t.

Problem 2:

A linear, time-invariant system has the input $x(t)$ and impulse response $h(t)$ shown below.



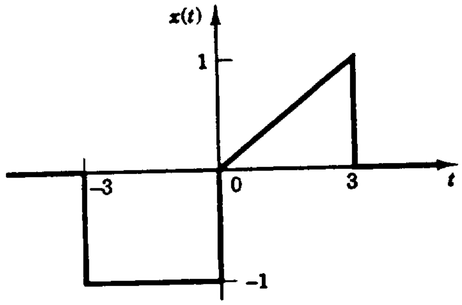


(a) Find the maximum value of the output.

(b) Find all the values of t for which the output is maximum.

Problem 3:

Given $x(t)$ as drawn below, sketch and label each of the following signals.



(a) $y(t) = x(3 - 3t)$

(b) $y(t) = 2 + 2x(t/3)$

Problem 4:

Classify the following two systems in terms of memory, linearity, time-invariance, causality, and stability.

(a) $y(t) = e^{tx(t)}$

(b) $y(t) = \int_{-\infty}^{t} x(2\tau)d\tau$

Problem 5:

Find the Fourier transforms of the following signals. Simplify your answers as much as possible.

(a) $x(t) = e^{-2t}[u(t) - u(t-5)]$

(b) $x(t) = t[u(t) - u(t-5)]$

TABLE 4.1 Fourier Transform Properties

Property	*Time domain*	*Frequency domain*
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Modulation	$e^{j\omega_0 t}x(t)$	$X[j(\omega - \omega_0)]$
Axis scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Axis reversal	$x(-t)$	$X(-j\omega)$
Duality	$X(jt)$	$2\pi x(-\omega)$
	$\frac{1}{2\pi}X(-jt)$	$x(\omega)$
Differentiation	$\frac{dx(t)}{dt}$	$j\omega X(j\omega)$
	$-jtx(t)$	$\frac{dX(j\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\,\delta(\omega)$
	$-\frac{1}{jt}x(t) + \pi x(0)\,\delta(t)$	$\int_{-\infty}^{\infty} X(j\lambda)\,d\lambda$
Convolution	$h(t) * x(t)$	$H(j\omega)X(j\omega)$
Multiplication	$x(t)p(t)$	$\frac{1}{2\pi}X(j\omega) * P(j\omega)$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Symmetries	$x(t)$ real	$\mathrm{Re}\{X(j\omega)\}$ even $\mathrm{Im}\{X(j\omega)\}$ odd $\lvert X(j\omega)\rvert$ even $\angle X(j\omega)$ odd
	$\mathrm{Ev}\{x(t)\}$ $\mathrm{Od}\{x(t)\}$	$\mathrm{Re}\{X(j\omega)\}$ $j\,\mathrm{Im}\{X(j\omega)\}$
Parseval's relation finite energy	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2\,d\omega$
periodic signal	$\frac{1}{T}\int_{T} \lvert x(t)\rvert^2\,dt$	$\sum_{k=-\infty}^{\infty} \lvert a_k\rvert^2$

TABLE 4.2 Common Fourier Transforms

Signal	*Time domain*	*Frequency domain*
Rectangular pulse	$x(t) = \begin{cases} 1, & \lvert t\rvert < T_p/2 \\ 0, & \lvert t\rvert > T_p/2 \end{cases}$	$T_p \operatorname{sinc} \frac{\omega T_p}{2}$
Sinc pulse	$\frac{\omega_b}{\pi}\operatorname{sinc}\omega_b t$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < \omega_b \\ 0, & \lvert\omega\rvert > \omega_b \end{cases}$
Impulse	$\delta(t)$	1, for all ω
Unit step	$u(t)$	$\frac{1}{j\omega} + \pi\,\delta(\omega)$
Delayed impulse	$\delta(t - t_0)$	$e^{-j\omega t_0}$
Causal exponential	$e^{-at}u(t),\ a > 0$	$\frac{1}{a + j\omega}$
Weighted exponential	$te^{-at}u(t),\ a > 0$	$\frac{1}{(a + j\omega)^2}$
dc signal	a_0, for all t	$2\pi a_0\,\delta(\omega)$
Complex sinusoid	$e^{j\omega_0 t}$	$2\pi\,\delta(\omega - \omega_0)$
Sine wave	$\sin\omega_0 t$	$(\pi/j)[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
Cosine wave	$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
Square wave	Figure 4.1 (odd symmetry)	$\sum_{k=-\infty,\ k \text{ odd}}^{\infty} \frac{4}{jk}\,\delta(\omega - k\omega_0)$
Impulse train	$\sum_{k=-\infty}^{\infty} \delta(t - kT)$	$\omega_0 \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$
Rectangular pulse train	Figure 4.2 (even symmetry)	$\sum_{k=-\infty}^{\infty} \frac{2}{k}\sin(k\omega_0 T_p/2)\,\delta(\omega - k\omega_0)$