GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #1

Assigned: 27 August 1999
Due Date: 3 September 1999 (FRIDAY)

Reading: In *DSP First*, Appendix A on *Complex Numbers*, pp. 378–398; and Ch. 2 on *Sinusoids*, pp. 9–43.

The web site for the course uses Web-CT:

`http://classweb.gatech.edu:8080/SCRIPT/ECE2025/scripts/serve_home`

Your initial password = SSN(4:8), the 4th through 8th digits of your SSN, but you should change it.

$\Longrightarrow$ Please check the "Bulletin Board" daily, or more often. All official course announcements will be posted there.

ALL of the **STARRED** problems will have to be turned in for grading.

Some of the problems have solutions that can be found on the CD-ROM. Next week a solution to all problems in this assignment will be posted to the web. After the beginning of your assigned lecture on Friday (either 11am or 12pm), the homework is considered late and will be given a zero.

Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$\begin{aligned} z &= (x, y) \\ &= x + jy \qquad\qquad \text{where } j = \sqrt{-1} \\ &= \Re e\{z\} + j\Im m\{z\} \end{aligned}$$

The pair (x, y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate.

In *polar form* we will use these notations:

$$\begin{aligned} z &= |z|e^{j \arg z} \\ &= re^{j\theta} \\ &= r\angle\theta \end{aligned}$$

where $|z| = r = \sqrt{x^2 + y^2}$ and $\arg z = \theta = \arctan(y/x)$. Again, in a vector drawing, r is the length and θ the direction of the vector.

Euler's Formula:

$$re^{j\theta} = r\cos\theta + j\,r\sin\theta$$

can be used to convert between Cartesian and polar forms.

Problems 1.1–1.4 are rwview problems. In these problems you will manipulate some complex numbers. A calculator will be useful for this purpose, especially if it is one with complex arithmetic capability. It is convenient to learn how to use this feature. However, it is also worthwhile to be able to do the calculations by hand; i.e., it is important to *understand* what your calculator is doing!

PROBLEM 1.1:

Convert the following to polar form:

(a) $z = j10$	(b) $z = -10$	(c) $z = (-10, -10)$
(d) $z = -2 + j2$	(e) $z = -3 + j\sqrt{3}$	(f) $z = 20$

Give numerical values for the magnitude and angle (phase).

PROBLEM 1.2:

Convert the following to rectangular form:

(a) $z = 3\sqrt{2}\, e^{-j(3\pi/4)}$	(c) $z = 4\,\angle\,(\pi/3)$
(b) $z = 5e^{j(\pi/2)}$	(d) $z = 5\,\angle\,(-61\pi)$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3:

Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -2 - j2$ and $z_2 = 3e^{-j3\pi/4}$.

(a) Conjugate: z_1^*	(d) z_2^2	(g) $z_1 + z_2^*$
(b) jz_2	(e) $z_1^{-1} = 1/z_1$	(h) $\lvert z_2\rvert^2 = z_2 z_2^*$
(c) z_2/z_1	(f) $z_1 z_2$	(i) $z_2 + z_2^*$

Note: z^* means the "conjugate" of z. Part (h) is the *magnitude-squared.*

PROBLEM 1.4:

Simplify the following complex-valued expressions. Give your answer in either rectangular or polar form, whichever is most convenient. In parts (a)-(d) assume that A, a, and ϕ are positive real numbers. Your answers to parts (a)-(d) will be in terms of these quantities.

(a) For $z = Ae^{j\pi/3}$, determine an expression for $\Im m\{z^*\}$.

(b) For $z = Ae^{-j\pi/3}$, determine an expression for $|z|/z$.

(c) For $z = 10e^{j\phi}$, determine an expression for $\Re e\{jz\}$.

(d) For $z = -a - ja$, determine an expression for z in polar form.

(e) For $z_1 = -2 + j2$ and $z_2 = 4e^{-j\pi/3}$, evaluate $z_3 = z_1 + z_2$ and **plot all three complex numbers in the complex plane**. In this case a numerical answer is required.

PROBLEM 1.5*:

Simplify the following and give the answer in polar form. Make a plot of the vectors involved in the complex addition.

(a) $z_a = 2e^{j(2\pi/3)} + e^{j(5\pi/4)}$

(b) $z_b = \sqrt{2}e^{j(\pi/4)} + \sqrt{2}e^{-j(\pi/4)} - 1$

(c) In addition, write the MATLAB statements that will perform the addition and also display the magnitude and phase of the result. Consult `help` on the DSP-First functions: `zprint`, `zvect`, etc. Use these to check your hand calculations in parts (a) and (b).

PROBLEM 1.6*:

Suppose that MATLAB is used to plot a sinusoidal signal. The following MATLAB code generates the signal and makes the plot. Draw a sketch of the plot that will be done by MATLAB. Determine the period of the sinusoid and label the period on your plot.

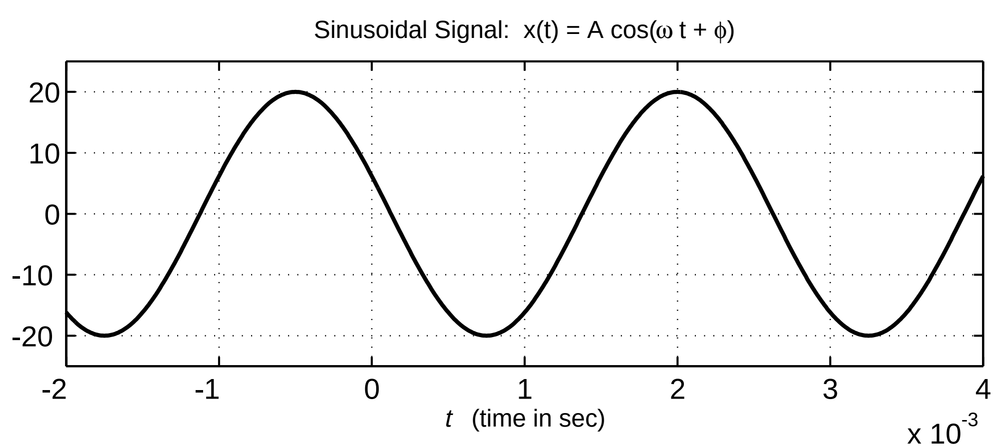
```
dt = 0.001;
tt = -.05 : dt : .15;
Fo = 10;
Z =sqrt(2)*(1+j);
xx = real( Z*exp( 2j*pi*Fo*tt ) );
%
plot( tt, xx ),   grid
title( 'SECTION of a SINUSOID' ),    xlabel('TIME   (sec)')
```

PROBLEM 1.7*:

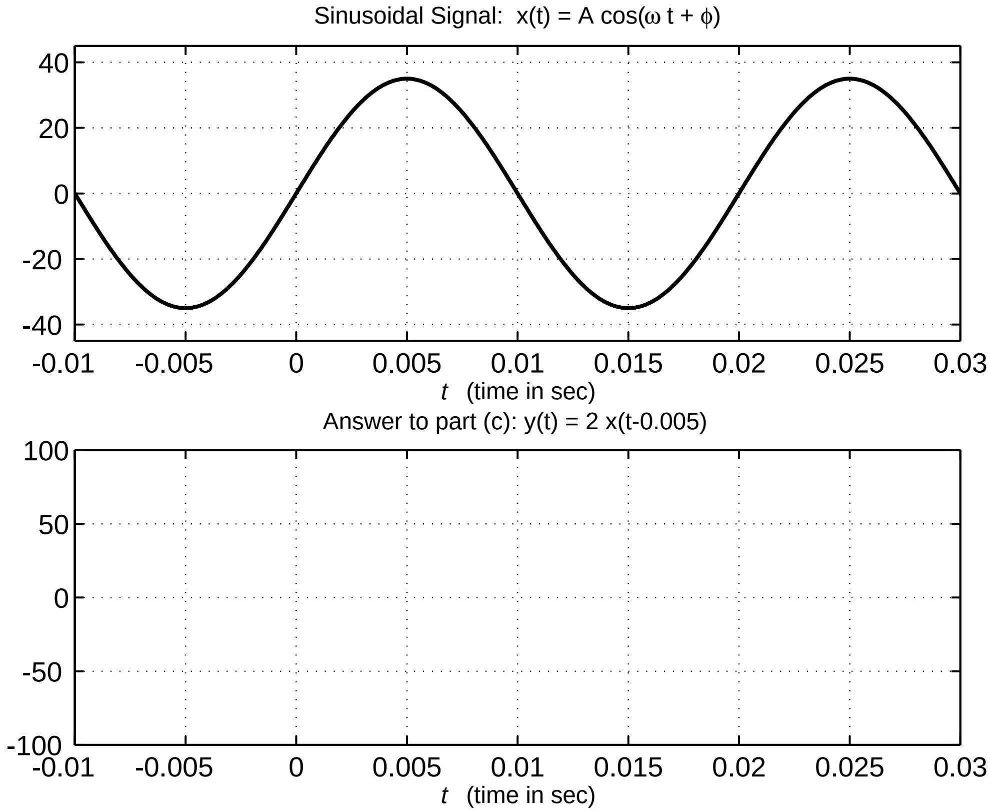
The waveform in the following figure can be expressed as

$$x(t) = A\cos[\omega_0(t - t_1)] = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, t_1, and ϕ such that $-\pi < \phi \leq \pi$.



PROBLEM 1.8*:



(a) The above figure shows a plot of a sinusoidal wave $x(t)$. From the plot, determine the values of A, ω_0, and $-\pi < \phi \le \pi$ in the representation

$$x(t) = A\cos(\omega_0 t + \phi)$$

Where appropriate, be sure to indicate the units of the sinusoidal signal parameters.

(b) Determine Z for the complex signal $z(t) = Ze^{j\omega_0 t}$ such that $x(t) = \Re e\{z(t)\}$.

(c) On the axes provided above, sketch the signal $y(t) = 2x(t - .005)$.

PROBLEM 1.9*:

Simplify the following and give the answer as a single sinusoid. Draw the vector diagram of the complex amplitudes (phasors) to show how you obtained the answer.

(a) $x_a(t) = 2\cos(2\pi t + 2\pi/3) - \cos(2\pi t + \pi/4)$

(b) $x_b(t) = \cos(41t + 17\pi) + \sqrt{2}\cos(41t + \pi/4) + \sqrt{2}\cos(41t - \pi/4)$

(c) $x_c(t) = \cos(200\pi t + 3\pi/4) + \cos(200\pi t + 5\pi/4) + 2\cos(200\pi t - \pi/4) + 2\cos(200\pi t + \pi/4)$