GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 1999
Problem Set #13

Assigned: 23 November 99
Due Date: 3 December 99 (FRIDAY)

The Final Exam Time depends on the time of your scheduled lecture: Period 9, Dec. 15 (Wed.) for the 11am section, and Period 13, Dec. 17 (Fri.) for the 12pm section. ***You must take the exam for your scheduled lecture section.

Reading: Finish reading Chapter 12 and begin reading Chapter 13.

$\Longrightarrow$ The five (5) **STARRED** problems will have to be turned in for grading.

Next week a solution will be posted.

PROBLEM 13.1*:

In each of the following cases, use the table of Fourier transform pairs together with the table of Fourier transform properties to complete the following Fourier transform pair relationships:

(a) $x(t) = 2\delta(t-2) \quad \Longleftrightarrow \quad X(j\omega) =$

(b) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = 20\pi\delta(\omega)$

(c) $x(t) = \dfrac{\sin 20\pi(t-1)}{\pi(t-1)} \quad \Longleftrightarrow \quad X(j\omega) =$

(d) $x(t) = \qquad\qquad \Longleftrightarrow \quad X(j\omega) = 2\delta(\omega - 0.1\pi) + 2\delta(\omega + 0.1\pi)$

(e) $x(t) = \dfrac{d}{dt}\left[\dfrac{\sin 20\pi t}{\pi t}\right] \quad \Longleftrightarrow \quad X(j\omega) =$

(f) $x(t) = \dfrac{\sin 400\pi t}{\pi t}\cos(20000\pi t) \quad \Longleftrightarrow \quad X(j\omega) =$

PROBLEM 13.2*:

The input to an LTI system is $\quad x(t) = 10 + 2\delta(t-2) + \dfrac{\sin(2000\pi t)}{\pi t}$

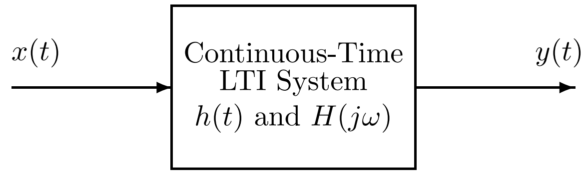
and the frequency response of the system is $\quad H(j\omega) = \begin{cases} 10 & |\omega| < 1000\pi \\ 0 & |\omega| > 1000\pi. \end{cases}$

(a) Determine $X(j\omega)$, the Fourier transform of $x(t)$. (You may give part of your answer as a sketch if that is most convenient.)

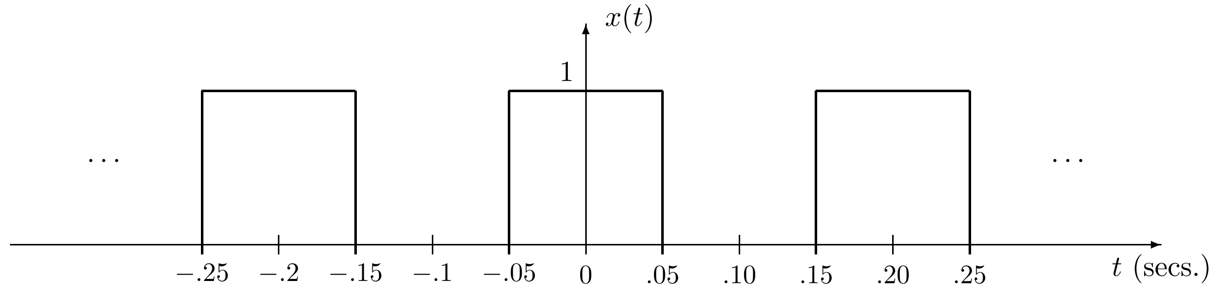
(b) Using the specific $X(j\omega)$ determined in (a), obtain a simple expression for $Y(j\omega)$, the Fourier transform of the output of the LTI system, in terms of $H(j\omega)$.

(c) Use the inverse Fourier transform to determine $y(t)$, the output of the system for input $x(t)$. *Do not leave your answer in terms of* $h(t)$.

PROBLEM 13.3*:



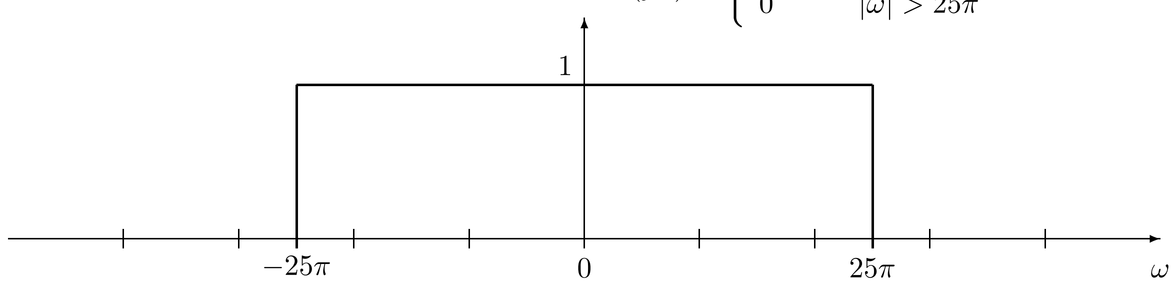
The input to the above LTI system is the periodic square wave $x(t)$ depicted below: **(Assume this same input for all parts below.)**



(a) Determine the Fourier transform of the input signal $x(t)$.

(b) Assume that the frequency response of the system is the ideal lowpass filter plotted below. Make a new plot of the Fourier transform of the input signal on the same graph for $-5\omega_0 \leq \omega \leq 5\omega_0$, and also plot the frequency response of the ideal lowpass filter on the same graph.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 25\pi \\ 0 & |\omega| > 25\pi \end{cases}$$



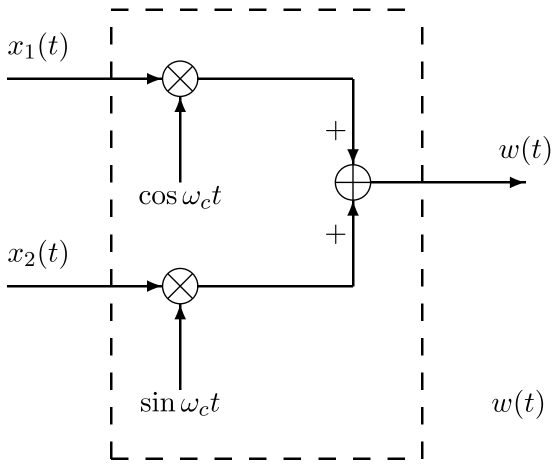
(c) Determine the Fourier transform of the output signal for the filter of part (b). Also, give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. **Your answer should be expressed in terms of only real quantities.**

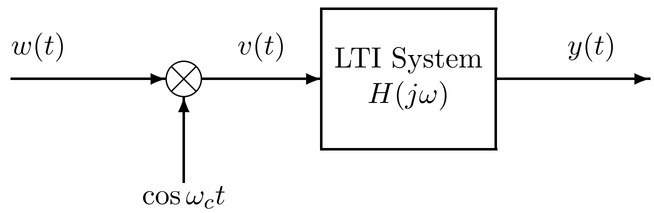
(d) Now assume that the frequency response of the filter is changed so that the "DC component" of the input signal is *removed* while keeping all other frequencies unchanged. Plot a new frequency response $H(j\omega)$ that will remove only the DC component without affecting the other frequencies of $x(t)$. Is your answer unique? If not, why not?

(e) Make a plot of the output signal $y(t)$ that would result from part (d); i.e., a plot of the input signal with the DC component removed.

PROBLEM 13.4*:

The system in the dashed box below is called a *quadrature modulation system*. It is a method of sending two bandlimited signals over the same channel.





$$w(t) = x_1(t)\cos\omega_c t + x_2(t)\sin\omega_c t$$

Quadrature Modulator

Demodulator = Mixer, then Filter

Assume that both input signals are bandlimited with highest frequency ω_m; i.e., $X_1(j\omega) = 0$ for $|\omega| \geq \omega_m$ and $X_2(j\omega) = 0$ for $|\omega| \geq \omega_m$, where $\omega_c \gg \omega_m$.

(a) Determine an expression for the Fourier transform $W(j\omega)$ in terms of $X_1(j\omega)$ and $X_2(j\omega)$. Make a sketch of $W(j\omega)$. Assume simple shapes (but different) for the bandlimited Fourier transforms $X_1(j\omega)$ and $X_2(j\omega)$, and use them in making your sketch of $W(j\omega)$.

(b) From the expression found in part (a) and the sketch that you drew, you should see that $W(j\omega) = 0$ for $|\omega| \leq \omega_a$ and for $|\omega| \geq \omega_b$. Determine ω_a and ω_b.

(c) Given the trigonometric identities $2\sin\theta\cos\theta = \sin 2\theta$ and $2\cos^2\theta = (1 + \cos 2\theta)$, show that in the "demodulator" figure on the right above, the output of the mixer is:

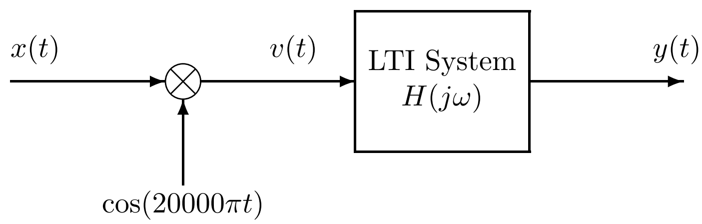
$$v(t) = \tfrac{1}{2}x_1(t)(1 + \cos 2\omega_c t) + \tfrac{1}{2}x_2(t)\sin 2\omega_c t$$

(d) The signal $v(t)$ as determined in part (c) is the input to an LTI system. Determine the frequency response of that system so that its output is $y(t) = x_1(t)$. Give your answer as a carefully labeled plot of $H(j\omega)$.
You may use the result of part (c) even if you were unable to derive it.

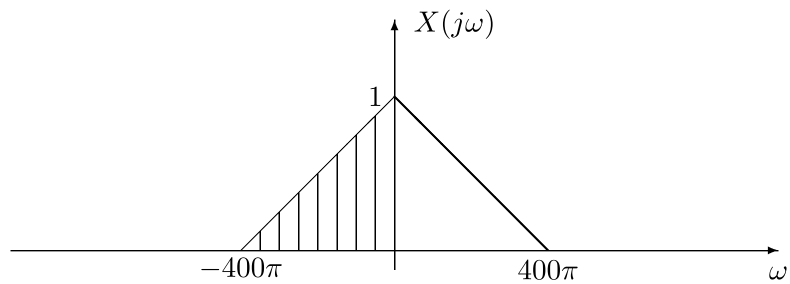
(e) (e) Draw a block diagram of a demodulator system whose output will be $x_2(t)$ when its input is $w(t)$. This requires that you change the mixer.

PROBLEM 13.5*:

Consider the following modulation system:



Assume that the input signal $x(t)$ has a bandlimited Fourier transform as depicted below



and the linear system has frequency response

$$H(j\omega) = \begin{cases} 2 & 19600\pi < |\omega| < 20000\pi \\ 0 & \text{otherwise.} \end{cases}$$

(a) Plot the Fourier transform $Y(j\omega)$ of the corresponding output signal $y(t)$. *Note that the negative frequency portion of the Fourier transform $X(j\omega)$ is shaded. Mark the corresponding region or regions in your plot of $Y(j\omega)$.*

(b) The resulting signal $y(t)$ is called an AM lower sideband single sideband (SSB) signal. Determine a system that would give an AM *upper sideband* single sideband signal as its output.

(c) It may not be obvious, but $x(t)$ can be recovered from the AM-SSB signal $y(t)$ that contains only the lower sideband. Draw a block diagram of the appropriate demodulator to recover $x(t)$ from $y(t)$. Specify all modulator frequencies and the frequency responses of any filters that are needed. It is sufficient to specify ideal filters when giving the frequency response of the filters, but any filter used must be the frequency response of a "real" system, i.e., a system whose impulse response is real-valued.

(d) Determine the signal $g(t)$ whose Fourier transform is "one-sided" $G(j\omega) = \begin{cases} 7 & 0 < \omega < 20\pi \\ 0 & \text{otherwise.} \end{cases}$

PROBLEM 13.6:

See problems under EE2201 for examples of filtering and modulation:

1. Spring & Fall-98, Winter & Spring 99, Problem Sets #3, #4 and #5.
2. Winter 98, Problem Sets #3, #4, #5 and #6.