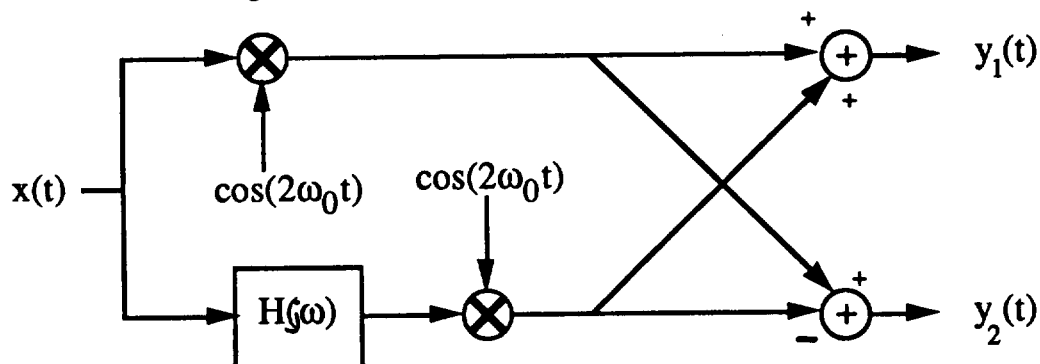
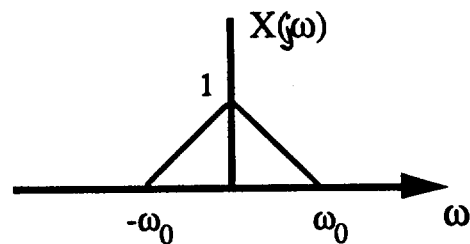
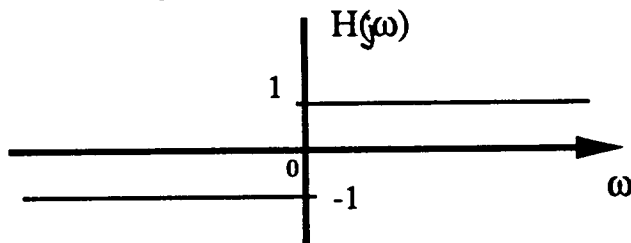


Problem 1: (20 points)

Consider the following modulation scheme:



where the frequency response of the system $H(j\omega)$ and the Fourier transform of $x(t)$ are:

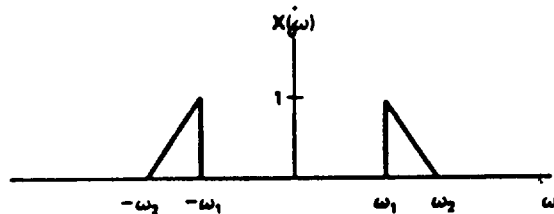


(a) Sketch $Y_1(j\omega)$, the Fourier transform of $y_1(t)$.

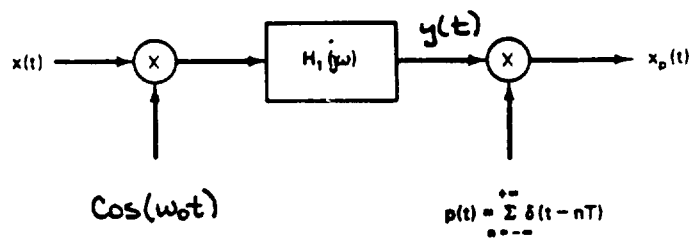
(b) Sketch $Y_2(j\omega)$, the Fourier transform of $y_2(t)$.

Problem 2: (20 points)

Consider a signal $x(t)$ that has the Fourier transform:



The Nyquist sampling theorem tells us that we have to sample $x(t)$ with a sampling frequency of more than $2\omega_2$ in order to be able to recover $x(t)$ from its samples. However, because $x(t)$ is a *bandpass* signal (i.e., $X(j\omega)$ is only non-zero for $\omega_1 \leq |\omega| \leq \omega_2$), with some additional work we can sample less often. Consider the following sampling system:



where $\omega_0 = \omega_1$ and the lowpass filter $H_1(j\omega)$ has cutoff frequency $\omega_2 - \omega_1$.

(a) Sketch $Y(j\omega)$, the Fourier transform of $y(t)$.

(b) Find the maximum sampling period T such that $y(t)$ (and consequently $x(t)$) is recoverable from $x_p(t)$.

Problem 3: (20 points)

The signal

$$y(t) = e^{-2t}u(t)$$

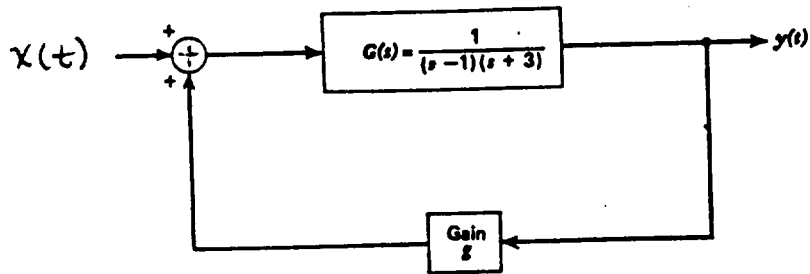
is the output of a causal linear, time-invariant system whose impulse response has the Laplace transform

$$H(s) = \frac{s-1}{s+1}$$

Find two possible inputs $x(t)$ that could produce this output.

Problem 4: (20 points)

Consider the feedback system shown below:



(a) Is the overall system stable if the gain g is zero?

(b) For what range of g is this overall system stable?